

MODERATED MEDIATION IN MULTILEVEL STRUCTURAL EQUATION MODELS: DECOMPOSING EFFECTS OF RACE ON MATH ACHIEVEMENT WITHIN VERSUS BETWEEN HIGH SCHOOLS IN THE UNITED STATES

Michael J. Zyphur, Zhen Zhang, Kristopher J. Preacher, and Laura J. Bird

At roughly similar times in the 1980s, social scientists formalized what have become enduring interests in multilevel modeling (e.g., Raudenbush & Bryk, 1986) and moderation and mediation (e.g., Baron & Kenny, 1986). Today, moderation and mediation models have been synthesized so that these effects can be combined and estimated in a wide variety of cases (Edwards & Lambert, 2007; Preacher, Rucker, & Hayes, 2007), including with latent variables and latent interactions (Cheung & Lau, 2015; Sardeshmukh & Vandenberg, 2016). In the multilevel arena, approaches now exist that allow assessing multilevel mediation (e.g., Preacher, Zyphur, & Zhang, 2010; Preacher, Zhang, & Zyphur, 2011) and multilevel moderation with latent variables (e.g., Preacher, Zhang, & Zyphur, 2016).

What remains to be offered, however, is a synthesis of these interests in a way that allows estimating moderation and mediation at multiple levels of analysis. Our chapter addresses this by first describing the logic of moderated mediation, including how to formalize it as a structural equation model (SEM). We then extend this logic to multilevel SEM (MSEM) to estimate level-specific moderated mediation. Our approach allows the typical random

coefficient prediction method for estimating cross-level moderation with random slopes (as outcomes), but our approach can also use a latent moderated structural equations (LMS) approach to estimate moderation, which requires latent variable interactions (see Preacher et al., 2016).

To avoid the high-dimensional numerical integration that often accompanies these interactions, we describe a Bayesian *plausible values* approach that multiply-imputes latent variable scores in the first step, then allowing researchers to form product terms as if they were observed to estimate moderated effects in a second step. This approach can be used for any model wherein latent interactions or power polynomials otherwise require numerical integration, and therefore, it is also applicable in single-level models (e.g., Sardeshmukh & Vandenberg, 2016). In the context of MSEM, our plausible values approach has the benefit of comparatively fast estimation while still allowing higher level product terms to be treated as if they were measured with error (e.g., unlike Leite & Zuo, 2011).

We offer a worked example using the well-known High School and Beyond (HS&B) data set (e.g., Raudenbush & Bryk, 1986, 2002), with

7,185 students nested in 160 schools. The HS&B data and Mplus program code for all models that we estimate can be downloaded from quantpsy.org. With these additional materials, the reader can estimate the models that we specify and modify them to experiment with multilevel moderated mediation.

To help the reader keep track of the parameters in our models, we use familiar mediation notation (as in Baron & Kenny, 1986), with regression coefficients as follows: a is the first path in a mediation relationship, b is the second path in a mediation relationship, c is the total effect of a predictor on an ultimate outcome without controlling for a mediator, and c' is the direct effect of a predictor on an ultimate outcome while controlling for a mediator. Furthermore, where appropriate, we use subscripts that indicate the outcome variable and the predictor variables associated with a regression coefficient. We illustrate this notation next but recommend that the unfamiliar reader first consult primary texts such as Baron and Kenny (1986), MacKinnon (2008), Preacher and Hayes (2004), Preacher et al. (2007), and Hayes (2013).

Our models estimate some effects of being Black in the United States. Given long-running racism and racial segregation in the United States (Bonilla-Silva, 2006), and because race is social and relational (see Lucal, 1996; Smedley & Smedley, 2005; Tsui, Egan, & O'Reilly, 1992), we treat individuals' self-identification as being Black or non-Black as indicating an important racial categorization in society—it matters. To be clear, as with feminist approaches to sex or gender (Haraway, 2006), our use of these terms and this categorization for our analyses is not meant to reify or otherwise reproduce racism. Instead, by showing negative effects of identifying as Black on math achievement both directly and indirectly via socioeconomic status (SES) within and between schools, our goal is to show racial inequalities so that they can be taken seriously and addressed.

In our Discussion section, we describe the benefits of our MSEM approach. They include an improved ability to conceptually reason and hypothesize about multilevel moderated mediation effects. Furthermore, the flexibility of MSEM allows random intercepts and random slopes that can be

used as predictors, outcomes, indicators, mediators, or moderators at higher levels of analysis. With this expanded toolbox, researchers can better conduct research that addresses worldly problems of concern, such as racism.

MODERATED MEDIATION

To preface our discussion of multilevel moderated mediation, we first introduce basic concepts associated with moderation and mediation. We then treat moderated mediation in a single-level SEM framework and offer an empirical example using the HS&B data set.

Moderation

Moderation refers to an interaction or a conditional effect, wherein the effect of a predictor variable x on an outcome variable y varies across the levels of another predictor w (Cohen, Cohen, West, & Aiken, 2013). This kind of effect is usually modeled by forming a product term xw among the two predictor variables as follows (see a conceptual model of this effect in Figure 20.1a; see a more statistically accurate depiction in Figure 20.1b):

$$y_i = v_y + c_{yx}x_i + c_{yw}w_i + c_{yxw}x_iw_i + \varepsilon_{y,i}, \quad (20.1)$$

wherein i is a unit of observation (e.g., an individual student); v is an intercept; each c is a regression coefficient; and ε is a residual. The conditional nature of the effects can be shown by rearranging Equation 20.1, which we do to illustrate the example of x 's effect on y across varying levels of w :

$$y_i = (v_y + c_{yw}w_i) + (c_{yx} + c_{yxw}w_i)x_i + \varepsilon_{y,i}. \quad (20.2)$$

Here, the first parenthetical term is a *simple intercept*, which equals the expected value of y when w takes on a specific value; whereas the second parenthetical term is a *simple slope* of x , which equals the expected value of y when w takes on a specific value (Preacher et al., 2007). To test for moderation, researchers typically examine the statistical significance of c_{yxw} , which is sensible because only if $c_{yxw} \neq 0$ will the coefficient on x in Equation 20.2 detectably deviate from c_x . Furthermore, the statistical significance of

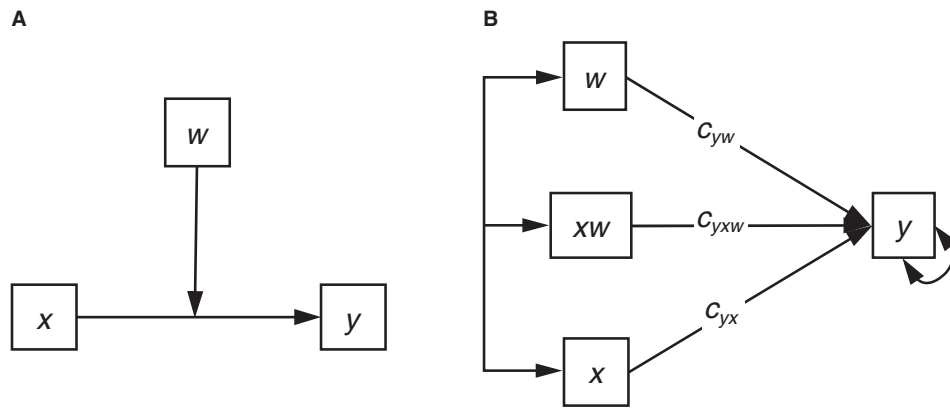


FIGURE 20.1. (a) A conceptual diagram of a moderation model. (b) A path diagram of a moderation model, wherein covariances among predictors are accounted for when deriving coefficients rather than explicitly estimated.

a given simple slope can be computed for any given value of w , or it can be computed continuously across a range of observed w values. All this is typically done on the basis of the standard error (SE) of c_{yxw} as computed under normal theory.

Mediation

By the term *mediation* we mean an indirect effect, such as the effect of x on y that is carried by a mediator m (Cohen et al., 2013). This kind of effect can be shown in equations for y and m as follows (see Figure 20.2):

$$m_i = v_m + a_{mx}x_i + \epsilon_{m,i} \quad (20.3)$$

$$y_i = v_y + b_{ym}m_i + c'_{yx}x_i + \epsilon_{y,i} \quad (20.4)$$

so that with substitution the indirect effect of x is shown to be a product term as follows:

$$y_i = v_y + b_{ym}(v_m + a_{mx}x_i + \epsilon_{m,i}) + c'_{yx}x_i + \epsilon_{y,i}, \quad (20.5)$$

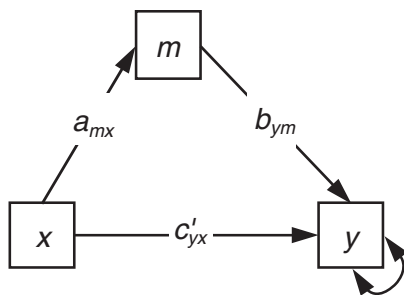


FIGURE 20.2. Path diagram of a mediation model.

which can be rearranged to show a traditional regression model structure as follows:

$$y_i = (v_y + b_{ym}v_m) + (a_{mx}b_{ym} + c'_{yx})x_i + b_{ym}\epsilon_{m,i} + \epsilon_{y,i}, \quad (20.6)$$

wherein the first parenthetical term is an intercept; the second parenthetical term is the total effect of x , composed of an indirect effect $a_{mx}b_{ym}$ and a direct effect c'_{yx} ; and the term $b_{ym}\epsilon_{m,i}$ is the direct effect of m that is independent of x . To test for mediation, researchers often modify traditional tests of statistical significance by estimating a confidence interval (CI) around $a_{mx}b_{ym}$ —although the literature on mediation has historically distinguished between *partial* and *full* mediation (e.g., Baron & Kenny, 1986), we do not reproduce this distinction here and instead focus on indirect effects. Because estimates of such effects are not normally distributed, normal theory does not apply, and therefore, bootstrapping, Monte Carlo, or Bayes procedures are typically used (e.g., Preacher & Hayes, 2008; Preacher & Selig, 2012; Wang & Preacher, 2015; Yuan & MacKinnon, 2009).

Moderated Mediation

The term *moderated mediation* refers to the dependence of an indirect effect on at least one moderator variable w , such that indirect effects are made conditional on values of a moderator (or moderators). Many specifications produce moderated mediation (Hayes, 2013), but a general type can be shown by combining the logic of Equations 20.1, 20.3,

and 20.4 as follows (see Figures 20.3a and 20.3b; see Model 59 in Hayes, 2018):

$$m_i = v_m + a_{mx}x_i + a_{mw}w_i + a_{mxw}x_iw_i + \epsilon_{m,i} \quad (20.7)$$

$$y_i = v_y + b_{ym}m_i + b_{ymw}m_iw_i + c'_{yx}x_i + c'_{yw}w_i + c'_{yxw}x_iw_i + \epsilon_{y,i}, \quad (20.8)$$

such that w moderates the effects of x on m and m on y (i.e., the “first stage” and “second stage” moderated mediation from Edwards & Lambert, 2007), and w moderates the direct effect of x on y (i.e., the traditional form of moderation). The effects involved in this model can be shown by substitution as follows:

$$y_i = v_y + b_{ym}(v_m + a_{mx}x_i + a_{mw}w_i + a_{mxw}x_iw_i + \epsilon_{m,i}) + b_{ymw}(v_m + a_{mx}x_i + a_{mw}w_i + a_{mxw}x_iw_i + \epsilon_{m,i})w_i + c'_{yx}x_i + c'_{yw}w_i + c'_{yxw}x_iw_i + \epsilon_{y,i}, \quad (20.9)$$

which can be rearranged as follows:

$$y_i = \left[v_y + b_{ym}v_m + \left(\begin{matrix} b_{ymw}v_m + a_{mw}b_{ym} \\ + a_{mw}b_{ymw}w_i + c'_{yw} \end{matrix} \right) w_i \right] + [(a_{mx} + a_{mxw}w_i)(b_{ym} + b_{ymw}w_i) + (c'_{yx} + c'_{yxw}w_i)]x_i + (b_{ym} + b_{ymw}w_i)\epsilon_{m,i} + \epsilon_{y,i}, \quad (20.10)$$

which has a similar interpretation as Equation 20.6, such that the first bracketed term is the simple intercept of y , which includes indirect and direct

effects of w ; the second bracketed term is the total effect of x , which is composed of first the indirect effect $(a_{mx} + a_{mxw}w_i)(b_{ym} + b_{ymw}w_i)$ and then the direct effect $(c'_{yx} + c'_{yxw}w_i)$; the first parenthetical term on the third line is the direct effect of m , which is moderated by w ; and the final term is the residual of y .

For the uninitiated reader to fluently understand moderated mediation in Equation 20.10, some explanation is in order. Focusing on the effect of x in the second bracketed term, the moderation coefficients are a_{mxw} and b_{ymw} . Both of these are multiplied by w so that when both coefficients are equal to zero, moderation is not present and Equation 20.10 is more like Equation 20.6 because the indirect effect of x reduces to $a_{mx}b_{ym}$. However, if $a_{mxw} \neq 0$ and/or $b_{ymw} \neq 0$, moderation is present. Specifically, a_{mxw} allows for moderation of the path linking the independent variable and the mediator (i.e., the a_{mx} path) and b_{ymw} allows for moderation of the path linking the mediator and the dependent variable (i.e., the b_{ym} path). But because b_{ym} is part of the indirect effects involving paths a_{mx} and a_{mxw} , both of these paths can be moderated by w when multiplied by b_{ymw} .

As the reader may intuit, there are many ways to specify moderated mediation (see Hayes, 2013, 2015, 2018). Instead of describing the many cases that are possible, we want to offer a general model structure for understanding moderated mediation that can be extended to the multilevel case. We now do this with a general SEM specification (e.g., Edwards & Lambert,

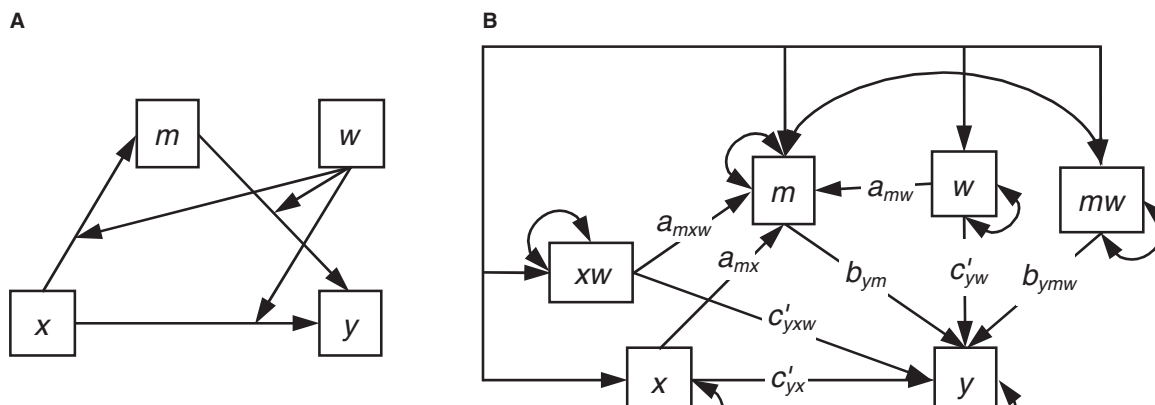


FIGURE 20.3. (a) A conceptual diagram of our single-level moderated mediation model. (b) A path diagram of our single-level moderated mediation model, wherein predictor covariances are explicitly estimated, including a covariance among m and mw . Path coefficients are labeled as in our equations, with a terms indicating initial paths in a mediation/indirect effects equation, b terms indicating second paths in a mediation/indirect effects equation, and c' paths indicating direct effects.

2007; Hayes & Preacher, 2013). Here, and in our multilevel models, we use variants and simplifications of the model in Muthén and Asparouhov (2008) as implemented in *Mplus* (see Muthén & Muthén, 2016; see also Preacher et al., 2010, 2016).

A General Structural Equation Model Specification and Estimation

To begin, we show an “all y ” SEM specification as follows:

$$\mathbf{y}_i = \mathbf{v} + \mathbf{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i \quad (20.11)$$

$$\boldsymbol{\eta}_i = \boldsymbol{\alpha} + \mathbf{B}\boldsymbol{\eta}_i + \boldsymbol{\zeta}_i, \quad (20.12)$$

wherein \mathbf{y}_i is a vector of observed scores on the dependent variables (often called *observed indicators*); \mathbf{v} is a vector of intercepts capturing the mean structure of the data; $\mathbf{\Lambda}$ is a matrix of factor loadings representing the strength and direction of relationships among latent variables and their observed indicators; $\boldsymbol{\varepsilon}_i$ is a vector of residuals with covariance matrix $\boldsymbol{\Theta}$ (typically with unrestricted diagonal elements, i.e., estimated variances); $\boldsymbol{\eta}_i$ is typically a vector of latent variables that are believed to cause the covariance structure of observed indicators, but it may also be used to reflect the actual observed variables if $\mathbf{\Lambda}$ contains unities that link each observed variable with a single latent variable and elements in \mathbf{v} and $\boldsymbol{\Theta}$ are fixed at zero; $\boldsymbol{\alpha}$ is a vector of intercepts or means corresponding to a latent variable mean structure (typically restricted to zero); \mathbf{B} is a matrix of regression coefficients often used to model causal effects among latent variables; and $\boldsymbol{\zeta}_i$ is a matrix of residuals with covariance matrix $\boldsymbol{\Psi}$ (typically with unrestricted diagonal elements, i.e., variances). In the case of latent interactions, $\boldsymbol{\eta}_i$ can be used to stack products of latent variables so that all observed, latent, and product-term variables can be understood as existing in $\boldsymbol{\eta}_i$ —this is useful for concision, reducing the complexity of our equations (for more technical treatments, see Klein & Moosbrugger, 2000; Klein & Muthén, 2007; Preacher et al., 2016).

The result is that in Equations 20.11 and 20.12, all moderation, mediation, and moderated mediation effects are either contained in \mathbf{B} , or they can be constructed from elements in \mathbf{B} . For example, Equations 20.7 and 20.8 can be shown in the form of Equations 20.11 and 20.12 as follows:

$$\mathbf{y}_i = \begin{bmatrix} m_i \\ y_i \\ x_i \\ w_i \\ xw_i \\ mw_i \end{bmatrix} = \mathbf{v} + \mathbf{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \eta_m \\ \eta_y \\ \eta_x \\ \eta_w \\ \eta_{xw} \\ \eta_{mw} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (20.13)$$

and

$$\boldsymbol{\eta}_i = \begin{bmatrix} \eta_m \\ \eta_y \\ \eta_x \\ \eta_w \\ \eta_{xw} \\ \eta_{mw} \end{bmatrix} = \boldsymbol{\alpha} + \mathbf{B}\boldsymbol{\eta}_i + \boldsymbol{\zeta}_i$$

$$= \begin{bmatrix} v_m \\ v_y \\ \alpha_x \\ \alpha_w \\ \alpha_{xw} \\ \alpha_{mw} \end{bmatrix} + \begin{bmatrix} 0 & 0 & a_{mx} & a_{mw} & a_{mxw} & 0 \\ b_{ym} & 0 & c'_{yx} & c'_{yw} & c'_{yxcw} & b_{ymw} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_m \\ \eta_y \\ \eta_x \\ \eta_w \\ \eta_{xw} \\ \eta_{mw} \end{bmatrix} + \begin{bmatrix} \varepsilon_{m,i} \\ \varepsilon_{y,i} \\ \zeta_{x,i} \\ \zeta_{w,i} \\ \zeta_{xw,i} \\ \zeta_{mw,i} \end{bmatrix} \quad (20.14)$$

Here, by constraining the values of ν , Λ , and ϵ_i , Equation 20.13 equates the terms in η_i to the observed variables in y_i . Furthermore, the constraints imposed in Equation 20.14 result in any variables that are both predictors of the same outcome and are not related in \mathbf{B} having an unrestricted relationship in Ψ . Given this ordering of variables in η_i , for example, the matrix Ψ might be shown as

$$\Psi = \begin{bmatrix} \Psi_{m,m} & & & & & & \\ 0 & \Psi_{y,y} & & & & & \\ 0 & 0 & \Psi_{x,x} & & & & \\ 0 & 0 & \Psi_{x,w} & \Psi_{w,w} & & & \\ 0 & 0 & \Psi_{x,xw} & \Psi_{w,xw} & \Psi_{xw,xw} & & \\ \Psi_{m,mw} & 0 & \Psi_{x,mw} & \Psi_{w,mw} & \Psi_{xw,mw} & \Psi_{mw,mw} & \end{bmatrix}, \quad (20.15)$$

wherein the diagonal elements are variances or residual variances, and off-diagonal elements are covariances or residual covariances.

Estimation can be accomplished with a variety of tools. SEM is typically estimated with maximum likelihood, which may be robust to nonnormality and the nonindependence of observations (as in Muthén & Muthén, 2016). However, Bayesian approaches are also possible (see Muthén & Asparouhov, 2012). In all cases, estimators of uncertainty such as CIs for indirect effects should not be based on normal theory.

Single-Level Moderated Mediation: Race in High School and Beyond

Using SEM, any combination of parameters can be estimated and used to examine moderated mediation (e.g., using *Mplus* features such as “model indirect” and “model constraint”; Muthén & Muthén, 2016). To show this, we use the HS&B data (see online *Mplus* files in “Single.Level.Modmed.zip”). For illustrative purposes, we treat the data as single-level and use a sandwich estimator to adjust *SEs* for nonindependence (with “Type=Complex” and “Cluster=school” in *Mplus*). To derive all CIs reported in the following text, we use a Monte Carlo approach that allows estimating CIs in the presence of clustered data

(which presents difficulty for more common nonparametric bootstrapping; see Preacher & Selig, 2012; online *Mplus* files in “MonteCarlo.CI.zip”). Throughout, when *SEs* conform to normal theory, we present *p*-values rather than CIs. We would not normally recommend a single-level approach for these data, but working with this example allows us to contrast single-level results with the multilevel results we present later.

The variables we use are: *student math achievement*, with higher scores implying greater achievement on a standardized test; *student SES*, with higher scores meaning higher parental income, education, occupational attainment, and education-related possessions such as books; *student race*, wherein 1 = Black and 0 = other; and *student gender*, wherein 0 = male and 1 = female. More details about the variables can be found in the publication on the HS&B data by National Opinion Research Center (1980).

We estimate a model wherein math achievement y is a function of SES m and race x , with SES m also being a function of race x . This model allows estimating some effects of being Black in the United States, which can negatively affect math achievement directly and indirectly via SES (Altonji & Blank, 1999; Bertrand & Mullainathan, 2004; Bonilla-Silva, 2006; Steele & Aronson, 1995). Similar negative effects are known in the epidemiology literature, pointing to negative effects of being Black in the United States on health and other outcomes both directly and via SES (Navarro, 1990; Ren, Amick, & Williams, 1999). Furthermore, this is a sensible model in terms of causality because we assume that changes in student math achievement cannot influence parental SES and that changes in math achievement or parental SES cannot influence individual race.

We allow gender w to moderate all relationships (as in Equations 20.7–20.10, 20.13, and 20.14). This moderator is sensitive to different forms of race-based differences for Black men versus Black women (see Galinsky, Hall, & Cuddy, 2013; Hall, Hall, & Perry, 2016; Thomas, Witherspoon, & Speight, 2008; Tomaskovic-Devey, 1993; Wingfield, 2007). Because of such gender differences, the direct and indirect effects of race may differ for males versus

females, and any direct effect of SES may differ. As with race, changes in other study variables cannot influence gender.

Table 20.1 displays model parameters. On the basis of Equation 20.10, we can define multiple effects that will be of interest. These are specified under “model constraint” in the *Mplus* input and shown in Table 20.2. To understand their construction, we formally define them and explain their substantive meaning. Although w is categorical, our logic also works for continuous moderators by choosing relevant values of w (e.g., 1 standard deviation above and below the mean) to make comparisons like those we describe.

First, we define the indirect effect for females, which is the simple slope of the indirect effect when the moderator gender = 1 (i.e., $w = 1$). This is

$$(a_{mx} + a_{mxw} w_i)(b_{ym} + b_{ymw} w_i) = (a_{mx} + a_{mxw})(b_{ym} + b_{ymw}), \quad (20.16)$$

which is the indirect effect of race on math achievement via SES for females and is -1.255 with 95% CI $[-1.634, -.867]$, indicating that Black females have

TABLE 20.1

Single-Level Moderated Mediation Model Parameters

Parameter	Estimate	Standard error	<i>p</i> value
SES parameters			
v_m (SES intercept)	.192	—	—
a_{mx} (race→SES)	-.505	.064	<.001
a_{mw} (gender→SES)	-.117	.039	.002
a_{mxw} (race*gender→SES)	.061	.078	.432
Math achievement parameters			
v_y (MA intercept)	14.31	—	—
b_{ym} (SES→MA)	2.515	.211	<.001
b_{ymw} (SES*gender→MA)	.312	.261	.232
c'_{yx} (race→MA)	-3.012	.368	<.001
c'_{yw} (gender→MA)	-1.466	.251	<.001
c'_{yxw} (race*gender→MA)	.318	.481	.509

Note. MA = math achievement; SES = socioeconomic status.

TABLE 20.2

Single-Level Moderated Mediation Model's Further Calculated Parameters

Parameter name	Lower 2.5%	Estimate	Upper 97.5%	Standard error	<i>p</i> value
Female					
Indirect race effect	-1.634	-1.255	-.867	—	—
Direct race effect	—	-2.694	—	.412	<.001
Total race effect	-4.926	-3.949	-2.947	—	—
SES effect	—	2.827	—	.173	<.001
Male					
Indirect race effect	-1.604	-1.27	-.954	—	—
Direct race effect	—	-3.012	—	.368	<.001
Total race effect	-5.117	-4.282	-3.432	—	—
SES effect	—	2.515	—	.211	<.001
Difference in effects (male–female)					
Indirect race effect	-0.444	-.015	.397	—	—
Direct race effect	—	-.318	—	.481	.509
Total race effect	-1.396	-.333	.719	—	—
SES effect (b_{ymw})	—	-.312	—	.261	.232

Note. Where parameters involve products of coefficients, confidence intervals are generated by Monte Carlo using parameter estimates and their asymptotic covariance matrix with 10,000 draws. SES = socioeconomic status.

lower math achievement due to the effect of their race on their SES. The equivalent conditional indirect effect for males is when gender = 0 (i.e., $w = 0$) or

$$(a_{mx} + a_{mxw} w_i)(b_{ym} + b_{ymw} w_i) = a_{mx} b_{ym}, \quad (20.17)$$

which defines the indirect effect of race on math achievement via SES for males and is -1.27 with 95% CI $[-1.604, -.954]$, indicating that Black males have lower math achievement due to the effect of their race on their SES. In turn, the difference in the indirect effects for men versus women is defined as follows:

$$\begin{aligned} & a_{mx} b_{ym} - (a_{mx} + a_{mxw})(b_{ym} + b_{ymw}) \\ &= -(a_{mxw} b_{ymw} + a_{mxw} b_{ym} + a_{mxw} b_{ymw}). \end{aligned} \quad (20.18)$$

This difference is interpreted as the moderating effect of gender on the indirect effect of race on math achievement via SES while holding the direct effect of race constant. This difference is $-.015$ with 95% CI $[-.444, .397]$, indicating no statistically significant difference between males and females in the indirect effect of race on math achievement via SES—although the point estimate of $-.015$ means the indirect effect of race appears to be slightly more negative for males, the CI centering almost on zero.

Comparing the direct effects of race on math achievement shows a similar pattern. For females ($w = 1$) this effect is

$$c'_{yx} + c'_{yxw} w_i = c'_{yx} + c'_{yxw}, \quad (20.19)$$

which describes the direct effect of race on math achievement for females and is -2.694 ($p < .001$), indicating that Black females have lower math achievement. The effect for males ($w = 0$) is

$$c'_{yx} + c'_{yxw} w_i = c'_{yx}, \quad (20.20)$$

which describes the direct effect of race on math achievement for males and is -3.012 ($p < .001$), indicating that Black males have lower math achievement. In turn, the difference in male versus female direct effects is

$$c'_{yx} - (c'_{yx} + c'_{yxw}) = -c'_{yxw}. \quad (20.21)$$

This difference is interpreted as the conditional interaction between race and gender (because SES

is controlled for) and is $-.318$ ($p = .509$), indicating no statistically significant difference between males and females in the direct effect of race on math achievement.

For total effects, the same logic applies, so that the effect for females ($w = 1$) is

$$\begin{aligned} & (a_{mx} + a_{mxw} w_i)(b_{ym} + b_{ymw} w_i) + (c'_{yx} + c'_{yxw} w_i) \\ &= (a_{mx} + a_{mxw})(b_{ym} + b_{ymw}) + (c'_{yx} + c'_{yxw}), \end{aligned} \quad (20.22)$$

which is the overall effect of race on math achievement for females and is -3.949 with 95% CI $[-4.926, -2.947]$, indicating that for females, race has an overall negative effect on math achievement. For males ($w = 0$) this is

$$\begin{aligned} & (a_{mx} + a_{mxw} w_i)(b_{ym} + b_{ymw} w_i) + (c'_{yx} + c'_{yxw} w_i) \\ &= a_{mx} b_{ym} + c'_{yx}, \end{aligned} \quad (20.23)$$

which describes the overall effect of race on math achievement for males and is -4.282 with 95% CI $[-5.117, -3.432]$, indicating that for males race has an overall negative effect on math achievement. In turn, the difference in these effects for males versus females is

$$\begin{aligned} & a_{mx} b_{ym} + c'_{yx} - [(a_{mx} + a_{mxw})(b_{ym} + b_{ymw}) + (c'_{yx} + c'_{yxw})] \\ &= -(a_{mxw} b_{ymw} + a_{mxw} b_{ym} + a_{mxw} b_{ymw} + c'_{yxw}). \end{aligned} \quad (20.24)$$

This difference is interpreted as the moderating effect of gender on the effect of race on math achievement. This difference is $-.333$ with 95% CI $[-1.396, .719]$, indicating no statistically significant difference between males and females in the overall effect of race on math achievement.

The direct effect of SES can also be estimated. For females ($w = 1$), this is

$$b_{ym} + b_{ymw} w_i = b_{ym} + b_{ymw}, \quad (20.25)$$

which describes the effect of SES on math achievement for females while holding race constant. This effect is 2.827 ($p < .001$), indicating that higher SES for females leads to higher math achievement. The same effect for males ($w = 0$) is

$$b_{ym} + b_{ymw} w_i = b_{ym}, \quad (20.26)$$

which describes the effect of SES on math achievement for males while holding race constant. This effect is 2.515 ($p < .001$), indicating that higher SES for males leads to higher math achievement. In turn, their difference is simply

$$b_{ym} - (b_{ym} + b_{ymw}) = -b_{ymw}, \quad (20.27)$$

which describes the moderating effect of gender on the effect of SES on math achievement while holding race constant. This is $-.312$ ($p = .232$), indicating no statistically significant difference between males and females in the effect of SES on math achievement while holding race constant.

Overall, the pattern of results for race is consistent with the long-running history of racism in the United States. The effects of being Black on math achievement are negative, both indirectly via SES and directly. This is predictable given the substantial literature on racism and its effects both generally (e.g., Bonilla-Silva, 2006) and on standardized test scores specifically (e.g., Steele & Aronson, 1995). However, contrary to what some research may suggest (e.g., Galinsky et al., 2013), we find no moderating effect of gender, even when testing direct effects of SES.

MULTILEVEL MODERATED MEDIATION IN MULTILEVEL STRUCTURAL EQUATION MODELS

Unfortunately, these models and analyses are insensitive to the clustering in our data except that they adjust *SEs*. When children are nested in schools or data are otherwise grouped, different variances and effects are mixed: between group and within group (Cronbach, 1976; Cronbach & Snow, 1977; Cronbach & Webb, 1975; Preacher et al., 2010, 2016; Zhang, Zyphur, & Preacher, 2009). Between-group variances and effects are related to group means, whereas within-group variances and effects are related to deviations away from the means (as in analysis of variance [ANOVA]). In turn, when analyzing data and making inferences, within-group terms represent individuals (e.g., students) and between-group terms represent groups (e.g., schools). To motivate this style of representation, we first justify it as follows.

Motivating the Study of Race Between and Within Schools

Our position (e.g., Preacher et al., 2010, 2011, 2016; Zhang et al., 2009) is that by ignoring clustering, single-level analyses create “uninterpretable blends” of variances and effects that are attributable to different kinds of things—students versus schools (Cronbach, 1976, p. 9.20). Furthermore, differences in the magnitudes of these terms across levels of analysis “occur with considerable regularity” (Raudenbush & Bryk, 2002, p. 140), and these differences have critical implications for substantive interventions that might be designed to target entire groups (e.g., schools) versus the individuals residing within them. The job of the data scientist who endeavors to inform intervention or policy planning is to decompose variances and effects so that inferences about different kinds of entities can be unambiguously made in light of the clustered structure of a data set.

Despite the long recognition of the need to decompose level-specific effects (e.g., Dansereau & Yammarino, 2000), there is some debate regarding this point, especially for mediation and moderation analyses (e.g., Pituch & Stapleton, 2012; Tofighi & Thoemmes, 2014). Therefore, we clarify our views and then connect them to the case of multilevel moderated mediation to study race in the HS&B data. First, consider a data set with information from students in a single school $j = 1$ and a simple bivariate model:

$$y_{i1} = \nu + \beta x_{i1} + \varepsilon_{i1}. \quad (20.28)$$

Here, β is the effect of x on y for all students in school $j = 1$. As with all single-level regression models, we and most other researchers would refer to β as an individual-level effect. However, what researchers mean is that β is a within-school effect, which becomes clear by rewriting Equation 20.28 in terms of variances, covariances, and means as follows:

$$y_{i1} = \mu_y + \frac{\sigma_{xy}}{\sigma_x^2}(x_{i1} - \mu_x) + \varepsilon_{i1}, \quad (20.29)$$

wherein μ terms are means, σ_{xy} is x - y covariance, and σ_x^2 is the variance of x .

As Equation 20.29 shows, group means play no part in deriving β in single-level analyses—the means for y and x could be changed by any value and β would be unchanged. Also, statistical inference with SEs is a function of ϵ_{i1} , which is a within-school term. In turn, so-called *individual-level* analyses with data from a single group estimate within-group effects, meaning that when researchers make inferences about individuals, they have all along been making within-group inferences (i.e., inferences about individuals that are always relative to the mean of the group). Therefore, consistent with typical regression practices, we recommend using within-group terms to make inferences about individuals.

To further illustrate this point, consider the HS&B data with N students in J schools. To make inferences about students, any confounding “unobserved heterogeneity” associated with schools should be controlled. This kind of “fixed-effects” model—in econometrics terms—can be constructed by creating $J-1$ indicator variables (e.g., dummy codes) in a vector \mathbf{z}_j as follows:

$$y_{ij} = \nu + \beta x_{ij} + \delta'_j \mathbf{z}_j + \epsilon_{ij}. \quad (20.30)$$

Here, the effect of school membership is accounted for by the coefficients in the vector δ_j and the effect of x on y for students is still β , which is a within-group term, as is ϵ_{ij} (which is used for statistical inference with SEs). Furthermore, by accounting for the “school effects” δ_j , what is really occurring is that the school means are being entirely accounted for, which is to say that δ_j is accounting for all between-school variance, which is associated with schools—to emphasize, in this kind of fixed-effects model, there is no remaining between-school variance that can be accounted for with any additional predictors. In other words, when attempting to make inferences about individuals while controlling for group effects, researchers focus on within-group variance to make inferences about individuals and control for between-group variance to model the effect of groups, as is done when “within-group centering” data by eliminating group means (Preacher et al., 2010).

Moreover, to motivate inferences about groups by using between-group variances and effects, consider

an experiment wherein researchers randomly assign participants to a control group $x_{j=1}$ or an experimental group $x_{j=2}$. To make an inference about the effect of interest, researchers must model the effect of group membership, which can be shown as follows:

$$y_{ij} = \nu + \beta x_j + \epsilon_{ij}, \quad (20.31)$$

wherein the model now reflects terms for an individual i and a group j , with x coding for group membership. Here, the effect of interest is β , which in an ANOVA framework is well known as a between-group effect, capturing the difference between the two group means. Here, researchers do not substantively care about ϵ_{ij} because this within-group term is typically regarded as being due to individual or subject-specific effects.

In all these cases—typical regression models, fixed-effects regression controlling for group effects, and ANOVA—researchers always make inferences about individuals using within-group variances or effects that model deviations away from group means, and inferences are made about groups using between-group variances or effects that model group means. Therefore, in all our models we decompose the between- and within-group parts of any observed variables measured at the “individual” level, so accurate inferences can be made about the appropriate kinds of things that are being assessed (e.g., students, schools, communities). Such decomposition of level-specific effects has long been recommended in the literature (e.g., Cronbach, 1976; Cronbach & Webb, 1975; Dansereau & Yammarino, 2000; Kozlowski & Klein, 2000), but this work is often overlooked.

For studying race, separating student versus school effects is key because different processes can influence students within schools versus schools as wholes (Benner & Graham, 2013), especially because schools are typically defined by local environments such as neighborhoods. Although Black students in a school may experience individualized forms of racism as noted previously (motivating a focus on within-school effects), there is evidence that collective “institutional” racism has profound effects. For example, formal and informal segregationist agendas in the United States drove Black individuals into poor and blighted neighborhoods (Massey &

Denton, 1993; Seitles, 1998; Williams & Collins, 2001)—an infamous example is the design of low bridge overpasses to keep Black bus passengers from crossing into upper-class White neighborhoods (Caro, 1974). In addition to being excluded from important social capital, institutional racism has had profound effects, including poorer nutrition, education, and employment rates, as well as community problems that make life unstable, stressful, and emotionally hard (Seaton & Yip, 2009; Umaña-Taylor, 2016; Williams, 1999; Williams & Williams-Morris, 2000).

In turn, institutional racism causes covariance between racial composition, such as the proportion of Black students in a school, and the collective testing outcomes at a school. This effect may operate directly, but it can also function indirectly through collective SES, which further reflects the problems of poorer neighborhoods and schools (e.g., Pickett & Pearl, 2001). As a moderator, gender composition could influence these effects because of the different ways that Black males and females are collectively treated (Hall et al., 2016; Wingfield, 2007).

Therefore, it is reasonable to decompose the between- versus within-school parts of observed variables to examine collective versus individual effects of race. To do so, we now introduce multilevel approaches to moderation, mediation, and moderated mediation.

Multilevel Moderation

To understand MSEM for the purposes of multilevel moderated mediation analyses, we begin by extending moderation, mediation, and moderated mediation models from Equations 20.1, 20.3, 20.4, 20.7, and 20.8 to the multilevel case. In these models, observed variables such as y will typically reflect within- and between-group components when data are clustered or otherwise nested. These components can be decomposed as follows:

$$y_{ij} = y_{Bj} + y_{Wij}, \quad (20.32)$$

wherein a B subscript indicates a between-group part (e.g., a school mean, sometimes referred to as a *random intercept*) and a W subscript indicates a within-group part (e.g., a student's relative standing after subtracting the school mean).

In turn, the moderation model in Equation 20.1 can be reformulated by decomposing the B and W parts of all relevant variables as follows (for concision, we omit random slopes as regression coefficients that vary across groups, which are possible in our MSEM approach):

$$y_{Bj} = \mathbf{v}_{By} + c_{Byx}x_{Bj} + c_{Byw}w_{Bj} + c_{Byxw}x_{Bj}w_{Bj} + \boldsymbol{\varepsilon}_{By,j} \quad (20.33)$$

$$y_{Wij} = c_{Wyx}x_{Wij} + c_{Wyw}w_{Wij} + c_{Wyxw}x_{Wij}w_{Wij} + \boldsymbol{\varepsilon}_{Wy,ij}, \quad (20.34)$$

wherein all terms are as before, but B terms denote between-school variables or effects and W terms denote within-school variables or effects. Notice that the intercept for y in Equation 20.33 is a B term, which is consistent with our arguments related to Equations 20.28 to 20.31. Notice also that the product terms $x_{Bj}w_{Bj}$ and $x_{Wij}w_{Wij}$ are not $(xw)_{Bj}$ and $(xw)_{Wij}$, because the latter implies first multiplying x and w and then decomposing the B and W parts of the product term, which is not the same as multiplying the B and W components (Preacher et al., 2016).

The point of Equations 20.33 and 20.34 is that the B coefficients can be used to make inferences about groups (e.g., schools), and the W coefficients can be used to make inferences about individuals, who are by design nested in the groups. The moderation effects c_{Byxw} and c_{Wyxw} have the same interpretation as previously, except they apply to moderation of B and W effects—similar operations as in Equation 20.2 can define B and W moderation (Preacher et al., 2016).

Multilevel Mediation

The same is true for the mediation model in Equations 20.3 and 20.4, which in a multilevel framework would be

$$m_{Bj} = \mathbf{v}_{Bm} + a_{Bmx}x_{Bj} + \boldsymbol{\varepsilon}_{Bm,j} \quad (20.35)$$

$$y_{Bj} = \mathbf{v}_{By} + b_{Bym}m_{Bj} + c'_{Byx}x_{Bj} + \boldsymbol{\varepsilon}_{By,j} \quad (20.36)$$

$$m_{Wij} = a_{Wmx}x_{Wij} + \boldsymbol{\varepsilon}_{Wm,ij} \quad (20.37)$$

$$y_{Wij} = b_{Wym}m_{Wij} + c'_{Wyx}x_{Wij} + \boldsymbol{\varepsilon}_{Wy,ij}, \quad (20.38)$$

wherein all terms are as before and with similar interpretations, except the B parts apply to groups (e.g., schools) and the W parts apply to individuals,

who are nested within groups. Furthermore, the same logic of mediation exists, with B and W indirect effects being $a_{Bmx}b_{Bym}$ and $a_{Wmx}b_{Wym}$, respectively—the reader can perform the same operations as in Equations 20.5 and 20.6 for both B and W mediation (Preacher et al., 2010).

Multilevel Moderated Mediation

Multilevel moderated mediation implies the same straightforward extension to the B and W case, so that Equations 20.7 and 20.8 become (again, for concision, we omit random slopes)

$$m_{Bj} = v_{Bm} + a_{Bmx}x_{Bj} + a_{Bmw}w_{Bj} + a_{Bmxw}x_{Bj}w_{Bj} + \epsilon_{Bm,j} \quad (20.39)$$

$$y_{Bj} = v_{By} + b_{Bym}m_{Bj} + b_{Bymw}m_{Bj}w_{Bj} + c'_{Byx}x_{Bj} + c'_{Byw}w_{Bj} + c'_{Byxw}x_{Bj}w_{Bj} + \epsilon_{By,j} \quad (20.40)$$

$$m_{Wij} = a_{Wmx}x_{Wij} + a_{Wmw}w_{Wij} + a_{Wmxw}x_{Wij}w_{Wij} + \epsilon_{Wm,ij} \quad (20.41)$$

$$y_{Wij} = b_{Wym}m_{Wij} + b_{Wymw}m_{Wij}w_{Wij} + c'_{Wyx}x_{Wij} + c'_{Wyw}w_{Wij} + c'_{Wyxw}x_{Wij}w_{Wij} + \epsilon_{Wy,ij}, \quad (20.42)$$

wherein moderated mediation has the same familiar form, except with separate B and W parts to allow inference to groups and individuals. In turn, the same operations we used to explain moderated mediation after Equation 20.8 apply to Equations 20.39 to 20.42, with B indirect and direct effects for groups as $(a_{Bmx} + a_{Bmxw}w_{Bj})(b_{Bym} + b_{Bymw}w_{Bj})$ and $(c'_{Byx} + c'_{Byxw}w_{Bj})$, respectively, and W indirect and direct effects for individuals as $(a_{Wmx} + a_{Wmxw}w_{Wij})(b_{Wym} + b_{Wymw}w_{Wij})$ and $(c'_{Wyx} + c'_{Wyxw}w_{Wij})$, respectively. Here, the reader can apply the same logic as with single-level analyses, keeping in mind that B effects apply to groups and W effects apply to individuals.

A General Multilevel Structural Equation Model Specification and Estimation

To estimate terms in Equations 20.33 to 20.42 with B and W moderated mediation parameters that mirror those in Equations 20.16 to 20.27 (and Table 20.1 and 20.2), we extend the SEM in Equations 20.11 to 20.15 to the multilevel case.

We do this succinctly as follows, but we note that the interested reader can consult complementary treatments in Preacher et al. (2010, 2011, 2016):

$$y_{ij} = \Lambda \eta_{ij} \quad (20.43)$$

$$\eta_{ij} = \alpha_j + B_j \eta_{ij} + \zeta_{ij} \quad (20.44)$$

$$\eta_j = \mu + \beta \eta_j + \zeta_j, \quad (20.45)$$

wherein the meaning of terms differs from Equations 20.11 to 20.15.

In Equation 20.43, y_{ij} is a vector of observed variables; Λ is a matrix indicating whether variables vary within groups, between groups, or both; and η_{ij} is a vector of latent variables that vary either within or between groups. For example, Equation 20.32 can be formulated as Equation 20.43 to clarify its meaning as follows:

$$y_{ij} = \Lambda \eta_{ij} = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} y_{Bj} \\ y_{Wij} \end{bmatrix} = y_{Bj} + y_{Wij}, \quad (20.46)$$

wherein the two elements in Λ for y_{ij} indicate that it has W and B parts, contained in η_{ij} .

In turn, Equation 20.44 contains the B part of observed variables in an intercept vector α_j (again, these are sometimes referred to as *random intercepts*), the W effects among the W parts of observed variables in a matrix B_j , and W residuals in a vector ζ_{ij} . In other words, the W structural model is in B_j , and therefore, B_j will contain W moderated mediation terms, such as those in Equations 20.41 and 20.42.

Alternatively, Equation 20.44 contains B model parts as follows: η_j contains all B variables of interest from η_{ij} , but this is done by stacking the B intercepts in α_j as well as any random slopes from B_j ; μ is a vector of intercepts or grand means; β is a matrix of B effects; and ζ_j is a vector of B residuals. In other words, the B structural model is in β , and therefore, β will contain B moderated mediation terms, such as those in Equations 20.39 and 20.40.

Hopefully, by now the reader can infer that multilevel moderated mediation merely requires applying familiar single-level concepts to the W and B model parts in Equations 20.44 and 20.45. However, before proceeding, there are a few caveats to mention. First, MSEM allows for more flexibility

than we can cover here, such as random slopes (e.g., for W effects in B_j , which would be stacked in η_j). Such slopes allow cross-level interactions, which may be a useful complement to the multilevel moderated mediation we examine here. The interested reader can easily pursue this using our logic and that in Preacher et al. (2010, 2016), which discusses special issues related to the use of random slopes and cross-level interactions.

Second, because the B and W parts of an observed variable are latent, they must be estimated. This estimation can be done by calculating school averages, but as in most multilevel models (e.g., Raudenbush & Bryk, 2002), MSEM does this using an empirical Bayes approach to account for sampling error. However, in some cases, this may be unwarranted, and researchers may prefer to compute group means for B parts as if they were observed (see Lüdtke et al., 2008; Lüdtke, Marsh, Robitzsch, & Trautwein, 2011; Marsh et al., 2009; Preacher et al., 2016). For HS&B data, both individuals and schools were randomly sampled, and therefore, empirical Bayes estimation is warranted.

Third, in Equations 20.33 and 20.34, we noted that $x_{Bj}w_{Bj}$ and $x_{wij}w_{wij}$ are not $(xw)_{Bj}$ and $(xw)_{wij}$. The implication for moderated mediation in MSEM is that the reader cannot simply compute observed product terms for interacting variables such as $x_{ij}w_{ij}$ and then specify these in y_{ij} from Equation 20.43. The reason is that MSEM decomposes the W and B parts of observed variables to account for uncertainty associated with sampling error (as implied by Equation 20.46).

To estimate W and B moderation requires computing product terms for latent B and W variables separately, as in $x_{Bj}w_{Bj}$ and $x_{wij}w_{wij}$ (Preacher et al., 2016). This is done in *Mplus* by putting a latent variable “behind” a set of B and W parts of observed variables and then forming product terms as latent variable interactions—which Preacher et al. (2016) specified in their online supplemental material. Figure 20.4 shows this, wherein the interacting B and W parts of observed variables are treated as latent, with the variances of their B and W parts fixed to .01 to facilitate convergence (as in Preacher et al., 2016). This allows estimating the

product terms required for multilevel moderation of various kinds.

Fourth, the recommended approach to latent interactions from Preacher et al. (2016) uses LMS, which is implemented in *Mplus* (see Klein & Moosbrugger, 2000; Klein & Muthén, 2007). However, this approach can encounter serious difficulties with convergence in the case of high-dimensional numerical integration. For example, the model in Figure 20.4 using the HS&B data was not estimable with adequate dimensions of numerical integration, and we could not achieve convergence using the “Integration=Montecarlo” approach to this integration as shown in Preacher et al.’s (2016) online supplemental material (the reader is invited to experiment with the *Mplus* input in “Multilevel.ModMed.zip”). This is because our model has many latent variables (i.e., three B terms, three W terms, as well as their associated latent product terms), which is not surprising when combining the approaches of Preacher et al. (2016) with that of Preacher et al. (2010). Such complexity is to be expected with multilevel moderated mediation (consider multilevel versions of Hayes, 2018, which could even include many random slopes of W model parts). Indeed, such complexity is to be expected even in the single-level case of moderated mediation with latent variables.

Therefore, to avoid numerical integration, we use a Bayesian plausible values approach to latent variable interactions in *Mplus* (see Asparouhov & Muthén, 2010a, 2010b, 2010c). This approach uses Bayesian estimation with default “diffuse” or “uninformative” prior probabilities to approximate maximum-likelihood estimation (see Muthén & Asparouhov, 2012). The key to this estimation is that it allows generating a Bayesian analog of factor scores for latent variables by sampling from their posterior distribution some number of times (20 in our case; see Mislavy et al., 1992, and Von Davier, Gonzalez, & Mislavy, 2009). Interestingly, this is equivalent to a multiple imputation method with latent variables treated as missing data, which overcomes the need for estimating latent variables and their interactions directly, which requires computationally difficult numerical integration.

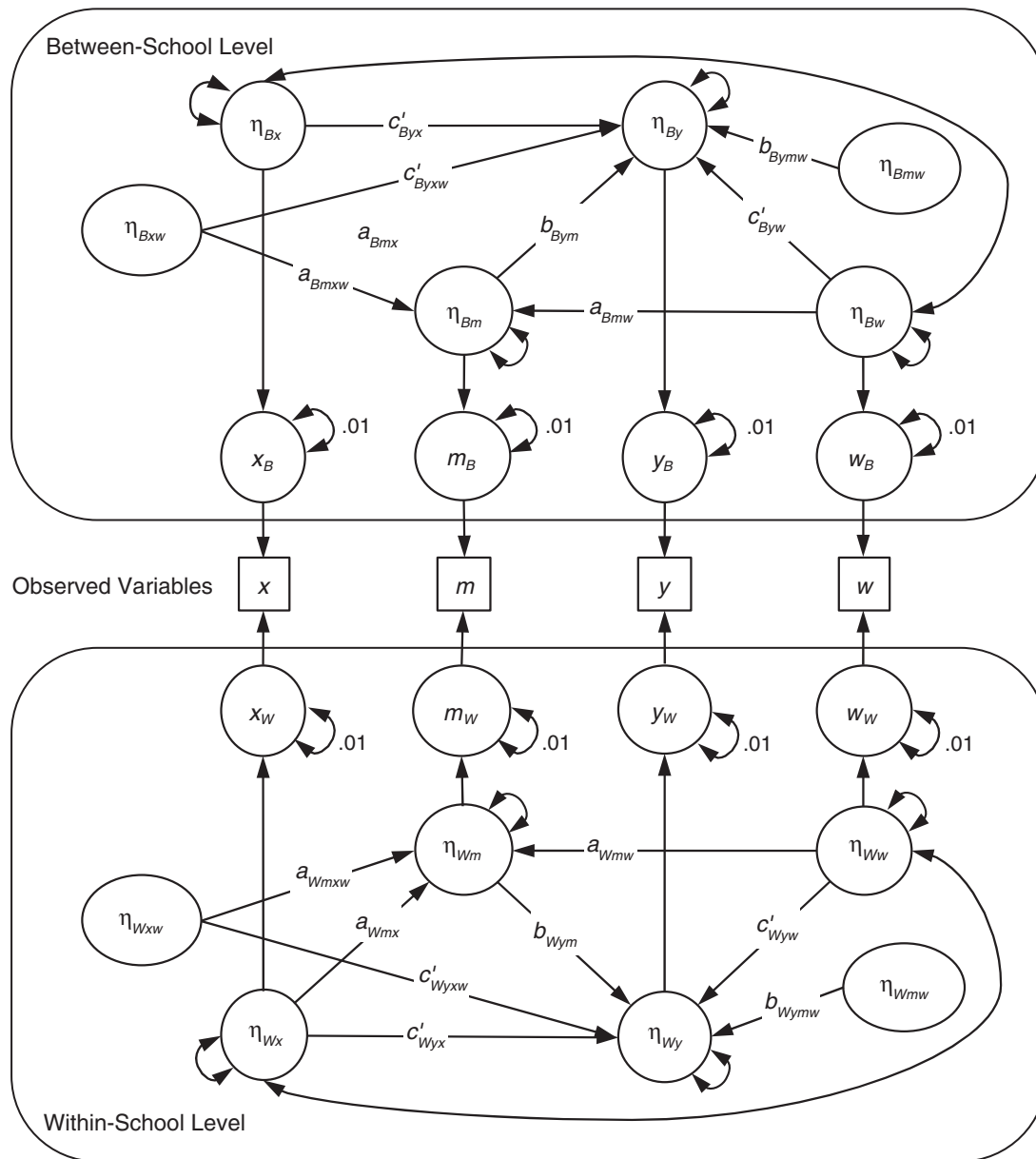


FIGURE 20.4. A path diagram of our multilevel moderated mediation model.

Although research on multiple imputation shows that interactions or nonlinear effects should be used for imputing observed data (e.g., Bartlett et al., 2015; Seaman, Bartlett, & White, 2012; von Hippel, 2009), including in the multilevel case (Goldstein, Carpenter, & Browne, 2014), our approach imputes missing unobserved variables rather than observed variables, and therefore, our method should capture some of the interaction and nonlinear patterns that are a function of the observed, nonmissing data. In the multilevel

case, we expect that latent W and B scores can be accurately estimated even without including latent products in the model—although there are conditions for this being the case, including having no missing data (or few missing data) along observed variables (for insight, see the previous citations). After generating the plausible values in a first step, a maximum likelihood procedure is used to estimate parameters and compute model fit in a second step (Asparouhov & Muthén, 2010a, 2010b; Enders, 2010), with estimates averaged

across the plausible values and *SEs* adjusted for the uncertainty they indicate (as in Rubin, 1987; Schafer, 1997).

Our approach has two steps. Step 1: Plausible values are generated in *Mplus* by estimating the model of interest without latent interactions, using a Bayes estimator with default diffuse/uninformative prior probability distributions (see *Mplus* files in “Multilevel.ModMed.Plausible.Values.zip”). Step 2: A model is estimated using plausible values as if they were multiple imputations (e.g., Rubin, 1987; Schafer, 1997), using a typical maximum-likelihood based approach, with product terms computed for the plausible values to approximate latent interactions (see Figure 20.5). This allows treating all *B* and *W* model parts as if they were observed, with uncertainty in latent variable values treated as variation across the multiple imputations (i.e., differences in the plausible values). To capture

this uncertainty we use 20 imputations, which is a common number for multiple imputations (e.g., Rubin, 1987; Schafer, 1997). As the reader can grasp by experimenting with the full multilevel model and the two-step plausible values approach, the latter drastically simplifies the estimation of models involving latent interactions.

Fifth, and finally, because testing mediation with indirect effects cannot use *SEs* derived from normal theory, alternative approaches are recommended that we previously described. In the multilevel case with latent interactions, the situation is also complicated (see Zyphur, Zammuto, & Zhang, 2016). Therefore, we use a Monte Carlo approach wherein parameter estimates and their asymptotic covariance matrix are used to generate 10,000 estimates of effects (as in Zyphur et al., 2016; see online *Mplus* files in “MonteCarlo.CI.zip”), which are then used to estimate CIs empirically.

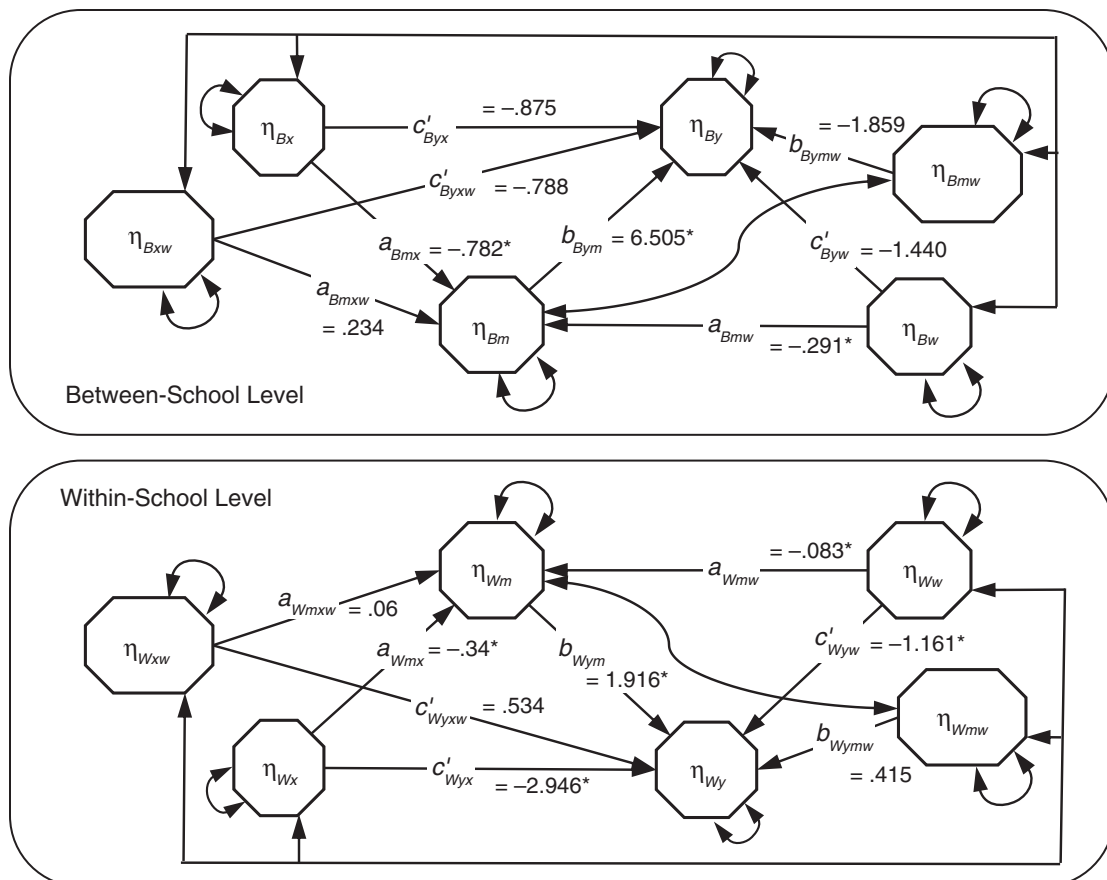


FIGURE 20.5. A path diagram of our multilevel mediation model with all latent variables as plausible values (i.e., multiply imputed), which we represent as octagons.

Between- and Within-School Race in High School and Beyond

Tables 20.3 and 20.4 display *B* and *W* model parameters, respectively (see also Figure 20.5). On the basis of Equation 20.10, *B* and *W* moderated mediation effects can be defined for Equations 20.16 to 20.27 (as applied to Equations 20.39–20.42). These are specified under “model constraint” in our online *Mplus* files and shown in Tables 20.5 and 20.6 for *B* and *W* parameters, respectively. Because of the many results that exist in these tables, we summarize them for the sake of concision as follows.

The first thing to observe is that the *W* effects for students follow the same pattern as the single-level results—the direct and indirect effects of race have CIs that do not contain zero, and there is no moderation by gender. This similarity is expected because single-level analyses allow *W* parameters to dominate when there are many more units of observation at the *W* level compared with the *B* level and when *W* variances are larger than *B* variances (Preacher et al., 2010, 2016). For findings on the effects of race, the implication is that for any given student, being Black in the United States is harmful for math achievement directly and by having a negative effect on SES—again, both effects are

TABLE 20.3

Within-Level Moderated Mediation Model Parameters

Parameter	Estimate	Standard error	<i>p</i> value
SES effects			
a_{Wmx} (race→SES)	-.34	.037	<.001
a_{Wmw} (gender→SES)	-.083	.018	<.001
a_{Wmxw} (race*gender→SES)	.06	.068	.377
Math achievement effects			
b_{Wym} (SES→MA)	1.916	.118	<.001
b_{Wymw} (SES*gender→MA)	.415	.277	.134
c'_{Wyx} (race→MA)	-2.946	.259	<.001
c'_{Wyw} (gender→MA)	-1.161	.183	<.001
c'_{Wyxw} (race*gender→MA)	.534	.514	.299

Note. MA = math achievement; SES = socioeconomic status.

TABLE 20.4

Between-Level Moderated Mediation Model Parameters

Parameter	Estimate	Standard error	<i>p</i> value
SES effects			
v_{Bm} (SES intercept)	.324	—	—
a_{Bmx} (race→SES)	-.782	.222	<.001
a_{Bmw} (gender→SES)	-.291	.166	<.001
a_{Bmxw} (race*gender→SES)	.234	.39	.149
Math achievement effects			
v_{By} (MA intercept)	13.761	—	—
b_{Bym} (SES→MA)	6.505	.99	<.001
b_{Bymw} (SES*gender→MA)	-1.859	1.46	.203
c'_{Byx} (race→MA)	-.875	1.279	.494
c'_{Byw} (gender→MA)	-1.440	1.085	.184
c'_{Byxw} (race*gender→MA)	-.788	2.105	.708

Note. MA = math achievement; SES = socioeconomic status.

consistent with the long-running history of racism in the United States. However, gender seems to make little difference in these effects (although Table 20.3 shows that gender does have a sizable effect on math achievement, though this effect is smaller than the effect of race).

The *B* effects for schools, however, tell a different story. The upper panel of Table 20.6 shows results for all-female schools. The indirect effect of race on math achievement via SES for a school is -2.553 when comparing a school of Black females versus a school of non-Black females (with a CI not containing zero). However, the direct effect of race on math achievement for a school is -1.664 when comparing a school of Black females versus a school of non-Black females (with a CI containing zero). We have two important notes regarding this result. First, because variables often have distinctive meanings, varying measurement metrics and different reliabilities at *W* versus *B* levels (Bliese, 2000; Kozlowski & Klein, 2000), their path coefficients and the associated indirect effects are not readily comparable across levels. For example, the *B* indirect effect of -2.553 is not directly comparable to the same *W* indirect effect of -.652 because the units of analysis are different. This said, at each level, the same variable’s direct

TABLE 20.5

Within-Level Moderated Mediation Model's Further Calculated Parameters

Parameter name	Lower 2.5%	Estimate	Upper 97.5%	Standard error	p value
Female					
Indirect race effect	-1.06	-0.652	-.305	—	—
Direct race effect	—	-2.412	—	.507	<.001
Total race effect	-4.218	-3.064	-1.915	—	—
SES effect	—	2.332	—	.300	<.001
Male					
Indirect race effect	-.824	-.651	-.497	—	—
Direct race effect	—	-2.946	—	.259	<.001
Total race effect	-4.164	-3.597	-3.032	—	—
SES effect	—	1.916	—	.118	<.001
Difference in effects (male–female)					
Indirect race effect	-.319	.001	.367	—	—
Direct race effect	—	-.534	—	.514	.299
Total race effect	-1.563	-.533	.476	—	—
SES effect (b_{Wymw})	—	.415	—	.277	.134

Note. Where parameters involve products of coefficients, confidence intervals are generated by Monte Carlo using parameter estimates and their asymptotic covariance matrix with 10,000 draws. SES = socioeconomic status.

TABLE 20.6

Between-Level Moderated Mediation Model's Further Calculated Parameters

Parameter name	Lower 2.5%	Estimate	Upper 97.5%	Standard error	p value
Female					
Indirect race effect	-4.145	-2.553	-.893	—	—
Direct race effect	—	-1.664	—	1.105	.132
Total race effect	-6.512	-4.217	-1.475	—	—
SES effect	—	4.646	—	.713	<.001
Male					
Indirect race effect	-7.357	-5.077	-2.913	—	—
Direct race effect	—	-.875	—	1.279	.494
Total race effect	-8.452	-5.953	-3.151	—	—
SES effect	—	6.505	—	.99	<.001
Difference in effects (male–female)					
Indirect race effect	-5.907	-2.524	.704	—	—
Direct race effect	—	.788	—	2.105	.708
Total race effect	-6.09	-1.736	2.698	—	—
SES effect (b_{Bymw})	—	-1.859	—	1.46	.203

Note. Where parameters involve products of coefficients, confidence intervals are generated by Monte Carlo using parameter estimates and their asymptotic covariance matrix with 10,000 draws. SES = socioeconomic status.

and indirect effects can be compared and the relative magnitude of these effects can be informative (e.g., the *W* direct effect for females is -2.412 and the *W* indirect effect is $-.652$). Also, the level-specific ratios of direct versus indirect effects are not influenced by measurement metrics and can be compared across *B* and *W* levels. Second, we used theoretically extreme values for the *B*-level moderator's high versus low conditions (i.e., a school's gender composition can be 100% female or 100% male). We similarly used extreme values for the school-level race variable such that a school can be 100% Black or 100% non-Black. Using extreme values helps our interpretation of the conditional indirect effects given the categorical predictor and moderator variables. When a continuous moderator is examined, the more conventional approach is to use one level-specific standard deviation above and below its level-specific mean to calculate indirect effects at *B* and *W* levels. Overall, these findings show that for all-female schools, school-level race appears to influence school-level math achievement via school-level SES. This finding might be driven by neighborhood-level and school-level variables related to poverty. However, there is no direct effect of school-level race on school-level math achievement.

The same pattern of findings holds for Black males in all-male schools. As shown in the middle panel of Table 20.6, the indirect effect of school-level race on school-level math achievement via school-level SES for a school consisting solely of Black males is -5.077 when comparing a school of Black males versus a school of non-Black males (with a CI not containing zero). However, the direct effect yields a different conclusion, namely that the effect of school-level race on school-level math achievement for a school composed solely of Black males is $-.875$ when compared with a school of non-Black males (with a CI including zero). Therefore, again, for an entire male-only school, the effect of race appears to operate only indirectly via SES. Comparing the all-male school versus all-female school conditions, the difference in indirect effects has a confidence interval including zero (i.e., $[-5.907, .704]$). Therefore, school-level gender does not moderate the indirect effect of race on math achievement via school-level SES.

In sum, these results indicate that being a Black student in the United States has negative effects on math achievement, both directly and indirectly via SES, with the direct effects being stronger than those via SES. However, for schools as wholes, this is not the case. Consistent with centuries of institutional racism and other forms of injustice that cause a correlation between race and SES at the neighborhood and community levels (Bonilla-Silva, 2006), lower test scores for all-Black schools (regardless of gender) appear to be entirely due to SES. The implication is that, at the school level, the poverty and socioeconomic exclusion associated with racial differences may explain the effect of race on a school's math achievement. Overall, to address differences in test scores, the United States—like other countries—must do more to create racial equality by reducing the relationship between racial categories and important outcomes and socioeconomic resources.

DISCUSSION

We have described a novel approach for investigating multilevel moderated mediation using MSEM. Both conceptually and by example, we explored just one of the many possibilities for estimating these models, including a novel plausible values approach that avoids numerical integration—which would have otherwise derailed our analyses. Future work can explore specific cases that include random slopes and variables that vary only at the between-groups levels of analysis (see thorough treatments in Preacher et al., 2010, 2016). In all cases, the between- and within-groups parts of observed variables can be used to make inferences to higher versus lower level entities.

Our plausible values approach can be used for any model wherein latent interactions or power polynomials would otherwise require numerical integration and observed data provide adequate information to estimate latent standings. Therefore, this might be useful for single-level models that rely on LMS or other computationally intensive methods (e.g., Sardeshmukh & Vandenberg, 2016). Our plausible values approach has the benefit of allowing comparatively fast estimation when the

number of multiple imputations is not overly large, but it does require various conditions being met that we will explore in a future paper. However, we do offer a word of caution when using asymptotic (co)variances of parameter estimates to compute CIs with a Monte Carlo procedure, as we use here. Because these covariances are meant to be asymptotic with similar estimation assumptions as a maximum likelihood estimator, researchers may be motivated to produce many multiple imputations, perhaps 1,000 or 10,000. For the purpose of our example and to make our online supplemental materials easier to download, we limited the number of imputations to 20, which is common in the multiple imputation and plausible values literatures (e.g., Mislevy et al., 1992; Rubin, 1987; Schafer, 1997; Von Davier et al., 2009).

Beyond concerns regarding the number of imputations, there are additional limitations with our approach and inferences that should be recognized. First, our data are from 1979 and therefore may no longer be a good representation of the United States in various ways, including the Black and non-Black composition of schools. Second, related to this composition, the inferences we make at the school level are based on some extrapolations from our data (i.e., theoretically extreme values for school-level race and school-level gender). Although there are schools in our sample and in the United States wherein students are almost all Black or all non-Black (indeed, the overall trend of this segregation is getting worse rather than better in public schools; see Frankenberg & Lee, 2002), no schools in our sample were composed of all-Black males or all-Black females. Therefore, our inferences regarding moderated mediation warrant some caution at the school level because they cannot be clearly mapped onto observed ranges in the data (this is a general problem for interpreting and reporting effects that involve moderation; see Hayes, 2013; Hayes & Preacher, 2013).

In conclusion, whether estimating multilevel moderated mediation or other effects, we hope that we have shown the potential power of statistical modeling to produce images of people and society that can motivate practical action. In terms of race, it is clear that the United States

and other nations have a long way to go before justice or equality will be realized, and therefore, additional steps should be taken to eliminate racial inequalities. In our view, this should be the goal of statistical analyses: to motivate changes that make a difference.

References

- Altonji, J. G., & Blank, R. M. (1999). Race and gender in the labor market. In O. Ashenfelter & D. Card (Eds.), *Handbook of labor economics* (3rd ed., pp. 3143–3259). New York, NY: Elsevier.
- Asparouhov, T., & Muthén, B. (2010a). *Chi-square statistics with multiple imputation* (Version 2). Retrieved from <https://www.statmodel.com/download/MI7.pdf>
- Asparouhov, T., & Muthén, B. (2010b). Multiple imputation with *Mplus*. Retrieved from <https://www.statmodel.com/download/Imputations7.pdf>
- Asparouhov, T., & Muthén, B. (2010c). Plausible values for latent variables using *Mplus*. Retrieved from <http://www.statmodel.com/download/Plausible.pdf>
- Baron, R. M., & Kenny, D. A. (1986). The moderator–mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology*, *51*, 1173–1182. <http://dx.doi.org/10.1037/0022-3514.51.6.1173>
- Bartlett, J. W., Seaman, S. R., White, I. R., & Carpenter, J. R. (2015). Multiple imputation of covariates by fully conditional specification: Accommodating the substantive model. *Statistical Methods in Medical Research*, *24*, 462–487. <http://dx.doi.org/10.1177/0962280214521348>
- Benner, A. D., & Graham, S. (2013). The antecedents and consequences of racial/ethnic discrimination during adolescence: Does the source of discrimination matter? *Developmental Psychology*, *49*, 1602–1613. <http://dx.doi.org/10.1037/a0030557>
- Bertrand, M., & Mullainathan, S. (2004). Are Emily and Greg more employable than Lakisha and Jamal? A field experiment on labor market discrimination. *The American Economic Review*, *94*, 991–1013. <http://dx.doi.org/10.1257/0002828042002561>
- Bliese, P. D. (2000). Within-group agreement, non-independence, and reliability: Implications for data aggregation and analysis. In K. J. Klein & S. W. Kozlowski (Eds.), *Multilevel theory, research, and methods in organizations* (pp. 349–381). San Francisco, CA: Jossey-Bass.
- Bonilla-Silva, E. (2006). *Racism without racists: Color-blind racism and the persistence of racial inequality in the United States*. Lanham, MD: Rowman & Littlefield.

- Caro, R. A. (1974). *The power broker: Robert Moses and the fall of New York*. New York, NY: Random House.
- Cheung, G. W., & Lau, R. S. (2015). Accuracy of parameter estimates and confidence intervals in moderated mediation models: A comparison of regression and latent moderated structural equations. *Organizational Research Methods, 20*, 746–769. <http://dx.doi.org/10.1177/1094428115595869>
- Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2013). *Applied multiple regression/correlation analysis for the behavioral sciences*. New York, NY: Routledge.
- Cronbach, L. J. (1976). *Research on classrooms and schools: Formulation of questions, design, and analysis*. Stanford, CA: Stanford University Evaluation Consortium.
- Cronbach, L. J., & Snow, R. (1977). *Aptitudes and instructional methods: A handbook for research on interactions*. New York, NY: Irvington.
- Cronbach, L. J., & Webb, N. (1975). Between-class and within-class effects in a reported aptitude * treatment interaction: Reanalysis of a study by G. L. Anderson. *Journal of Educational Psychology, 67*, 717–724. <http://dx.doi.org/10.1037/0022-0663.67.6.717>
- Dansereau, F., & Yammarino, F. J. (2000). Within and between analysis: The variant paradigm as an underlying approach to theory building and testing. In K. J. Klein & S. W. J. Kozlowski (Eds.), *Multilevel theory, research, and methods in organizations: Foundations, extensions, and new directions* (pp. 425–466). San Francisco, CA: Jossey-Bass.
- Edwards, J. R., & Lambert, L. S. (2007). Methods for integrating moderation and mediation: a general analytical framework using moderated path analysis. *Psychological methods, 12*, 1–22. <http://dx.doi.org/10.1037/1082-989X.12.1.1>
- Enders, C. K. (2010). *Applied missing data analysis*. New York, NY: Guilford Press.
- Frankenberg, E., & Lee, C. (2002). *Race in American public schools: Rapidly resegregating school districts*. Retrieved from: <http://files.eric.ed.gov/fulltext/ED468063.pdf>
- Galinsky, A. D., Hall, E. V., & Cuddy, A. J. (2013). Gendered races: Implications for interracial marriage, leadership selection, and athletic participation. *Psychological Science, 24*, 498–506. <http://dx.doi.org/10.1177/0956797612457783>
- Goldstein, H., Carpenter, J. R., & Browne, W. J. (2014). Fitting multilevel multivariate models with missing data in responses and covariates that may include interactions and non-linear terms. *Journal of the Royal Statistical Society: Series A. Statistics in Society, 177*, 553–564. <http://dx.doi.org/10.1111/rssa.12022>
- Hall, A. V., Hall, E. V., & Perry, J. L. (2016). Black and blue: Exploring racial bias and law enforcement in the killings of unarmed black male civilians. *American Psychologist, 71*, 175–186. <http://dx.doi.org/10.1037/a0040109>
- Haraway, D. (2006). A cyborg manifesto: Science, technology, and socialist-feminism in the late 20th century. In J. Weiss, J. Nolan, J. Hunsinger, & P. Trifonis (Eds.), *The international handbook of virtual learning environments* (pp. 117–158). Amsterdam, Netherlands: Springer. http://dx.doi.org/10.1007/978-1-4020-3803-7_4
- Hayes, A. F. (2013). *Introduction to mediation, moderation, and conditional process analysis: A regression-based approach*. New York, NY: Guilford Press.
- Hayes, A. F. (2015). An index and test of linear moderated mediation. *Multivariate Behavioral Research, 50*, 1–22. <http://dx.doi.org/10.1080/00273171.2014.962683>
- Hayes, A. F. (2018). *Model templates for PROCESS for SPSS and SAS*. Retrieved from <http://docplayer.net/408084-Model-templates-for-process-for-spss-and-sas-c-2013-2015-andrew-f-hayes-and-the-guilford-press-model-1-conceptual-diagram-statistical-diagram.html>
- Hayes, A. F., & Preacher, K. J. (2013). Conditional process modeling: Using structural equation modeling to examine contingent causal processes. In G. R. Hancock & R. O. Mueller (Eds.), *Structural equation modeling: A second course* (2nd ed., pp. 219–266). Charlotte, NC: Information Age.
- Klein, A. G., & Moosbrugger, H. (2000). Maximum likelihood estimation of latent interaction effects with the LMS method. *Psychometrika, 65*, 457–474. <http://dx.doi.org/10.1007/BF02296338>
- Klein, A. G., & Muthén, B. O. (2007). Quasi-maximum likelihood estimation of structural equation models with multiple interaction and quadratic effects. *Multivariate Behavioral Research, 42*, 647–673. <http://dx.doi.org/10.1080/00273170701710205>
- Kozlowski, S. W. J., & Klein, K. J. (2000). A multilevel approach to theory and research in organizations: Contextual, temporal, and emergent processes. In K. J. Klein & S. W. J. Kozlowski (Eds.), *Multilevel theory, research, and methods in organizations: Foundations, extensions, and new directions* (pp. 3–90). San Francisco, CA: Jossey-Bass.
- Leite, W. L., & Zuo, Y. (2011). Modeling latent interactions at level 2 in multilevel structural equation models: An evaluation of mean-centered and residual-centered unconstrained approaches. *Structural Equation Modeling, 18*, 449–464. <http://dx.doi.org/10.1080/10705511.2011.582400>
- Lucal, B. (1996). Oppression and privilege: Toward a relational conceptualization of race. *Teaching Sociology, 24*, 245–255. <http://dx.doi.org/10.2307/1318739>

- Lüdtke, O., Marsh, H. W., Robitzsch, A., & Trautwein, U. (2011). A 2×2 taxonomy of multilevel latent contextual models: Accuracy-bias trade-offs in full and partial error correction models. *Psychological Methods, 16*, 444–467. <http://dx.doi.org/10.1037/a0024376>
- Lüdtke, O., Marsh, H. W., Robitzsch, A., Trautwein, U., Asparouhov, T., & Muthén, B. (2008). The multi-level latent covariate model: A new, more reliable approach to group-level effects in contextual studies. *Psychological Methods, 13*, 203–229. <http://dx.doi.org/10.1037/a0012869>
- MacKinnon, D. P. (2008). *Introduction to statistical mediation analysis*. Mahwah, NJ: Taylor & Francis.
- Marsh, H. W., Lüdtke, O., Robitzsch, A., Trautwein, U., Asparouhov, T., Muthén, B., & Nagengast, B. (2009). Doubly-latent models of school contextual effects: Integrating multilevel and structural equation approaches to control measurement and sampling error. *Multivariate Behavioral Research, 44*, 764–802. <http://dx.doi.org/10.1080/00273170903333665>
- Massey, D. S., & Denton, N. A. (1993). *American apartheid: Segregation and the making of the underclass*. Cambridge, MA: Harvard University Press.
- Mislevy, R. J., Johnson, E. G., & Muraki, E. (1992). Scaling procedures in NAEP. *Journal of Educational Statistics, 17*, 131–154.
- Muthén, B. O., & Asparouhov, T. (2008). Growth mixture modeling: Analysis with non-Gaussian random effects. In G. Fitzmaurice, M. Davidian, G. Verbeke, & G. Molenberghs (Eds.), *Longitudinal data analysis* (pp. 143–165). Boca Raton, FL: Chapman & Hall/CRC.
- Muthén, B., & Asparouhov, T. (2012). Bayesian structural equation modeling: A more flexible representation of substantive theory. *Psychological Methods, 17*, 313–335. <http://dx.doi.org/10.1037/a0026802>
- Muthén, L. K., & Muthén, B. O. (2016). *Mplus user's guide* (7th ed.). Los Angeles, CA: Author.
- National Opinion Research Center. (1980). *High School and Beyond information for users: Base year (1980) data*. Chicago, IL: National Opinion Research Center.
- Navarro, V. (1990). Race or class versus race and class: Mortality differentials in the United States. *The Lancet, 336*, 1238–1240. [http://dx.doi.org/10.1016/0140-6736\(90\)92846-A](http://dx.doi.org/10.1016/0140-6736(90)92846-A)
- Pickett, K. E., & Pearl, M. (2001). Multilevel analyses of neighbourhood socioeconomic context and health outcomes: A critical review. *Journal of Epidemiology and Community Health, 55*, 111–122. <http://dx.doi.org/10.1136/jech.55.2.111>
- Pituch, K. A., & Stapleton, L. M. (2012). Distinguishing between cross- and cluster-level mediation processes in the cluster randomized trial. *Sociological Methods & Research, 41*, 630–670. <http://dx.doi.org/10.1177/0049124112460380>
- Preacher, K. J., & Hayes, A. F. (2004). SPSS and SAS procedures for estimating indirect effects in simple mediation models. *Behavior Research Methods, Instruments & Computers, 36*, 717–731. <http://dx.doi.org/10.3758/BF03206553>
- Preacher, K. J., & Hayes, A. F. (2008). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. *Behavior Research Methods, 40*, 879–891. <http://dx.doi.org/10.3758/BRM.40.3.879>
- Preacher, K. J., Rucker, D. D., & Hayes, A. F. (2007). Addressing moderated mediation hypotheses: Theory, methods, and prescriptions. *Multivariate Behavioral Research, 42*, 185–227. <http://dx.doi.org/10.1080/00273170701341316>
- Preacher, K. J., & Selig, J. P. (2012). Advantages of Monte Carlo confidence intervals for indirect effects. *Communication Methods and Measures, 6*, 77–98. <http://dx.doi.org/10.1080/19312458.2012.679848>
- Preacher, K. J., Zhang, Z., & Zyphur, M. J. (2011). Alternative methods for assessing mediation in multilevel data: The advantages of multilevel SEM. *Structural Equation Modeling, 18*, 161–182. <http://dx.doi.org/10.1080/10705511.2011.557329>
- Preacher, K. J., Zhang, Z., & Zyphur, M. J. (2016). Multilevel structural equation models for assessing moderation within and across levels of analysis. *Psychological Methods, 21*, 189–205. <http://dx.doi.org/10.1037/met0000052>
- Preacher, K. J., Zyphur, M. J., & Zhang, Z. (2010). A general multilevel SEM framework for assessing multilevel mediation. *Psychological Methods, 15*, 209–233. <http://dx.doi.org/10.1037/a0020141>
- Raudenbush, S., & Bryk, A. S. (1986). A hierarchical model for studying school effects. *Sociology of Education, 59*, 1–17. <http://dx.doi.org/10.2307/2112482>
- Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods*. New York, NY: Sage.
- Ren, X. S., Amick, B. C., & Williams, D. R. (1999). Racial/ethnic disparities in health: The interplay between discrimination and socioeconomic status. *Ethnicity & Disease, 9*, 151–165.
- Rubin, D. B. (1987). *Multiple imputation for nonresponse in surveys*. New York, NY: Wiley. <http://dx.doi.org/10.1002/9780470316696>
- Sardeshmukh, S. R., & Vandenberg, R. J. (2016). Integrating moderation and mediation: A structural equation modeling approach. *Organizational Research Methods, 20*, 721–745. <http://dx.doi.org/10.1177/1094428115621609>

- Schafer, J. L. (1997). *Analysis of incomplete multivariate data*. London, England: Chapman & Hall. <http://dx.doi.org/10.1201/9781439821862>
- Seaman, S. R., Bartlett, J. W., & White, I. R. (2012). Multiple imputation of missing covariates with non-linear effects and interactions: An evaluation of statistical methods. *BMC Medical Research Methodology*, *12*, 46. <http://dx.doi.org/10.1186/1471-2288-12-46>
- Seaton, E. K., & Yip, T. (2009). School and neighborhood contexts, perceptions of racial discrimination, and psychological well-being among African American adolescents. *Journal of Youth and Adolescence*, *38*, 153–163. <http://dx.doi.org/10.1007/s10964-008-9356-x>
- Seitles, M. (1998). The perpetuation of residential racial segregation in America: Historical discrimination, modern forms of exclusion, and inclusionary remedies. *Journal of Land Use & Environmental Law*, *14*, 89–124.
- Smedley, A., & Smedley, B. D. (2005). Race as biology is fiction, racism as a social problem is real: Anthropological and historical perspectives on the social construction of race. *American Psychologist*, *60*, 16–26. <http://dx.doi.org/10.1037/0003-066X.60.1.16>
- Steele, C. M., & Aronson, J. (1995). Stereotype threat and the intellectual test performance of African Americans. *Journal of Personality and Social Psychology*, *69*, 797–811. <http://dx.doi.org/10.1037/0022-3514.69.5.797>
- Thomas, A. J., Witherspoon, K. M., & Speight, S. L. (2008). Gendered racism, psychological distress, and coping styles of African American women. *Cultural Diversity and Ethnic Minority Psychology*, *14*, 307–314. <http://dx.doi.org/10.1037/1099-9809.14.4.307>
- Tofighi, D., & Thoemmes, F. (2014). Single-level and multilevel mediation analysis. *The Journal of Early Adolescence*, *34*, 93–119. <http://dx.doi.org/10.1177/0272431613511331>
- Tomaskovic-Devey, D. (1993). *Gender & racial inequality at work: The sources and consequences of job segregation*. Ithaca, NY: Cornell University Press.
- Tsui, A. S., Egan, T. D., & O'Reilly, C. A. (1992). Being different: Relational demography and organizational attachment. *Administrative Science Quarterly*, *37*, 549–579. <http://dx.doi.org/10.2307/2393472>
- Umaña-Taylor, A. J. (2016). A post-racial society in which ethnic-racial discrimination still exists and has significant consequences for youths' adjustment. *Current Directions in Psychological Science*, *25*, 111–118. <http://dx.doi.org/10.1177/0963721415627858>
- Von Davier, M., Gonzalez, E., & Mislevy, R. (2009). What are plausible values and why are they useful. *IERI monograph series*, *2*, 9–36.
- von Hippel, P. T. (2009). How to impute interactions, squares, and other transformed variables. *Sociological Methodology*, *39*, 265–291. <http://dx.doi.org/10.1111/j.1467-9531.2009.01215.x>
- Wang, L., & Preacher, K. J. (2015). Moderated mediation analysis using Bayesian methods. *Structural Equation Modeling*, *22*, 249–263. <http://dx.doi.org/10.1080/10705511.2014.935256>
- Williams, D. R. (1999). Race, socioeconomic status, and health. The added effects of racism and discrimination. *Annals of the New York Academy of Sciences*, *896*, 173–188. <http://dx.doi.org/10.1111/j.1749-6632.1999.tb08114.x>
- Williams, D. R., & Collins, C. (2001). Racial residential segregation: A fundamental cause of racial disparities in health. *Public Health Reports*, *116*, 404–416. [http://dx.doi.org/10.1016/S0033-3549\(04\)50068-7](http://dx.doi.org/10.1016/S0033-3549(04)50068-7)
- Williams, D. R., & Williams-Morris, R. (2000). Racism and mental health: The African American experience. *Ethnicity & Health*, *5*, 243–268. <http://dx.doi.org/10.1080/713667453>
- Wingfield, A. H. (2007). The modern mammy and the angry Black man: African American professionals' experiences with gendered racism in the workplace. *Race, Gender, & Class*, *14*, 196–212.
- Yuan, Y., & MacKinnon, D. P. (2009). Bayesian mediation analysis. *Psychological Methods*, *14*, 301–322. <http://dx.doi.org/10.1037/a0016972>
- Zhang, Z., Zyphur, M. J., & Preacher, K. J. (2009). Testing multilevel mediation using hierarchical linear models: Problems and solutions. *Organizational Research Methods*, *12*, 695–719. <http://dx.doi.org/10.1177/1094428108327450>
- Zyphur, M. J., Zammuto, R. F., & Zhang, Z. (2016). Multilevel latent polynomial regression for modeling (in)congruence across organizational groups: The case of organizational culture research. *Organizational Research Methods*, *19*, 53–79. <http://dx.doi.org/10.1177/1094428115588570>