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# A State Space Modeling Approach to Mediation Analysis

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*Mediation is a causal process that evolves over time. Thus, a study of mediation requires data collected throughout the process. However, most applications of mediation analysis use cross-sectional rather than longitudinal data. Another implicit assumption commonly made in longitudinal designs for mediation analysis is that the same mediation process universally applies to all members of the population under investigation. This assumption ignores the important issue of ergodicity before aggregating the data across subjects. We first argue that there exists a discrepancy between the concept of mediation and the research designs that are typically used to investigate it. Second, based on the concept of ergodicity, we argue that a given mediation process probably is not equally valid for all individuals in a population. Therefore, the purpose of this article is to propose a two-faceted solution. The first facet of the solution is that we advocate a single-subject time-series design that aligns data collection with researchers' conceptual understanding of mediation. The second facet is to introduce a flexible statistical method—the state space model—as an ideal technique to analyze single-subject time series data in mediation studies. We provide an overview of the state space method and illustrative applications using both simulated and real time series data. Finally, we discuss additional issues related to research design and modeling.*

Keywords: *mediation, state space model*

## 1. Introduction

Mediation is a causal process that evolves over time. In the simplest case, the causal variable ($X$) exerts an effect on the outcome variable ($Y$) partially or completely through a mediator variable ($M$) over time. Clearly, time plays an explicit role in the mediation process. In one of the earliest articles devoted specifically to mediation, Judd and Kenny (1981) emphasized the role of time, even placing the words "process analysis" in the title of their article. More recently, Schmitz (2006) provided an overview of the necessity of process analyses in the context of learning and instruction. Ideally, the empirical study of mediation requires (1) data collected throughout the process and (2) pertinent statistical methods that can capture the dynamic mechanism underlying the causal process. However, process-oriented methods that explicitly consider the role of time are not common in educational and psychological research (Schmitz, 2006). In the context of mediation, Cole and Maxwell (2003) discussed that most applications of mediation analysis use cross-sectional rather than longitudinal data, and not all methodological treatments acknowledge the necessity of considering time (see Maxwell & Cole, 2007; Maxwell, Cole, & Mitchell, 2011).

Another implicit assumption commonly made in mediation analysis is that the same mediation process universally applies to all members of the population under investigation. This assumption is made in a handful of longitudinal models recently developed within the structural equation modeling (SEM) framework proposed to study mediation processes. Cheong, MacKinnon, and Khoo (2003) used a parallel process latent growth curve model to investigate the effect of a causal variable on the change in an outcome variable through change in a mediator variable. Gollob and Reichardt (1991) and Cole and Maxwell (2003) implemented a cross-lagged panel model emphasizing longitudinal relations between the absolute level of the causal variable and the outcome variable through the mediator variable. Another variant of a longitudinal SEM approach is the latent difference score model, in which differences between adjacent observations are treated as latent variables (e.g., Hamagami & McArdle, 2007; MacKinnon, 2008; McArdle, 2001; McArdle & Nesselroade, 1994; Selig & Preacher, 2009). In the traditional regression framework, Judd, Kenny, and McClelland (2001) advocated the use of within-subject designs (in contrast to between-subjects designs) to assess mediation and moderation, where individuals are put into both treatments. In all models mentioned previously, parameters are estimated by pooling information across subjects. Although the longitudinal designs in which these models are applied take the role of time into consideration, they are still limited in that none of them considers the important issue of ergodicity before aggregating the data across subjects (ergodicity is described later). As we will show shortly, the conclusions drawn from pooling across subjects may not be informative about how single subjects behave (e.g., Ferrer & Widaman, 2008; Molenaar, 2004).

In this article, we contend that (a) there is a discrepancy between the concept of mediation and the research designs that are typically undertaken to investigate it in practice and (b) based on the concept of ergodicity, there is not necessarily a universal mediation process that is equally valid for everyone in the population.

Therefore, the purpose of this article is to propose a two-faceted solution to the persistent problem of using cross-sectional designs in mediation analysis. The first facet of the solution is to encourage researchers to rethink how they approach the research design for mediation studies; that is, we advocate moving from the traditional cross-sectional design to a single-subject time-series design that aligns data collection with researchers' conceptual understanding of mediation. The second facet is to introduce a flexible statistical method—the state space model (SSM)—as an ideal technique to analyze single-subject[1] time-series data in mediation studies. In Section 2, we elaborate on the theoretical foundation of the single-subject time-series design for mediation analysis. In Section 3, we provide an overview of state space methods and two illustrative applications using simulated and empirical data sets. In Section 4, we discuss additional issues related to research design and modeling.

## 2. Foundation of the Single-Subject Time-Series Design

In a cross-sectional design, the focus is on the analysis of *interindividual* variation—differences among different units at a single point in time. According to a survey in 2005 of the five American Psychological Association journals publishing the most articles studying mediation, Maxwell and Cole (2007) reported that more than half of the mediation studies were based on cross-sectional data. However, according to the concept of mediation as well as the basic requirements for causal inference, mediation must involve at least two relations that unfold over time. Specifically, the effect of a causal variable ($X$) is first exerted on the mediator variable ($M$, i.e., $X \rightarrow M$), and then, this effect is carried over to the outcome variable ($Y$, i.e., $M \rightarrow Y$). Thus, it is immediately clear that a certain amount of time must elapse for the effect of $X$ to reach $Y$. Therefore, a requirement for any mediation analysis is the consideration of the role of time, that is, the necessity of the analysis of *intraindividual* variation—changes in the same unit over time. By comparing the concept of mediation and how mediation analysis was conducted in the literature, it is evident that there exists a large discrepancy between how mediation is theoretically conceptualized and how it has *actually* been modeled in the past. This discrepancy raises an important validity issue concerning the equivalence between the analysis of interindividual variation and the analogous analysis of intraindividual variation.

Cole and Maxwell (2003) argued and demonstrated that very restrictive conditions are required to ensure accurate results from mediation analysis based on cross-sectional data. In reality, such restrictive conditions almost never occur, and bias in cross-sectional analyses of longitudinal mediation has been amply demonstrated (Maxwell & Cole, 2007; Maxwell et al., 2011). The divide between interindividual variation and intraindividual variation not only exists in mediation analysis but also appears in areas such as test theory, factor analysis, and developmental psychology (Molenaar, 2004, 2008a, 2008b).

Given the existence of this divide in various areas, an important question becomes whether this division between orientations is justified. Unfortunately, as a direct consequence of the classical ergodic theorems, the answer to the question is "no." In fact, equivalence between the analysis of inter- and intraindividual variation is established only for ergodic processes (Molenaar, 2004). It is worth noting that, outside the context of mediation and in the broader sense of studying behavior, the inter-/intraindividual debate can be traced back to an older distinction between *nomothetic* lawfulness, emphasizing generality in the population, and *idiographic* characterization, emphasizing the uniqueness of the individual (e.g., Allport, 1937; Lamiell, 1981, 1988; Molenaar, 2004; Rosenzweig, 1958; van Kampen, 2000; Zevon & Tellegen, 1982). When individuals differ qualitatively rather than quantitatively, "Qualitative differences mistaken for quantitative differences can seriously distort relationships and are a prescription for diluted nomothetic relationships" (Nesselroade, Gerstorf, Hardy, & Ram, 2007, p. 219). The following subsection gives a brief, heuristic description of the concept of ergodicity as the foundation of the single-subject time-series design.

## *2.1. Ergodicity*

From the perspective of dynamical systems, a process is said to be ergodic if the average of a single trajectory over time (structure of intraindividual variation) is equal to the average of the ensemble of trajectories at a single point in time (structure of interindividual variation by pooling across subjects). In order to understand the consequences of ergodicity in psychology, the development of human behavior over time is conceived of as a unique high-dimensional space that contains dynamic processes and all the relevant information about the subject (cf. Molenaar, 1994, 2004, 2008a, 2008b; Molenaar & Ram, 2009, 2010; Sinclair & Molenaar, 2008). For a particular individual, a finite sample of the dynamic process over consecutive time points (usually evenly spaced) constitutes a trajectory in his or her behavior space. This trajectory carries information about intraindividual variation. Correspondingly, a finite sample of the same behavior space from a group of individuals at a single point in time represents an ensemble of trajectories, carrying information about interindividual variation from this group of individuals. Hence, the question of whether the divide between orientations is justified becomes a question of whether the developmental trajectory of human behavior is ergodic. This, in turn, reduces to the question of whether stationarity and homogeneity hold for a Gaussian process. Therefore, examining the stationarity and homogeneity criteria for a given process are essential empirical steps.

For a Gaussian process, two criteria are required to be met simultaneously for a process to be considered ergodic: stationarity and homogeneity. In terms of the first criterion, a stationary process refers to a stochastic process whose joint probability distribution is time-invariant. For a Gaussian process, stationarity[2] requires that the first two moments of the process are time-invariant. That is, the mean function of the time series is a constant, and the covariance function of the time series depends only on relative time differences (i.e., "lag"). Nonstationary processes, however, are the norm in psychology, for example, learning and developmental trajectories.

Regarding the second criterion, homogeneity refers to the situation in which each member of a population obeys the same dynamic law and follows the same statistical model, constituting exchangeable replications of each other, much as molecules of a homogeneous gas. The reality, however, is quite the opposite. In fact, heterogeneity is a general characteristic of human populations. Furthermore, besides the widely recognized genetic and environmental effects that cause heterogeneity, Molenaar, Boomsma, and Dolan (1993) argued that there exists a third source of developmental differences: self-organization of nonlinear epigenetic processes.

Since nonstationarity, heterogeneity, or both are thought to be the rule rather than the exception in most psychological processes, such processes are then nonergodic, which means that the structure of interindividual variation is not equivalent to the structure of intraindividual variation. Therefore, we conclude that there is no equivalence between measurement orientations in the majority of cases. This implies that there are not necessary lawful relationships between the analysis of inter- and intraindividual variation. Thus, in cases concerning processes that unfold over time, statistical analyses should focus on intraindividual variation, with greater emphasis on single-subject time-series designs. This conclusion, therefore, also applies to mediation analysis.

Discussions of the implications and consequences of ergodicity started to appear in many areas of psychological research about two decades ago (e.g., Molenaar, 1994; Molenaar, 2004, 2008a, 2008b; Molenaar & Campbell, 2009; Molenaar & Ram, 2009, 2010; Molenaar, Sinclair, Rovine, Ram, & Corneal, 2009; Nesselroade & Molenaar, 1999; Sinclair & Molenaar, 2008, and most recently Hamaker, 2012). However, there is virtually no mention of ergodicity in the mediation literature (but see Roe, 2012).

## 3. Overview of State Space Methods

After establishing the theoretical foundation of the time-series design, process-oriented methods are required to analyze the time series data. In this section, we discuss the second facet of our proposed solution, that is, an overview of SSM. This is, we believe, the first effort to utilize SSM to investigate mediation. As the first application of SSM in the context of mediation, we provide some essential basics of the most straightforward and frequently discussed variant of SSM in the time-series and econometrics literature, that is, the linear Gaussian SSM. Due to space limitations, our introduction is brief. For more comprehensive treatments of the state space methodology in general, we refer readers to Commandeur and Koopman (2007), Harvey (1989), and Durbin and Koopman (2001).

### 3.1 History of State Space Modeling in Psychology and Its Application to Mediation Analysis

State space methods have their origin in control theory, beginning with the groundbreaking article by Kalman (1960). Applications in astronautics were initially developed (and are still used) for accurately tracking the position and velocity of moving objects such as aircraft, missiles, and rockets. Shortly after its application in engineering, SSM also found application in time series analysis and econometrics (e.g., Aoki, 1987; Harvey, 1989). More recently, quantitative social and behavioral scientists have begun applying SSM because of its statistical flexibility for evaluating both the measurement properties and the lead–lag relationships among latent variables in psychological processes. Analytic similarities and differences between the currently dominant SEM and the relatively newly emerging SSM in the psychology literature are discussed by several authors. Specifically, MacCallum and Ashby (1986) noted that SSM is a special case of SEM, while Otter (1986) showed the reverse. Chow, Ho, Hamaker, and Dolan (2010) reconciled the two approaches and provided a more detailed discussion of the relative strengths and weaknesses of both approaches vis-à-vis their use in representing intraindividual dynamics and interindividual differences.

Another line of research involving SSM in psychology is built upon the state space representation of the dynamic factor model (DFM; Molenaar, 1985). Dynamic factor analysis was proposed to combine P-technique factor analysis (Cattell, 1963; Cattell, Cattell, & Rhymer, 1947) and time series analysis (Browne & Nesselroade, 2005; Molenaar, 1985; Molenaar, de Gooijer, & Schmitz, 1992; Nesselroade, McArdle, Aggen, & Meyers, 2002). Some recent work devoted to methodological discussions and substantive applications of DFM can be found in the psychology literature (e.g., Chow, Nesselroade, Shifren, & McArdle, 2004; Ferrer & Nesselroade, 2003; Hershberger, Corneal, & Molenaar, 1994; Nesselroade & Molenaar, 1999; Sbarra & Ferrer, 2006; Shifren, Hooker, Wood, & Nesselroade, 1997; Wood & Brown, 1994; Zhang & Browne, 2006). Given the strong similarity between SSM and DFM, the two terms frequently are used interchangeably, and DFMs often are expressed in state space form to exploit better parameter estimation properties (Hamaker, Dolan, & Molenaar, 2005; Ho, Ombao, & Shumway, 2005; Ho, Shumway, & Ombao, 2006; Molenaar, 1994; Song & Ferrer, 2009, 2012; Zhang, Hamaker, & Nesselroade, 2008).

Although SSM models have been used in many areas of research, their application in questions about mediation is not available in the literature. Statistically, probably all existing longitudinal models for studying mediation can be represented in their state space form, and identical results can be obtained. However, we do not pursue this direction because of the consequences of the ergodic theorems stated before. Our proposed approach, instead, is based on the specification of an SSM for analyzing time series data from a single subject (especially when the number of measurement occasions is large).

In practice, modeling the mediation process as an SSM has important benefits. First, it allows the researcher to investigate time-related sequences among variables (i.e., predictor → mediator → outcome), as the process represented by these variables unfolds over time. Although some longitudinal structural equation models also allow such investigation, the state space approach provides a better and more thorough depiction because some mediation processes need a longer time to unfold. Second, SSM can accommodate complex specifications such as measurement structures and second-order factors.[3] Third, state space analyses at the individual level provide a theoretically sound, bottom-up approach to create homogeneous subpopulations. If a hypothetical model fits separately the time series data from several subjects, a homogeneous subpopulation can be created from the analyses at the individual level, and generalized conclusions can be drawn for this subpopulation. The bottom-up approach, however, can be labor-intensive. More discussion of multiple-subject time series is given in subsequent sections.

### *3.2. The Linear Gaussian SSM*

Currently, there is no standard notation in the literature for SSM, and different authors have different preferences. Based on the similarity between SEM and SSM, we choose to use LISCOMP notation to present the formulation of SSM. The benefit of using LISCOMP notation is that each matrix has gained a standard interpretation in the literature, thus providing a convenient and familiar notation. Let $y_t$ be a $p$-variate vector representing $p$ manifest variables, $\eta_t$ a $q$-variate vector representing $q$-latent variables ($p \geq q \geq 1$), and $t = 1, 2, \ldots, T$ denotes the time point for the corresponding vector. The general linear Gaussian SSM contains a measurement equation

$$y_t = \tau_t + \Lambda_t \eta_t + \varepsilon_t, \;\; \varepsilon_t \sim MVN(0, \Theta_t),$$

and a transition equation,

$$\eta_t = \alpha_t + \mathrm{B}_t \eta_{t-1} + \zeta_t, \;\; \zeta_t \sim MVN(0, \Psi_t),$$

where $\tau_t$ is a $p \times 1$ vector for intercepts, $\Lambda_t$ is a $p \times q$ loading matrix, $\varepsilon_t$ is a $p \times 1$ vector for measurement errors (also known as innovations in the time series literature), $\Theta_t$ is a $p \times p$ diagonal covariance matrix, $\alpha_t$ is a $q \times 1$ vector for means, $\mathrm{B}_t$ is a $q \times q$ transition matrix, $\zeta_t$ is a $q \times 1$ vector for residuals, and $\Psi_t$ is a $q \times q$ covariance matrix. The measurement errors and residuals are assumed to be serially independent and independent of each other at all time points. We denote as $\theta_t$ the vertical vector that collects all parameters in $\tau_t$, $\Lambda_t$, $\Theta_t$, $\alpha_t$, $\mathrm{B}_t$, and $\Psi_t$, and the subscript $t$ means that $\theta_t$ is time varying, thus resulting in the time-varying SSM. Applications of the time-varying SSM can be found in recent articles by Molenaar, Sinclair, Rovine, Ram, and Corneal (2009), Sinclair and Molenaar (2008), and Chow, Zu, Shifren, and Zhang (2011).

Here we consider only the time-invariant SSM, with the understanding that the model could be extended to include time-varying parameters. The measurement and transition equations are thus simplified to

$$\begin{aligned} y_t &= \tau + \Lambda\eta_t + \varepsilon_t,\ \ \varepsilon_t \sim MVN(0, \Theta) \\ \eta_t &= \alpha + \mathrm{B}\eta_{t-1} + \zeta_t,\ \ \zeta_t \sim MVN(0, \Psi), \end{aligned}$$

in which the parameter vector, θ, is time-invariant.

### 3.3. Parameter Estimation and the Kalman Filter

Unknown parameters of the linear Gaussian SSM are estimated via a recursive algorithm, called the *Kalman filter* (KF). The KF[4] algorithm is initialized with the latent variable, $\eta_{0|0}$, and the associated covariance matrix, $\mathrm{P}_{0|0}$, and proceeds with the prediction and filtering steps iteratively at each time point. When $t = 1$, the prediction step gives the predicted latent variable and its covariance matrix, that is,

$$\begin{aligned} \eta_{1|0} &= \alpha + \mathrm{B}\eta_{0|0} \\ \mathrm{P}_{1|0} &= \mathrm{BP}_{0|0}\mathrm{B}' + \Psi. \end{aligned}$$

As a byproduct, the one-step-ahead prediction error and its associated covariance matrix are obtained, that is,

$$\begin{aligned} e_1 &= y_1 - y_{1|0} = y_1 - (\tau + \Lambda\eta_{1|0}) \\ \mathrm{D}_1 &= \Lambda\mathrm{P}_{1|0}\Lambda' + \Theta. \end{aligned}$$

Then, the filtering step uses the observed value at $t = 1$ to update the predicted values, giving

$$\begin{aligned} \mathrm{K}_1 &= \mathrm{P}_{1|0}\Lambda'\mathrm{D}_1^{-1}(\text{Kalman gain matrix}) \\ \eta_{1|1} &= \eta_{1|0} + \mathrm{K}_1 e_1 = \eta_{1|0} + \mathrm{P}_{1|0}\Lambda'\mathrm{D}_1^{-1}e_1 \\ \mathrm{P}_{1|1} &= \mathrm{P}_{1|0} - \mathrm{K}_1\mathrm{D}_1\mathrm{K}_1' = \mathrm{P}_{1|0} - \mathrm{P}_{1|0}\Lambda'\mathrm{D}_1^{-1}\Lambda\mathrm{P}_{1|0}. \end{aligned}$$

When $t = 2$, $\eta_{1|1}$ and $\mathrm{P}_{1|1}$ are used in the prediction step, followed by the filtering step to calculate $\eta_{2|2}$ and $\mathrm{P}_{2|2}$, and so on. In sum, for $t = 1, 2, \ldots, \mathrm{T}$, the recursive KF algorithm can be written as

$$\begin{aligned} \eta_{t|t-1} &= \alpha + \mathrm{B}\eta_{t-1|t-1} \\ \mathrm{P}_{t|t-1} &= \mathrm{BP}_{t-1|t-1}\mathrm{B}' + \Psi \\ e_t &= y_t - y_{t|t-1} = y_t - (\tau + \Lambda\eta_{t|t-1}) \\ \mathrm{D}_t &= \Lambda\mathrm{P}_{t|t-1}\Lambda' + \Theta \\ \mathrm{K}_t &= \mathrm{P}_{t|t-1}\Lambda'\mathrm{D}_t^{-1} \\ \eta_{t|t} &= \eta_{t|t-1} + \mathrm{K}_t e_t = \eta_{t|t-1} + \mathrm{P}_{t|t-1}\Lambda'\mathrm{D}_t^{-1}e_t \\ \mathrm{P}_{t|t} &= \mathrm{P}_{t|t-1} - \mathrm{K}_t\mathrm{D}_t\mathrm{K}_t' = \mathrm{P}_{t|t-1} - \mathrm{P}_{t|t-1}\Lambda'\mathrm{D}_t^{-1}\Lambda\mathrm{P}_{t|t-1}. \end{aligned}$$

Inserting $e_t$ and $D_t$ at each time point into the log-density function of the multivariate normal distribution and summing all log-density functions, the *prediction error decomposition* (PED; Schweppe, 1965) function is obtained:

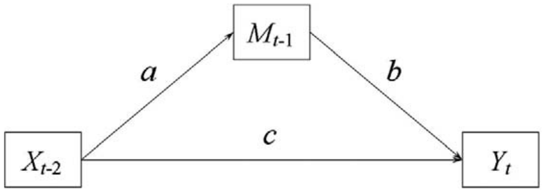


FIGURE 1. *Direct and indirect effects in the simplest mediation model.*

$$\text{PED} = \frac{1}{2}\sum_{t=1}^{\mathrm{T}} \left[-p\log(2\pi) - \log|D_t| - e_t' D_t^{-1} e_t\right].$$

Giving certain starting values in θ and maximizing the PED function with respect to θ provides the parameter estimates.

### *3.4. Testing Mediation: Bootstrapping the Time Series Data*

As described in the previous sections, early definitions of direct and indirect effects in mediation analysis were based on cross-sectional designs (Baron & Kenny, 1986). These definitions were theoretically inaccurate because of the lack of consideration of time. Modern definitions of the concepts advocate that the causal variable, the mediator variable, and the outcome variable of a mediation process should be obtained at different occasions (Collins, Graham, & Flaherty, 1998; Gollob & Reichardt, 1991). Figure 1 illustrates an example of the simplest three-variate model in which time is taken into account. This figure illustrates the concepts of direct and indirect effects in mediation analysis. In this model, $a$ is defined as the direct effect of $X_{t-1}$ on $M_t$, $b$ is the direct effect of $M_t$ on $Y_{t+1}$, and $c$ is the direct effect of $X_{t-1}$ on $Y_{t+1}$. The indirect effect of $X_{t-1}$ on $Y_{t+1}$ is defined as the product of $a$ and $b$, that is, $ab$. For more complicated models, the indirect effect can be a product of more parameters linking several different occasions.

In order to evaluate the direct effects, several statistical tests are available, for example, the Wald test and the likelihood ratio test. Testing the significance of the indirect effects is also of great importance but more difficult, and the common direct methods just mentioned are not appropriate because of the nonnormality of the sampling distribution of the indirect effect. As an alternative, the use of bootstrap confidence intervals (CIs) is recommended (Bollen & Stine, 1990; Hayes, 2009; MacKinnon, Lockwood, & Williams, 2004; Preacher & Hayes, 2004, 2008a, 2008b; Shrout & Bolger, 2002). The standard nonparametric bootstrap involves two steps. In the first step, a *resample* of size $N$ is drawn with replacement from the original sample. In the second step, model parameters are estimated from this resample. The two steps are replicated $B$ times (where $B$ is large), so that the sampling distribution of the statistic of interest can be obtained.

At the .95 level, the 2.5th and 97.5th percentiles are chosen to construct the CI to conduct the significance test of a single parameter, or of a product of parameters, by examining whether the CI excludes 0 (indicating a significant effect). Bootstrapping the time series data, however, poses yet another difficulty because of the temporal dependence of the time series data (Zhang & Browne, 2006). In general, the standard nonparametric bootstrap is not appropriate for time series data because it destroys the inherent time dependency in the data. In this section, we introduce two bootstrap methods appropriate for SSM, namely the parametric bootstrap and the residual-based bootstrap. Besides these two methods, there are other approaches appropriate for SSM (e.g., Zhang & Chow, 2010).

The parametric bootstrap is essentially a Monte Carlo simulation, in which repeated bootstrap samples are simulated from a specified model, where the estimates from the original sample are treated as parameters. The underlying assumption is that the specified model is correct in the population. To generate each random sample, a number of steps are followed:

1. Generate $\eta_0$ from *MVN* $(0, 100 \times \mathbf{I}_q)$.
2. Set the iteration number $t = 1$.
3. Generate $\zeta_t$ from *MVN* $(0, \hat{\Psi})$.
4. Calculate $\eta_t$ using $\eta_t = \hat{\alpha} + \hat{\mathrm{B}}\eta_{t-1} + \zeta_t$.
5. Generate $\varepsilon_t$ from *MVN* $(0, \hat{\Theta})$.
6. Calculate $y_t$ using $y_t = \hat{\tau} + \hat{\Lambda}\eta_t + \varepsilon_t$.
7. Set $t = t + 1$ and return to Step 3.
8. Repeat Steps 3 to 6 until $t > \mathrm{T} + 1{,}000$.
9. Save the data from 1,001 to $\mathrm{T} + 1{,}000$.

Although not always necessary, the first 1,000 observations are typically discarded as the burn-in period.

The residual-based bootstrap was first applied to linear Gaussian SSM by Stoffer and Wall (1991) in assessing the precision of maximum likelihood estimates, and it is considered a semiparametric approach. Similar to the parametric bootstrap, population parameters are taken to be sample estimates in the residual-based bootstrap, and the underlying assumption of a correctly specified model is also made. On the other hand, random samples are drawn, with replacement, from the standardized residuals as in the standard nonparametric bootstrap. Specifically, the residual-based bootstrap procedure is based on the innovations form of the KF:

$$\begin{aligned}
\varepsilon_t &= y_t - \tau - \Lambda\eta_{t|t-1} \\
\mathrm{D}_t &= \Lambda \mathrm{P}_{t|t-1}\Lambda' + \Theta \\
\mathrm{K}_t &= \mathrm{P}_{t|t-1}\Lambda' \mathrm{D}_t^{-1} \\
\eta_{t+1|t} &= \mathrm{B}\eta_{t|t-1} + \mathrm{B}\mathrm{K}_t\varepsilon_t \\
y_t &= \tau + \Lambda\eta_{t|t-1} + \varepsilon_t.
\end{aligned}$$

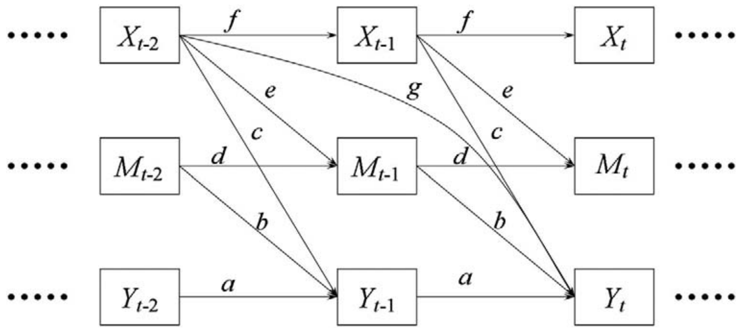


FIGURE 2. *A temporal slice of the lag-2 model.*

Then, the algorithm proceeds as follows:

1. Calculate standardized innovations using $\hat{\mathrm{D}}_t^{-1/2}\hat{\varepsilon}_t$, denoted $\tilde{\varepsilon}_t$.
2. Draw, with replacement, a random sample from $\tilde{\varepsilon}_t$ to obtain $\tilde{\varepsilon}_t^*$.
3. Construct a bootstrap sample by fixing the initial conditions of the KF and iteratively using the following two equations:

$$\eta_{t+1|t} = \hat{B}\eta_{t|t-1} + \hat{\mathrm{B}}\hat{\mathrm{K}}_t\hat{\mathrm{D}}_t^{1/2}\tilde{\varepsilon}_t^*$$
$$y_t = \hat{\tau} + \hat{\Lambda}\eta_{t|t-1} + \hat{\mathrm{D}}_t^{1/2}\tilde{\varepsilon}_t^*.$$

The idea behind the residual-based bootstrap is that the standardized residuals are independent and identically distributed, and therefore exchangeable, after all the dynamic and measurement relationships have been accounted for by the model. This procedure, however, is not robust against model misspecification (Stoffer & Wall, 1991, 2004; Zhang & Chow, 2010).

For the examples considered in this article, we set $B = 2{,}000$ for both bootstrap procedures to approximate the sampling distribution of the product. As a general rule, large numbers are required to allow enough simulated cases in both tails of the sampling distribution of the indirect effect so that the percentiles can be accurately estimated for constructing a CI (Yung & Chan, 1999).

### *3.5. Illustration 1: A Simulated Lag-2 Example*

In order to illustrate the parameter estimation in SSM and the two bootstrap procedures just described, a three-variate, single-subject time series data set is generated from a lag-2 model (depicted in Figure 2). Note that the figure represents a temporal slice of three time points from the entire process. Three equations are involved in this model:

$$\begin{aligned} Y_t &= aY_{t-1} + bM_{t-1} + cX_{t-1} + gX_{t-2} + \varepsilon_{Yt} \\ M_t &= dM_{t-1} + eX_{t-1} + \varepsilon_{Mt} \\ X_t &= fX_{t-1} + \varepsilon_{Xt}, \end{aligned}$$

where $a$, $d$, and $f$ are autoregressive parameters of the outcome variable, the mediator variable, and the causal variable, separately; $b$, $c$, and $e$ are the lag-1 cross-regressive parameters; $g$ is the lag-2 cross-regressive parameter; and $\varepsilon_{Yt}$, $\varepsilon_{Mt}$, and $\varepsilon_{Xt}$ are residuals in each equation. If $g$ is equal to 0, it reduces to the lag-1 model, which is a particular case of the lag-2 model. In addition, if $c$ is also equal to 0, it corresponds to one of the models in Cole and Maxwell (2003, model 5). As we will show immediately, the lag-2 model can be expressed in state space form. In principle, as long as we can write the model in state space form, the parameter estimation and bootstrap procedures can be readily applied.

By defining the following measurement equation and the transition equation, the state space form of the lag-2 model is obtained:

$$\begin{pmatrix} Y_t \\ M_t \\ X_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} Y_t \\ M_t \\ X_t \\ Y_{t-1} \\ M_{t-1} \\ X_{t-1} \end{pmatrix}, \text{and } \Theta = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} Y_t \\ M_t \\ X_t \\ Y_{t-1} \\ M_{t-1} \\ X_{t-1} \end{pmatrix} = \begin{pmatrix} a & b & c & 0 & 0 & g \\ 0 & d & e & 0 & 0 & 0 \\ 0 & 0 & f & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ M_{t-1} \\ X_{t-1} \\ Y_{t-2} \\ M_{t-2} \\ X_{t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{Yt} \\ \varepsilon_{Mt} \\ \varepsilon_{Xt} \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\text{and } \Psi = \begin{pmatrix} \psi_Y & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_M & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_X & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

We simulated four time series data sets with lengths equal to 50, 100, 150, and 200. Compared with the typical lengths in the time series literature, the lengths considered here are relatively short. Although most social scientists typically collect at best only a handful of repeated measures, intensive longitudinal data are highly desirable for SSM analyses. Some precedent examples have already emerged in emotion studies using daily diary data (Chow, Hamaker, Fujita, &

TABLE 1
*Parameter Estimates From a Lag-2 Model Fitted to Simulated Data*

| Parameter | True Value | T = 50 | | T = 100 | | T = 150 | | T = 200 | |
|---|---|---|---|---|---|---|---|---|---|
| $a$: autoreg of $Y$ | .5 | .440 | | .460 | | .469 | | .432 | |
| $b$: $Y_t$ on $M_{t-1}$ | .4 | .295 | | .356 | | .351 | | .338 | |
| $c$: $Y_t$ on $X_{t-1}$ | .4 | .373 | | .342 | | .360 | | .386 | |
| $d$: autoreg of $M$ | .5 | .414 | | .444 | | .463 | | .476 | |
| $e$: $M_t$ on $X_{t-1}$ | .4 | .458 | | .462 | | .443 | | .400 | |
| $f$: autoreg of $X$ | .8 | .820 | | .741 | | .778 | | .792 | |
| g: $Y_t$ on $X_{t-2}$ | .3 | .298 | | .327 | | .281 | | .309 | |
| $\psi_Y$ | .1 | .110 | | .082 | | .091 | | .096 | |
| $\psi_M$ | .4 | .282 | | .318 | | .332 | | .349 | |
| $\psi_X$ | .9 | .753 | | .758 | | .815 | | .996 | |
| $eb$ | .16 | .135 | | .164 | | .155 | | .135 | |
| | | resid | parm | resid | parm | resid | parm | resid | parm |
| Number converged | | 2,000 | 2,000 | 2,000 | 2,000 | 2,000 | 2,000 | 2,000 | 2,000 |
| Lower bound of *eb* | | .067 | .055 | .119 | .110 | .117 | .115 | .104 | .105 |
| Upper bound of *eb* | | .220 | .245 | .228 | .237 | .209 | .208 | .176 | .178 |
| Range of the 95% CI | | .153 | .190 | .109 | .127 | .092 | .093 | .072 | .073 |

*Note*. CI = confidence interval; resid = residual-based bootstrap; parm = parametric bootstrap; *eb* is the indirect effect from $X_{t-2}$ to $Y_t$.

Boker, 2009; Song & Ferrer, 2009, 2012), a cognitive study using daily cognitive assessment data (Chow, Hamaker, & Allaire, 2009), and a neuropsychology study using functional magnetic resonance imaging data (Ho et al., 2005).

We estimated the parameters using the KF algorithm with results shown in Table 1. The estimated parameters in four different length conditions roughly show that length of the time series is inversely related to the magnitude of parameter bias. A challenge, however, is that when the sampling frequency (i.e., the time elapsed between consecutive measurement occasions) is fixed, longer time series data are more expensive and difficult to collect. One strategy to obtain longer time series data is to increase the sampling frequency. The issue of length and sampling frequency will be discussed briefly in the last section.

The indirect effect from $X$(t−2) to $Y$(t) is the product of $e$ and $b$. The two bootstrap methods are used to construct 95% CIs for this product. For the four simulated examples, the 95% CIs from both the parametric bootstrap and the residual-based bootstrap are similar, and none of the CIs contains zero, indicating a significant lag-2 indirect effect from $X$(t−2) to $Y$(t). In addition, the CIs from the parametric bootstrap are a bit larger than those from the residual-based bootstrap when T = 50 and 100; this difference almost disappears when T = 150 and 200.

### *3.6. Illustration 2: An Empirical Study*

In this section, we fit the lag-2 model displayed in Figure 2 to the time series of two man–woman dyads. Each series contains self-reported daily stress and affect for 91 days. These data are part of a larger study designed to examine dyadic interactions (for more details see, e.g., Ferrer, Steele, & Hsieh, 2012; Ferrer & Widaman, 2008). The variables used in these analyses are *female perceived stress* ($X$), *male positive affect specific to his relationship* ($M$), and *female negative affect specific to her relationship* ($Y$). Relationship-specific affect (RSA) was measured using the RSA scale (Ferrer et al., 2012), 18 items intended to tap into the participants' positive and negative emotional experiences specific to their relationships. Examples of the positive items include "emotionally intimate," "trusted," and "loved." Examples of the negative items include "sad," "trapped," and "discouraged." The stress construct was measured using 5 items from the Positive and Negative Affect Schedule (Watson, Clark, & Tellegen, 1988) including "distress," "upset," "scared," "nervous," and "afraid." For all analyses, we created unit-weighted composites for each person, using all the items in each of the scales.

The results of fitting the lag-2 models to these data are reported in Table 2. For Dyad 1, the estimated values for parameters $b$ and $e$ are statistically significant, indicating reliable evidence to support positive relations between the male partner's positive affect on a given day and his female partner's negative affect the following day, as well as between the female partner's perceived stress on a given day and the male partner's positive affect the following day. However, the estimated $c$ and $g$ parameters are not statistically significant. The nonsignificant estimate of $g$ may suggest a more parsimonious model that does not include the lag-2 structure between the female partner's stress and her subsequent negative affect.

For Dyad 2, the estimated $b$ parameter is not statistically significant (.011/.031 = .355), whereas the estimated $e$ and $g$ are both significant. It is not unreasonable to expect that the nonsignificant estimate of $b$ may result in a model with a nonsignificant indirect effect linking the causal variable at $t - 2$ to the outcome variable at $t$. For researchers interested in emotion, these preliminary results provide motivation for further investigation of each dyad's data separately. Particularly, the counterintuitive sign of the estimated $b$ for both dyads may flag some problems or uncertainties in the model, the data, or both. In reality, the affective processes underlying different dyads may be qualitatively different (e.g., including a feedback process).

For illustrative purposes, we also present the 95% CIs from the two bootstrap methods for both dyads, but we acknowledge that the lag-2 model may not be plausible for either dyad. The caveat is that these CIs are not readily interpretable because neither bootstrap method is robust to model misspecification. Further efforts are necessary to determine the best final model for each dyad and, in order to make proper statistical inferences, the two bootstrap methods need to be

TABLE 2
*Parameter Estimates From a Lag-2 Model Fitted to Two Dyads' Time Series Over 91 Days*

| Parameter | Dyad 1 | | Dyad 2 | |
|---|---|---|---|---|
| *a*: autoreg of *Y* (female negative affect relationship specific) | .420 (.137) | | .267 (.121) | |
| *b*: $Y_t$ on $M_{t-1}$ | .220 (.043) | | .011 (.031) | |
| *c*: $Y_t$ on $X_{t-1}$ | .053 (.132) | | .473 (.214) | |
| *d*: autoreg of *M* (male positive affect relationship specific) | .707 (.049) | | .736 (.066) | |
| *e*: $M_t$ on $X_{t-1}$ | .669 (.110) | | .792 (.205) | |
| *f*: autoreg of *X* (stress) | .927 (.040) | | .973 (.020) | |
| *g*: $Y_t$ on $X_{t-2}$ | −.042 (.099) | | .274 (.174) | |
| $\psi_Y$: Var(pagf) | .184 (.027) | | .113 (.017) | |
| $\psi_M$: Var(nasf) | .385 (.057) | | .588 (.088) | |
| $\psi_X$: Var(pasf) | .309 (.046) | | .049 (.007) | |
| *eb* | .147 | | .009 | |
| | resid | parm | resid | parm |
| Number converged | 2,000 | 2,000 | 2,000 | 2,000 |
| Lower bound of *eb* | .100 | .095 | −.029 | −.028 |
| Upper bound of *eb* | .225 | .246 | .076 | .049 |
| Range of the 95% CI | .125 | .151 | .105 | .077 |

*Note*. Standard errors are in parentheses. CI = confidence interval; resid = residual-based bootstrap; parm = parametric bootstrap; *eb* is the indirect effect from $X_{t-2}$ to $Y_t$.

applied to each final model separately. This is beyond the scope of the current illustration, but is deserving of separate study. In summary, the message from this illustration for mediation researchers is clear; that is, mediation analyses should be conducted for each subject (i.e., individual, dyad, or other unit of analysis) separately to accommodate heterogeneity in the units.

## 4. Additional Issues

### *4.1. Causal Inference*

Establishing causality is an important component of longitudinal research. There are several recent treatments of causal inference in mediation analysis, most inspired by Rubin's causal model (or the *potential outcomes* framework; see Albert, 2008; Ten Have & Joffe, 2010). In this framework, causality is defined with reference to potential outcomes that might have been obtained under different counterfactual conditions. Because it is not possible to observe outcomes under all possible conditions, certain assumptions are commonly invoked to permit causal inference, for example, the assumption that key paths composing an indirect effect are not confounded by omitted variables, and the assumption that $X$ does not moderate the effect of $M$ on $Y$, among others (Imai, Keele, & Tingley, 2010; Pearl, 2010, 2012; VanderWeele & Vansteelandt, 2009).

The single-subject design is not an attempt to establish causal relationships per se. Instead, it provides a temporally plausible way to model and test a hypothetical mediation process. That is, the parameters associated with the mediator/mediators can be tested against a null hypothesis using a sampling distribution (perhaps obtained by bootstrapping), so that the researcher can gain more insights into the data and determine to what degree the data are consistent with the hypothesized underlying process. Because it is not explicitly couched in a potential outcomes framework, researchers should be cautious in making causal inferences using the state space modeling approach to mediation analysis. However, it should be noted that SSM has in its favor that key effects are within-subject rather than between-subject, and measurements on key variables are necessarily separated in time. These features provide a stronger basis for causal inference than other methods that are designed for use with between-subject and/or cross-sectional data. Extending the logic of the potential outcomes framework to single-subject longitudinal designs is an interesting avenue for future research.

### *4.2. Extension to the Multiple-Subject Time Series*

Gathering time series data simultaneously on multiple persons (as in our empirical study) is a common practice. In terms of modeling strategy, it can be considered a straightforward extension to the model presented here. We call this extension the multiple-subject time-series design. Since the introduction of DFM (Molenaar, 1985), the single subject is always emphasized as the unit of analysis. This emphasis may give researchers the impression that DFM is restricted to analyzing the time series data of a single subject. This impression is not incorrect for the *standard* application of DFM. As is demonstrated in the modern literature, DFM and SSM can and should be extended to the multiple-subject time-series design (e.g., Chow, Hamaker, & Allaire, 2009; Chow, Hamaker, Fujita, & Boker, 2009; Chow, Zu et al., 2011; Hamaker et al., 2005; Molenaar, 2010b; Nesselroade, 2010; Song & Ferrer, 2012).

The term *panel model* is often used to refer to models used to analyze time-series data (of any length from short to long) collected from a group of subjects (of any sample size). In mediation analysis, panel models under the SEM framework usually take very short lengths (e.g., a handful of repeated measurements) and large sample sizes (e.g., Cole & Maxwell, 2003).

Methodologically, multiple-subject modeling can be implemented easily by fitting a *qualitatively* and *quantitatively* identical model to multiple subjects, treating each individual as a group. By qualitatively identical, we mean that each subject can be characterized by the same dynamic process implied by the specified model; whereas by quantitatively identical, we mean that the parameters of the specified model are equated across different subjects. Then, it is possible to test parameter invariance across subjects by means of likelihood ratio tests. This procedure is akin to the standard multiple-group SEM analysis of interindividual variation in searching for commonality across subjects (nomothetic lawfulness). Following the new definition of parameter invariance proposed by Nesselroade, Gerstorf, Hardy, and Ram (2007), we suggest that parameter invariance tests can be better examined at an appropriate level of abstraction. Specifically, parameters in the measurement equation can differ to some arbitrary degree to recognize and isolate idiosyncrasy, while those in the transition equation can be equated and tested for similarities that reflect nomothetic lawfulness.

However, as described previously, some empirical examples suggest that heterogeneity is the rule rather than the exception. In addition to our empirical example, for instance, economists studying workers' levels of satisfaction encountered the problem that each individual anchors his or her scale at a different level (Winkelmann & Winkelmann, 1998). This renders interindividual comparisons of responses meaningless in a cross-sectional study. Another example, as discussed by Nesselroade (2010), concerns participants' idiosyncratic use of language. Specifically, one debriefed participant from Mitteness and Nesselroade (1987) reported that she interpreted the term "anxious" to mean "eager."

This empirical evidence further supports the general conclusion of heterogeneity across people. On the other hand, Kelderman and Molenaar (2007) provided counterintuitive evidence of the insensitivity of the standard factor analysis of interindividual variation to the presence of extreme qualitative heterogeneity of the factor loadings in the population of subjects. This proven insensitivity can have serious practical and ethical consequences, yielding individual assessments and decisions that are biased to unknown degrees (Molenaar, 2008a, 2008b). Given these considerations, great caution should be used if homogeneity is to be assumed.

Another popular modeling strategy for multiple-subject time series data that allows some degree of heterogeneity is the multilevel modeling framework. Song and Ferrer (2012) recently proposed a random coefficient DFM (which can be conceptualized as a multilevel SSM) to investigate both intra- and interindividual variation. Specifically, they assumed that the parameters in the transition matrix are drawn from some distribution, so that differences in dynamics at the appropriate level of abstraction can be accommodated. A further extension of their model is to allow the parameters in the loading matrix to be random to capture heterogeneity in the between-subject factor structure. Such an extension, however, is complex and difficult in terms of model specification, and estimation may render a model infeasible and/or indefensible in practice.

### 4.3. Length and Sampling Frequency

Longer time series are desired for state space analysis and other time-series techniques in general. Whereas increasing sampling frequency is a way to obtain more data points for a given time span, the optimal balance of length and sampling frequency is almost always context-dependent (Brose & Ram, 2012; Sliwinski & Mogle, 2008). Researchers should consider carefully the relative benefits of extending the length of a study versus making more observations within a fixed length. A thorough discussion of the issue is beyond the scope of this article; see Collins and Graham (2002), Nesselroade (1991), Nesselroade and Boker (1994), Nesselroade and Jones (1991), and Windle and Davies (1999).

### 4.4. Use of Latent Variables in Mediation Analysis

According to Cole and Maxwell (2003, question 6: What are the effects of random measurement error?), unmodeled measurement error variance can cause both under- and overestimation of other model parameters. A natural extension of the simple mediation model that we have illustrated involves the use of latent variables, each of which can be indicated by multiple manifest variables. By explicitly modeling error variance ($\Theta$) in the measurement equation, the bias in parameter estimation can be almost completely resolved, provided that the model is otherwise correctly specified. Further, psychometric properties of the measurement instruments (e.g., tau-equivalence) can be evaluated by equating some loading parameters in the $\Lambda$ matrix. As for studying mediation, dynamics of the process can be examined at the latent level while taking into consideration the factorial structure of the data.

### 4.5. Use of Exogenous Variables

Exogenous variables, or fixed external inputs, may enter into the observation equation, the transition equation, or both (e.g., Lütkepohl, 2005, p. 613; Shumway & Stoffer, 2011, p. 320). In this case, the linear Gaussian SSM is extended to

$$\begin{aligned} y_t &= \tau_t + \Lambda_t \eta_t + \Gamma_t x_t + \varepsilon_t,\ \ \varepsilon_t \sim MVN(0, \Theta_t) \\ \eta_t &= \alpha_t + \mathrm{B}_t \eta_{t-1} + \Upsilon_t x_t + \zeta_t,\ \ \zeta_t \sim MVN(0, \Psi_t). \end{aligned}$$

Perhaps the most common purpose for including exogenous variables in statistical models is to account for the variance in some random component (e.g., Snijders & Bosker, 2011). Molenaar, de Gooijer, and Schmitz (1992) used a discrete time variable in the transition equation to accommodate a linear time trend. Beyond parameter estimation in time-varying SSM, Molenaar (1994, 2010a, 2010b) presented the theory, methods, and application of optimal control in psychopathological processes. Basically, a feedback function is derived after the

SSM parameters are estimated to determine the optimal level of the external inputs such that it is possible to manipulate the external inputs (e.g., the insulin dose) by the controller (e.g., the therapist) to guarantee that the outcome variable (e.g., the blood glucose level of a patient) will be as close as possible to the desired level. This type of application presents a grand opportunity to develop more effective personalized treatment, as opposed to the general dosage of the clinical medication which is not optimal, less effective, and often brings some undesired side effects.

Moreover, it is possible to collect data with exogenous variables in the multiple-subject time-series design. If the exogenous variable is time varying within a person, the multiple-group analysis discussed before might be applied (e.g., Molenaar, 2010b). If the exogenous variable is time-invariant within a person but varying between persons (i.e., a Level-2 variable), the multilevel (or random coefficient) state space framework can be applied (Song & Ferrer, 2012). In both scenarios, moderated mediation (or other complex interactions) can be examined (Card, 2012; Preacher, Rucker, & Hayes, 2007). Future research on this topic is warranted.

### *4.6. Missing Data*

An attractive feature of SSM is that missing values in the time series data are easily handled by the KF algorithm. The full-information maximum likelihood (FIML) procedure varies the dimension of the data vector for observations that contain one or more missing values. A computationally easier variant of the FIML procedure is to zero out the missing values and retain the same dimension of the data vector throughout the observations. No additional effort is needed to preprocess the missing values as in multiple imputation procedures. Shumway and Stoffer (2011, chapter 6, subsection 6.4) outline the details of the FIML procedure and its easier variant, and they also describe the necessary modifications for the expectation–maximization optimization algorithm.

### *4.7. Software Implementation*

Estimating time-varying SSMs is often a difficult task in that extensive programming skills are required to write one's own software. EKFIS, a Fortran program developed by Peter Molenaar to implement the Extended KF with Iteration and Smoothing algorithm has been used to illustrate the examples in several articles and chapters (e.g., Molenaar et al., 2009; Molenaar & Ram, 2009, 2010; Sinclair & Molenaar, 2008). However, the flexibility of time-varying SSM can become a liability as well, as EKFIS requires the ability to write and compile Fortran code (Molenaar & Ram, 2010). Other authors have implemented the (extended) KF algorithm using MATLAB and Ox/Ssfpack to estimate time-varying SSMs (e.g., Chow, Hamaker, & Allaire, 2009; Chow, Hamaker, Fujita, & Boker, 2009; Chow, Zu et al., 2011; Zu, 2008). The programming efforts involved are still, unfortunately and inevitably, demanding, which is one of the reasons for the scarcity of modeling work along these lines.

The programming task is relatively easier for time-invariant SSM than for its time-varying counterpart. In 2011, several articles were published to illustrate different software packages (e.g., EViews, MATLAB, R, SAS, Stata, and several others) in a special volume (Vol. 41) of the *Journal of Statistical Software*. However, each software package has certain limitations. Readers are urged to consult this special volume to get a flavor of the package with which they are the most familiar.

It is worth noting that MKFM6, a Fortran program provided by Dolan (2005), and a SAS/IML program, provided by Gu and Yung (2013), are available to estimate time-invariant linear Gaussian SSMs. MKFM6 is free and has been used by several authors (e.g., Chow, Ho, Hamaker, & Dolan, 2010; Hamaker et al., 2005; Zhang et al., 2008). The programming tasks in this article are implemented in SAS/IML, which was developed by modifying and extending the SAS/IML code provided by Gu and Yung. All the programs can be obtained by request from the first author.

Finally, a Bayesian approach to parameter estimation for DFM and SSM is emerging (e.g., Bhattacharya, Ho, & Purkayastha, 2006; Bhattacharya & Maitra, 2011; Chow, Tang, Yuan, Song, & Zhu, 2011; Song & Ferrer, 2012; Zhang & Nesselroade, 2007). This newer approach requires estimation methods that are computationally heavy (e.g., Markov chain Monte Carlo). The rapid development of specialized software programs (e.g., WinBUGS, Mplus, OpenBUGS), however, makes use of Bayesian methods both manageable and appealing.

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## Notes

1. Application of the state space model (SSM) is not restricted to single-subject time series data. For example, Chow, Ho, Hamaker, and Dolan (2010) provide an example in which SSM is applied to cross-sectional data. Moreover, we will discuss the extension of multiple-subject time series data in Subsection 4.1.
2. Note that our definition of stationarity is consistent with that used in the time-series literature, but not strictly parallel to the same term described in the mediation literature (Cole & Maxwell, 2003; Kenny, 1979).
3. More discussion is provided in Subsection 4.4.
4. Proof of the Kalman filter (KF) algorithm can be found, for instance, in Lütkepohl (2005, pp. 630–631) or Shumway and Stoffer (2011, pp. 326–327).

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