Journal of Educational and Behavioral Statistics
2015, Vol. 40, No. 6, pp. 579–603
DOI: 10.3102/1076998615606098

Factor Rotation and Standard Errors in Exploratory Factor Analysis

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In this article, we report a surprising phenomenon: Oblique CF-varimax and oblique CF-quartimax rotation produced similar point estimates for rotated factor loadings and factor correlations but different standard error estimates in an empirical example. Influences of factor rotation on asymptotic standard errors are investigated using a numerical exploration method. The results are (a) CF-varimax, CF-quartimax, CF-equamax, and CF-parsimax produced similar asymptotic standard errors when the factor loading matrix is an independent cluster solution and (b) the four rotation methods produced different asymptotic standard errors when the factor loading matrix has a more complex structure. In addition, properties of the CF family are explored with a full range of κ values.

Keywords: *factor analysis; factor loadings; factor rotation; standard error*

Introduction

Exploratory factor analysis (EFA) is a widely used statistical procedure in the social and behavioral sciences. EFA allows researchers to study unobservable constructs like intelligence and the big five personality traits when their measurements are contaminated by measurement error. A key step in EFA is factor rotation, which transforms the unrotated factor loading matrix to a rotated factor loading matrix and factor correlation matrix while keeping model fit unchanged.

Factor rotation is conducted to aid the interpretation of EFA results and involves identifying several large factor loadings for each factor. The large loadings are called salient factor loadings. The determination of salient factor loadings requires consideration of their sampling variation, because a large factor loading in the current sample is not necessarily a salient factor loading in another sample. Sampling variation of a factor loading can be captured by its standard error. Standard errors for rotated factor loadings and factor correlations were first derived by Jennrich (Archer & Jennrich, 1973; Jennrich, 1973b).

Cudeck and O'Dell (1994) discussed the importance of standard errors in EFA research. In one of their examples (p. 479, table 1), an overall test of the EFA model suggests a three-factor model, but one of the factors does not have any statistically significant loadings. In addition, standard errors can substantially vary within a factor loading matrix. Thus, different factor loadings have different levels of accuracy. This contradicts a rule of thumb of interpreting factor loadings larger than 0.3. The importance of standard errors is recognized by methodologists (Asparouhov & Muthén, 2009; Widaman, 2012) and substantive researchers (Currier, Kim, Sandy, & Neimeyer, 2012; Huprich, Schmit, Richard, Chelminski, & Zimmerman, 2010).

Yuan, Cheng, and Zhang (2010) studied how different aspects of confirmatory factor analysis (CFA) models influence standard errors for factor loadings. Their results are informative in the current context of EFA but may not be directly applicable. CFA differs from EFA in that factor rotation is not allowed in CFA, but it is essential in EFA. In addition, manifest variables typically load on just one factor in CFA. Factor loading matrices are often far more complex in EFA. Sampling variation of parameter estimates and statistical power in EFA were investigated by MacCallum and colleagues (MacCallum & Tucker, 1991; MacCallum, Widaman, Zhang, & Hong, 1999). They identified communality as an important factor that affects accuracy of parameter estimates.

The current study was originally motivated by an empirical study (Luo et al., 2008) where two rotation methods produced similar rotated factor loadings and factor correlations but substantially different standard error estimates. Our study makes two unique contributions. First, we examine the influence of different factor rotation criteria on standard errors. Second, we consider more flexible factor loading patterns where a manifest variable can have salient loadings on multiple factors.

The rest of the article is organized as follows: We first briefly review the EFA model and factor rotation. We then report an empirical example that motivated the current study. In the empirical example, oblique CF-varimax rotation and oblique CF-quartimax rotation produced similar rotated factor loadings but different standard error estimates. These two rotation methods result in different interpretations in the empirical example. We next describe a formula to compute standard errors in EFA and investigate factor rotation and standard errors using a numerical exploration method. The article concludes with several remarks.

The EFA Model and Factor Rotation

Model Specification and Estimation

The EFA model specifies that manifest variables are weighted sums of common factors and unique factors,

$$\boldsymbol{y} = \boldsymbol{\Lambda z} + \boldsymbol{u}. \tag{1}$$

TABLE 1.
Factor Loadings and Factor Correlations of Luo et al. Data Set.

	CF-Varimax				CF-Quartimax			
	Socpot	Depend	Accom	Interper	Socpot	Depend	Accom	Interper
Novelty	.65	.05	.08	.12	.66	.04	.09	.09
Diversity	.49	−.15	.23	.40	.51	−.15	.23	.38
Diverse thinking	.44	−.03	−.04	.27	.46	−.01	−.04	.25
Leadership	.65	.03	−.35	.04	.66	.05	−.35	.00
Logical vs. affective	.33	.12	.06	.39	.35	.13	.07	.36
Aesthetics	.35	−.25	−.11	.23	.37	−.23	−.12	.22
Extroversion–introversion	.58	.03	.00	−.09	.58	.02	.01	−.11
Enterprise	.53	.45	.11	−.22	.51	.43	.14	−.26
Responsibility	.06	.57	−.09	.23	.05	.59	−.08	.18
Emotionality	.01	−.75	.00	.02	.03	−.76	−.03	.07
Inferiority vs. self-acceptance	−.21	−.51	−.37	−.11	−.20	−.49	−.40	−.08
Practical mindedness	−.06	.56	.06	.29	−.06	.58	.08	.25
Optimism vs. pessimism	.21	.59	.14	−.01	.19	.59	.17	−.05
Meticulousness	−.11	.40	−.11	.30	−.11	.43	−.10	.26
Face	.06	−.26	−.29	.14	.08	−.23	−.31	.14
Internal vs. external control	.11	.19	.42	−.13	.10	.16	.44	−.14
Family orientation	.07	.39	.25	.28	.07	.39	.27	.25
Defensiveness	.07	−.19	−.73	−.07	.08	−.15	−.76	−.07
Graciousness vs. meanness	.00	.39	.54	.05	−.01	.36	.57	.04
Interpersonal tolerance	.22	.16	.53	.10	.22	.13	.56	.09
Self vs. social orientation	.08	−.06	−.63	.10	.08	−.01	−.66	.08
Veraciousness vs. slickness	−.06	.18	.54	.27	−.06	.16	.55	.26
Traditionalism vs. modernity	−.06	.09	−.52	.10	−.06	.14	−.53	.08

(continued)

TABLE 1. *(continued)*

	CF-Varimax				CF-Quartimax			
	Socpot	Depend	Accom	Interper	Socpot	Depend	Accom	Interper
Relationship orientation	.13	.05	−.02	.64	.15	.08	−.02	.61
Social sensitivity	.36	−.04	−.15	.56	.39	.00	−.16	.53
Discipline	.00	.18	−.58	.47	.01	.24	−.59	.43
Harmony	−.02	.24	.31	.57	−.01	.26	.32	.54
Thrift vs. extravagance	−.22	.00	−.15	.45	−.20	.04	−.16	.44
Depend	.22				.26			
Accom	.05	.04			.06	.46		
Interper	.29	.31	−.01		.27	.32	−.02	

Note. Socpot = social potency; depend = dependability; accom = accommodation; interper = interpersonal relatedness.

Here $\boldsymbol{y}$ is a $p \times 1$ vector of manifest variables, $\boldsymbol{z}$ is an $m \times 1$ vector of common factors, and $\boldsymbol{u}$ is a $p \times 1$ vector of unique factors. The $p \times m$ factor loading matrix $\boldsymbol{\Lambda}$ reflects the influence of common factors on manifest variables. The factor analysis data model of Equation 1 implies the factor analysis correlation structure,

$$\boldsymbol{P} = \boldsymbol{\Lambda\Phi\Lambda}' + \boldsymbol{D}_{\psi}. \tag{2}$$

Here $\boldsymbol{P}$ is a $p \times p$ manifest variable correlation matrix, $\boldsymbol{\Phi}$ is an $m \times m$ factor covariance matrix, and $\boldsymbol{D}_{\psi}$ is a $p \times p$ unique factor covariance matrix. The contributions of common factors $\boldsymbol{z}$ to manifest variables $\boldsymbol{y}$ are reflected by $\boldsymbol{\Lambda\Phi\Lambda}'$. In particular, a diagonal element of $\boldsymbol{\Lambda\Phi\Lambda}'$ represents the proportion of standardized manifest variance accounted for by $\boldsymbol{z}$, referred to as communality.

The EFA model is often estimated using a two-step procedure. The first step is factor extraction and the second step is factor rotation. In factor extraction, a $p \times m$ factor loading matrix $\boldsymbol{A}$ is obtained from minimization of a discrepancy function value $f(\boldsymbol{P}, \boldsymbol{R})$. The factor loading matrix $\boldsymbol{A}$ is referred to as the unrotated factor loading matrix, $\boldsymbol{R}$ is a $p \times p$ sample manifest variable correlation matrix, and the factor analysis correlation structure is expressed as $\boldsymbol{P} = \boldsymbol{AA}' + \boldsymbol{D}_{\psi}$. A popular choice for the discrepancy function is the maximum likelihood (ML) discrepancy function,[1]

$$f_{\mathrm{ML}}(\boldsymbol{P}, \boldsymbol{R}) = \log_e|\boldsymbol{P}| - \log_e|\boldsymbol{R}| + \mathrm{trace}[(\boldsymbol{R} - \boldsymbol{P})\boldsymbol{P}^{-1}]. \tag{3}$$

Here $\log_e|\boldsymbol{P}|$ is the natural log function of the determinant of $\boldsymbol{P}$, $\boldsymbol{P}^{-1}$ is the inverse of $\boldsymbol{P}$, and "trace" is an operator summing diagonal elements of a square matrix. Because the unrotated factor loading matrix $\boldsymbol{A}$ is obtained for mathematical convenience, it is seldom interpretable. The second step of EFA is to rotate the unrotated factor loading matrix $\boldsymbol{A}$ with the aim of improving interpretability.

Factor Rotation

Factor rotation produces a rotated factor loading matrix $\boldsymbol{\Lambda}$ and a factor covariance matrix $\boldsymbol{\Phi}$,

$$\boldsymbol{\Lambda} = \boldsymbol{A}\boldsymbol{T}^{-1} \text{ and } \boldsymbol{\Phi} = \boldsymbol{T}\boldsymbol{T}'. \tag{4}$$

Here $\boldsymbol{T}$ is an $m \times m$ transformation matrix. Both $\boldsymbol{\Lambda}$ and $\boldsymbol{A}$ fit data equally well, but $\boldsymbol{\Lambda}$ is usually more interpretable than $\boldsymbol{A}$. Because variances of common factors are arbitrary, it is convenient to require them to equal one. Thus, $\boldsymbol{\Phi}$ is also a factor correlation matrix. Factor rotation is called orthogonal rotation if common factors are uncorrelated, and oblique rotation if common factors are correlated. Oblique rotation is preferable to orthogonal rotation, because it tends to provide more interpretable factor loading patterns without the unrealistic restriction that common factors are uncorrelated.

Factor rotation is often carried out by optimization of a scalar function $Q(\boldsymbol{\Lambda})$ of the rotated factor loading matrix $\boldsymbol{\Lambda}$. The function $Q(\boldsymbol{\Lambda})$ is called the rotation criterion function, for example, the varimax criterion (Kaiser, 1958), the direct quartimin criterion (Jennrich & Sampson, 1966), and the target rotation criterion (Browne, 1972). Readers are referred to Browne (2001) for a comprehensive list of rotation criteria.

A popular choice of $Q(\boldsymbol{\Lambda})$ is the Crawford–Ferguson (CF) family (Browne, 2001; Crawford & Ferguson, 1970, equation (7)),

$$Q(\boldsymbol{\Lambda}) = (1-\kappa)\sum_{i=1}^{p}\sum_{j=1}^{m}\sum_{l\neq j}^{m}\lambda_{ij}^2\lambda_{il}^2 \quad + \quad \kappa\sum_{j=1}^{m}\sum_{i=1}^{p}\sum_{h\neq i}^{p}\lambda_{ij}^2\lambda_{hj}^2. \tag{5}$$

Equation 5 is the sum of two components: The first one measures row parsimony and the second one measures column parsimony. The row parsimony component reaches its lowest value of zero if each and every row of the rotated factor loading matrix $\boldsymbol{\Lambda}$ has at most one nonzero element. The column parsimony component reaches its lowest value of zero if each and every column of the rotated factor loading matrix $\boldsymbol{\Lambda}$ has only one nonzero element. The relative contributions of the row parsimony component and the column parsimony component are controlled by κ, which is specified between 0 and 1.

The CF family of Equation 5 is a complexity function, whose minimization produces the rotated factor loading matrix $\boldsymbol{\Lambda}$ and factor correlation matrix $\boldsymbol{\Phi}$. The CF family includes other rotation criteria as special cases. For orthogonal rotation, minimizing Equation 5 with $\kappa = 0$, $\kappa = 1/p$, $\kappa = m/(2p)$, and $\kappa = (m-1)/(p+m-2)$ is equivalent to quartimax rotation, varimax rotation, equamax rotation, and parsimax rotation, respectively. Thus, the members of the CF family with $\kappa = 0$, $\kappa = 1/p$, $\kappa = m/(2p)$, and $\kappa = (m - 1)/(p + m - 2)$ are referred to as CF-quartimax rotation, CF-varimax rotation, CF-equamax rotation, and CF-parsimax rotation, respectively (Browne, 2001, table 1). This equivalence does not carry over to oblique rotation, however. For example, maximizing the varimax criterion does not provide satisfactory results for oblique rotation, but minimizing the CF-varimax criterion tends to provide satisfactory results for oblique rotation (Browne, 2001).

Interpretation of Factor Loadings

A factor loading matrix is easy to interpret if each manifest variable has just one nonzero loading, and these nonzero loadings are in different columns. This is called an independent cluster solution in which all manifest variables are marker variables. An independent cluster solution does not exist in many applications of EFA, however. A well-known guideline for interpreting factor loading matrices is Thurstone's (1947) simple structure, which is more inclusive than an independent cluster solution. It consists of five rules:

1. Each row should contain at least one zero.
2. Each column should contain at least m zeros.
3. Each pair of columns should have several rows with zeros in one column but not the other.
4. If $m \geq 4$, every pair of columns should have several rows with zeros in both columns.
5. Every pair of columns of $\mathbf{\Lambda}$ should have few rows with nonzero loadings in both columns.

Thurstone's simple structure allows manifest variables to have nonzero loadings on more than one factor. These nonzero loadings are salient loadings and they help to define latent factors. Therefore, the interpretation of EFA involves distinguishing factor loadings as "zero" loadings or nonzero loadings. "Zero" is a misnomer if taken literally. It is unlikely that a factor loading is exactly zero even in the population. Nevertheless, the value of zero represents an ideal case for a trivial factor loading. Because such an ideal case is helpful in exemplifying an interpretable factor loading matrix and it is routinely used in practice, we use the term and acknowledge its lack of precision. Nonzero loadings should be statistically different from zero. Their confidence intervals should not include zero. Hypothesis tests or confidence intervals for factor loadings can be constructed with standard error estimates. Statistical significance does not imply practical significance, however. A factor loading is highly statistically significant if the point estimate is 0.04 and the standard error estimate is 0.01, but such a factor loading does not help understand latent factors. One strategy may be to complement rotated factor loadings by their corresponding semipartial correlations. The practical significance of factor loadings is then indicated by squared semipartial correlations. For example, 0.01, 0.09, and, 0.25 are considered small, medium, and large effects. We could interpret a factor loading that reflects less than a small effect as essentially a "zero" loading.

An Empirical Example of Personality Assessment

Our study was originally motivated by a surprising phenomenon we encountered in an empirical study (Luo et al., 2008). Two factor rotation methods produced similar rotated factor loadings but substantially different standard errors. The empirical study was on marital satisfaction, and the participants were 537 urban Chinese couples. Our analysis involves 28 facet scores of the Chinese Personality Assessment Inventory (Cheung et al., 1996) from the 537 husbands. We extracted four factors from the sample correlation matrix using ML. The 90% confidence interval for the root mean square error of approximation is [.044, .054], which indicates close fit for the four-factor EFA model (Browne & Cudeck, 1993). The unrotated factor loading matrix was rotated using two oblique rotation criteria: CF-varimax and CF-quartimax. These two factor rotation criteria are members of the CF family with $\kappa = 0$ for CF-quartimax and $\kappa = 1/28$ for CF-varimax. Factor extraction, factor rotation, and estimation of standard errors were carried out using the software CEFA 3.03 (Browne, Cudeck, Tateneni, & Mels, 2008).

Rotated Factor Loadings and Factor Correlations

Table 1 presents the rotated factor loading matrices and factor correlation matrices. The CF-varimax rotated factor loadings and factor correlations and the CF-quartimax rotated factor loadings and factor correlations are similar but not identical. For example, CF-varimax rotated factor loadings of "novelty" on "social potency," "dependability," "accommodation," and "interpersonal relatedness" are 0.65, 0.05, 0.08, and 0.12, respectively. Their CF-quartimax counterparts are 0.66, 0.04, 0.09, and 0.10, respectively. To quantify the similarity between CF-varimax rotated factor loadings and CF-quartimax rotated factor loadings, we computed coefficients of congruence (Gorsuch, 1983, p. 285) between these two sets of rotated factor loadings. They are 0.999, 0.996, 1.000, and 0.997 for the four factors social potency, dependability, accommodation, and interpersonal relatedness, respectively.

Standard Error Estimates for Rotated Factor Loadings and Factor Correlations

Table 2 presents standard error estimates for factor loadings and factor correlations. Although CF-varimax rotated factor loadings and factor correlations and CF-quartimax rotated factor loadings and factor correlations are similar, their standard error estimates differ substantially. A comparison of the left half and the right half of Table 2 reveals that standard error estimates for CF-quartimax rotated factor loadings on social potency and interpersonal relatedness tend to be larger

TABLE 2.
Standard Errors for Factor Loadings and Factor Correlations of Luo et al. Data Set.

	CF-Varimax				CF-Quartimax			
	Socpot	Depend	Accom	Interper	Socpot	Depend	Accom	Interper
Novelty	.04	.05	.04	.08	.10	.06	.04	.27
Diversity	.08	.04	.05	.10	.25	.05	.05	.28
Diverse thinking	.06	.05	.05	.07	.17	.06	.05	.22
Leadership	.04	.05	.04	.07	.06	.06	.04	.23
Logical vs. affective	.06	.05	.05	.06	.19	.06	.06	.19
Aesthetics	.07	.06	.05	.08	.18	.06	.05	.19
Extroversion–introversion	.04	.06	.05	.07	.04	.07	.05	.21
Enterprise	.07	.08	.05	.05	.14	.09	.05	.16
Responsibility	.04	.05	.05	.06	.04	.05	.05	.06
Emotionality	.06	.04	.04	.05	.14	.04	.04	.08
Inferiority vs. self-acceptance	.04	.04	.04	.04	.04	.07	.06	.09
Practical mindedness	.04	.05	.05	.06	.05	.06	.06	.06
Optimism vs. pessimism	.06	.05	.04	.04	.09	.07	.05	.06
Meticulousness	.05	.07	.06	.07	.07	.07	.06	.08
Face	.06	.06	.05	.06	.11	.06	.06	.07
Internal vs. external control	.05	.06	.05	.06	.08	.07	.05	.07
Family orientation	.05	.06	.05	.06	.08	.07	.05	.07
Defensiveness	.03	.04	.03	.06	.03	.05	.04	.05
Graciousness vs. meanness	.04	.04	.04	.05	.04	.06	.05	.04
Interpersonal tolerance	.04	.05	.04	.06	.07	.06	.05	.13
Self vs. social orientation	.04	.05	.04	.06	.05	.05	.04	.06
Veraciousness vs. slickness	.04	.05	.05	.06	.11	.06	.06	.05
Traditionalism vs. modernity	.04	.06	.05	.07	.04	.06	.05	.08

(continued)

TABLE 2. *(continued)*

	CF-Varimax				CF-Quartimax			
	Socpot	Depend	Accom	Interper	Socpot	Depend	Accom	Interper
Relationship orientation	.08	.05	.06	.04	.27	.05	.06	.13
Social sensitivity	.09	.04	.06	.07	.28	.05	.06	.22
Discipline	.05	.06	.07	.08	.15	.05	.07	.08
Harmony	.06	.06	.07	.05	.21	.06	.07	.07
Thrift vs. extravagance	.07	.07	.07	.06	.17	.07	.07	.07
Depend	.04				.15			
Accom	.05	.04			.07	.04		
Interper	.03	.05	.04		.06	.11	.04	

Note. Socpot = social potency; depend = dependability; accom = accommodation; interper = interpersonal relatedness.

than their CF-varimax counterparts. This phenomenon is more evident for the first seven manifest variables and the last five manifest variables. The first seven manifest variables were designed to measure the factor social potency, and the last five manifest variables were designed to measure the factor interpersonal relatedness. On the other hand, standard error estimates of these two rotation methods are similar for factor loadings on factors dependability and accommodation.

The substantially different standard error estimates of CF-varimax rotation and CF-quartimax rotation can lead to different interpretations. For example, the CF-varimax rotated factor loading of the manifest variable "diversity" on the factor interpersonal relatedness is 0.40 with a standard error estimate 0.10, and the corresponding CF-quartimax rotated factor loading is 0.38 with a standard error estimate 0.28. The 95% confidence interval for the CF-varimax rotated factor loading is [0.20, 0.60], and the 95% confidence interval for the CF-quartimax rotated factor loading is [−0.17, 0.93]. Therefore, zero is a plausible value for the factor loading if CF-quartimax is considered, but zero is not a plausible value if CF-varimax is considered.

Standard Errors for Rotated Factor Loadings

This section contains technical details of computing standard errors for EFA parameters. Readers who are unfamiliar with matrix algebra can skip it with minimum loss of continuity. The asymptotic covariance matrix of rotated factor loadings and factor correlations in EFA can be computed by inverting an augmented information matrix (Jennrich, 1974),

$$acov\ (\hat{\boldsymbol{\theta}}) = \begin{bmatrix} \boldsymbol{I}(\boldsymbol{\theta}) & \boldsymbol{L}'(\boldsymbol{\theta}) \\ \boldsymbol{L}(\boldsymbol{\theta}) & \mathbf{0} \end{bmatrix}^{-1}. \tag{6}$$

Here $\boldsymbol{I}(\theta)$ is the information matrix measuring the curvature of the ML discrepancy function. The matrix $\boldsymbol{L}(\theta)$ is added to deal with factor rotation. It collects partial derivatives of rotation constraints with respect to rotated factor loadings and factor correlations. When oblique rotation is conducted, the rotation constraints (Jennrich, 1973b, equation (28)) require the matrix

$$\boldsymbol{\Lambda}' \frac{\partial Q}{\partial \boldsymbol{\Lambda}} \boldsymbol{\Phi}^{-1}, \tag{7}$$

to be diagonal. Here $\frac{\partial Q}{\partial \boldsymbol{\Lambda}}$ is a $p \times m$ matrix of partial derivatives of the rotation criterion $Q(\boldsymbol{\Lambda})$ with respect to rotated factor loadings $\boldsymbol{\Lambda}$.

Computing standard errors for EFA with sample data involves two issues. First, asymptotic variances computed using Equation 6 are theoretical values that are appropriate when the sample size is infinity. Second, it requires population parameter values θ that are unavailable in practice. The first issue is a conceptual one. EFA is always carried out with a finite sample size (N) in practice. Analytic results for parameter estimate variances for EFA at N do not exist. However, asymptotic variances often provide satisfactory approximations to parameter estimate variances EFA at sample sizes routinely used in applied research. Parameter estimate variances appropriate for a sample size N can be approximated by dividing diagonal elements of the asymptotic covariance matrix by $n = N - 1$; taking the square roots of these variances produces the corresponding approximation to finite sample standard errors σ_n. The second issue is resolved by replacing population values with their sample estimates. The resulting standard errors are estimates of asymptotic standard errors scaled properly for a finite sample size N.

Most applications of EFA in the social and behavioral sciences are conducted with manifest variable correlation matrices. Statistical properties of correlation structures are more complex than those of covariance structures. The issues of estimating EFA models with correlation matrices versus covariance matrices were recognized by early factor analysts (Lawley & Maxwell, 1963). Shapiro and Browne (1990) described conditions where statistical properties of covariance structures could be used for correlation structures. In particular, they provided a formula which could be used to compute elements of the information matrix $\boldsymbol{I}(\theta)$ in Equation 6.

Alternatively, asymptotic standard errors for rotated factor loadings and factor correlations can be computed using the delta method (Cudeck & O'Dell, 1994). The delta method requires derivatives of rotated factor loadings and factor correlations with respect to unrotated factor loadings. Numerical differentiation could be used to approximate such derivatives.

Although we can use Equation 6 to compute asymptotic standard errors for rotated factor loadings and factor correlations, examining how patterns of factor loadings affect their asymptotic standard errors is a separate question. We are unaware of any study that systematically investigates how the pattern and magnitude of factor loadings and factor correlations affect their asymptotic standard errors in EFA. In their study on sampling variations and power in EFA with correlation matrices, MacCallum et al. (1999) found that communality is the most important factor for determining parameter estimate accuracy. The higher the communality is, the more accurate the parameter estimates are. Yuan et al. (2010) studied asymptotic standard errors in CFA with manifest covariance matrices. They reported that the asymptotic standard error for a factor loading increases when (1) the magnitude of the factor loading increases, (2) the magnitudes of other factor loadings decrease, and (3) the magnitudes of unique variances increase. The asymptotic standard error for a factor loading could increase or decrease when factor correlations increase.

Yuan et al. (2010, p. 641, section 2.3) pointed out that the task of examining the influence of patterns and magnitudes of factor loadings on their asymptotic standard errors is analytically intractable for CFA models with more than two factors. They resorted to a numerical method for exploring how patterns and magnitudes of factor loadings affect their asymptotic standard errors. Asymptotic standard errors in EFA are more complex than asymptotic standard errors in CFA for two reasons. First, factor rotation is an essential step in EFA, but it is unnecessary in CFA. Second, nearly all elements[2] of the rotated factor loading matrix are model parameters in EFA, but only a selection of elements of the factor loading matrix are model parameters in CFA. In particular, manifest variables can load on multiple factors in EFA, but they almost always load on just one factor in CFA. Thus, we employ a numerical method similar to one employed by Yuan et al. (2010) to assess the influences of factor loadings on their asymptotic standard errors in EFA.

Our study differs from Yuan et al. (2010) in two ways. First, we study EFA and they studied CFA. Second, our estimation of EFA is with manifest variable correlation matrices, and their estimation was with EFA with manifest variable covariance matrices. Our study extends MacCallum et al. (1999) in two ways. First, we consider more complex factor loading patterns where manifest variables can load on multiple factors. Second, we examine the influence of different rotation methods on asymptotic standard errors.

Numerical Explorations of Standard Errors in EFA

This section reports numerical explorations which were conducted to answer three questions. First, do different rotation methods affect asymptotic standard errors? Second, do factor loading patterns affect asymptotic standard errors? Third, do factor correlations affect asymptotic standard errors?

TABLE 3.
Typical Values of Factor Loadings Under Models I–III.

	Model I			Model II			Model III		
Sources	F1	F2	F3	F1	F2	F3	F1	F2	F3
MV1	.60	.00	.00	.60	.00	.50	.60	.00	.50
MV2	.70	.00	.00	.70	.00	.00	.70	.00	.00
MV3	.80	.00	.00	.80	.00	.00	.80	.00	.00
MV4	.00	.70	.00	.90	.00	.00	.90	.00	.00
MV5	.00	.80	.00	.00	.80	.00	.00	.80	.00
MV6	.00	.90	.00	.00	.90	.00	.00	.90	.00
MV7	.00	.00	.50	.00	.70	.00	.00	.70	.00
MV8	.00	.00	.50	.00	.60	.50	.00	.60	.50
MV9	.00	.00	.50	.00	.00	.50	.00	.50	.50

The Design of the Numerical Explorations

Three EFA models. The numerical explorations were carried out with three models. Typical factor loading values are given in Table 3. All three models have nine manifest variables and three factors. The factor loading matrix in Model I is an independent cluster pattern, and the factor loading matrices in Model II and Model III are more complex.

A factor loading is either “zero” or nonzero. A manifest variable is a marker variable if it has only one nonzero loading. In Model I, the values of nonzero factor loadings were chosen such that marker variables of the first factors (F1 and F2) have higher communalities than marker variables of the last factor (F3). Therefore, we refer to F1 and F2 as strong factors and to F3 as a weak factor. Factors F1 and F2 are stronger than factor F3 in the other two models as well. In Model II, the weak factor has only one marker variable. In Model III, the weak factor has no marker variables. Although the factor loading matrices in Models II and III are not independent cluster patterns, they are still in agreement with Thurstone’s simple structure.

To examine whether factor loading patterns affect asymptotic standard errors, we divide manifest variables into three types. If a manifest variable is a marker variable and the corresponding nonzero factor loading is on a strong factor, it is a marker variable of a strong factor; if a manifest variable is a marker variable and the corresponding nonzero factor loading is on a weak factor, it is a marker variable of a weak factor; and if a manifest variable has more than one nonzero loading, it is a nonmarker variable.

To examine whether factor correlations affect asymptotic standard errors for factor loadings, we considered five levels of factor correlations: .0, .1, .3, .5, and mixed. In the .0, .1, .3, and .5 conditions, correlations among the three factors are the same. In the mixed condition, correlations among the three factors are of different values: $\phi_{12} = .1$, $\phi_{13} = .3$, and $\phi_{23} = .5$.

Generation of manifest variable correlation matrices. An advantage of EFA is its flexibility: "Zero" loadings are not restricted to be exactly zero. Although zero is an appealing value for a trivial factor loading, such an ideal case is not plausible even in populations. To make our numerical exploration more relevant to applied research, we add small random quantities to all factor loadings described in Table 3. The random quantities have a uniform distribution (−0.05, 0.05). Manifest variable correlation matrices are generated according to Equation 2. Unique variances are chosen to ensure that manifest variable variances are one. For each of the three EFA models, 100 factor loading matrices were generated. Pairing the 100 factor loading matrices and five factor correlation levels produced 500 manifest variable correlation matrices for each of the three EFA models.

Factor extraction and factor rotation. Three factors were extracted from the manifest variable correlation matrices using ML. Oblique rotation was conducted with four criteria (CF-quartimax, CF-varimax, CF-equamax, and CF-parsimax). Asymptotic standard errors for rotated factor loadings were computed using Equation 6. Factor extraction, factor rotation, and computation of asymptotic standard errors were carried out using the software package CEFA 3.03 (Browne et al., 2008).[3]

It is important to note that none of the manifest variable correlation matrices contain any random sampling error. The small random quantities added to factor loadings are not sampling error but a procedure of generating more realistic population factor loading patterns.[4] The numerical exploration was carried out to study the influence of factor loading patterns on asymptotic standard errors. It is a theoretical investigation that is highly relevant to application of EFA. We computed ML standard errors, which are valid under correctly specified EFA models for normally distributed variables. If a procedure works well in this ideal situation, it may not work well in more realistic situations where manifest variables are often nonnormal and the EFA model is misspecified. If a procedure does not work well in this ideal situation, it is very unlikely that it would work well in more realistic situations. ML estimation is chosen because it is the most popular estimation method used in practice. Because EFA models are correctly specified in the numerical explorations, another estimation method like ordinary least squares estimation produces equivalent unrotated factor loading matrices. If asymptotic standard error inflation is introduced by factor rotation, ordinary least squares standard errors are inflated as well.

Model I

Figure 1 displays the averages of asymptotic standard errors[5] for rotated factor loadings under Model I. We can examine the three questions raised at the beginning of the section. First, different choices of factor rotation criteria do not affect asymptotic standard errors when the factor loading matrix has an independent

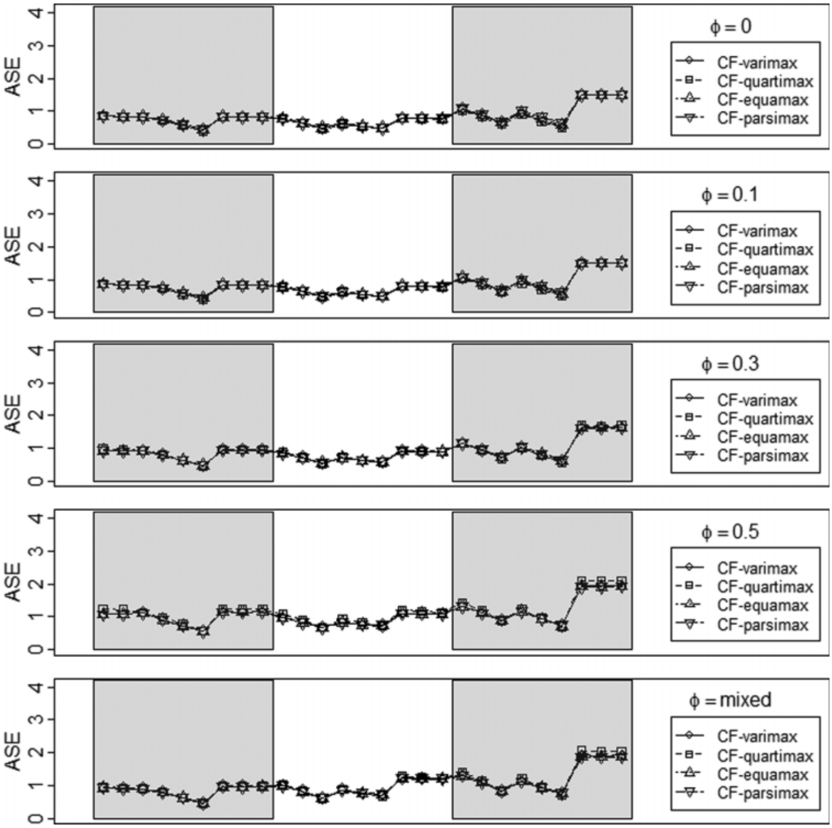

FIGURE 1. *Asymptotic standard errors for maximum likelihood rotated factor loadings of Model I. The factor loadings on Factors 1–3 are in the left shaded area (λ_{11}, λ_{21}, . . . , λ_{91}), the middle unshaded area (λ_{12}, λ_{22}, . . . , λ_{92}), and the right shaded area (λ_{13}, λ_{23}, . . . , λ_{93}), respectively.*

cluster pattern. CF-varimax, CF-quartimax, CF-equamax, and CF-parsimax gave essentially the same asymptotic standard errors for all parameters. Second, factor loading patterns do influence asymptotic standard errors. In particular, asymptotic standard errors for λ_{73}, λ_{83}, and λ_{93} are larger than those for other factor loadings. These three factor loadings are nonzero loadings of the three marker variables for the weak factor. Factor loadings of marker variables of strong factors have smaller standard errors than factor loadings of marker variables of weak factors. This result agrees with our prediction based on MacCallum et al. (1999): Large communalities result in smaller asymptotic standard errors for factor loadings. Because nearly all factor loadings are free parameters in EFA, we need to examine the influence of communalities on both nonzero loadings and "zero" loadings. When a manifest variable is a marker variable (e.g., MV7) of a weak factor, asymptotic standard errors for nonzero loadings (e.g., λ_{73}) are much larger than those for "zero" loadings (e.g., λ_{71} and λ_{72}). When a manifest variable (e.g., MV1) is a marker variable of a strong factor, asymptotic standard errors for large loadings (e.g., λ_{11}) and asymptotic standard errors for "zero" loadings (e.g., λ_{12} and λ_{13}) are close. The influence of communality on "zero" loadings is a non-issue in CFA, because "zero" loadings are constrained to be exactly zero in CFA.

To further explore the influence of communality on asymptotic standard errors for factor loadings, we manipulated the size of λ_{11} and observed how asymptotic standard errors for λ_{11}, λ_{12}, and λ_{13} change. These three factor loadings include a nonzero loading on a strong factor (λ_{11}), a "zero" loading on a strong factor (λ_{12}), and a "zero" loading on a weak factor (λ_{13}). Values of λ_{11} range from 0.5 to 0.99, and the corresponding communalities range from 0.25 to 0.98. Because there is little difference between the four rotation criteria, we report only CF-varimax rotation results. Figure 2 displays the asymptotic standard errors and their derivatives with respective to λ_{11}. The general trend for the three factor loadings is the same: Higher communalities lead to smaller asymptotic standard errors. The corresponding derivatives are all negative, which further indicates that asymptotic standard errors for factor loadings decrease as the communality increases. Although the three lines representing asymptotic standard errors are almost linear, the three lines representing derivatives are nonlinear. Therefore, the benefits of increasing communalities on asymptotic standard errors are different for different factor loadings.

Yuan et al. (2010) reported that asymptotic standard errors increase as the factor loadings increase, but asymptotic standard errors decrease as factor loadings increase in our numerical exploration. The apparent contradictory findings are due to the differences in how we estimate the models. They estimated CFA models with manifest covariance matrices, and we estimate EFA models with manifest correlation matrices. The factor loadings in the two studies are not directly comparable. One could standardize factor loadings with manifest variable covariance matrices to produce factor loadings with manifest variable correlation matrices. The direction of how factor loading sizes affect their asymptotic standard errors could be reversed by the process of standardization.[6]

Finally, to answer the question of whether different levels of factor correlations affect standard errors, we compare the five panels corresponding to different levels of factor correlations. We computed the means of five correlation levels across all parameters and the four rotation methods: They are .736, .747, .825, .997, and .946 when the factor correlations are .0, .1, .3, .5, and of mixed values, respectively. Although it is tempting to conclude that larger factor correlations result in smaller standard errors for rotated factor loadings, this result may not generalize to other sets of parameter estimates. In the context of CFA, Yuan et al. (2010) found that increasing factor correlations can result in either increased or decreased standard errors for factor loadings. The effect depends on particular parameter values of factor loadings and factor correlations.

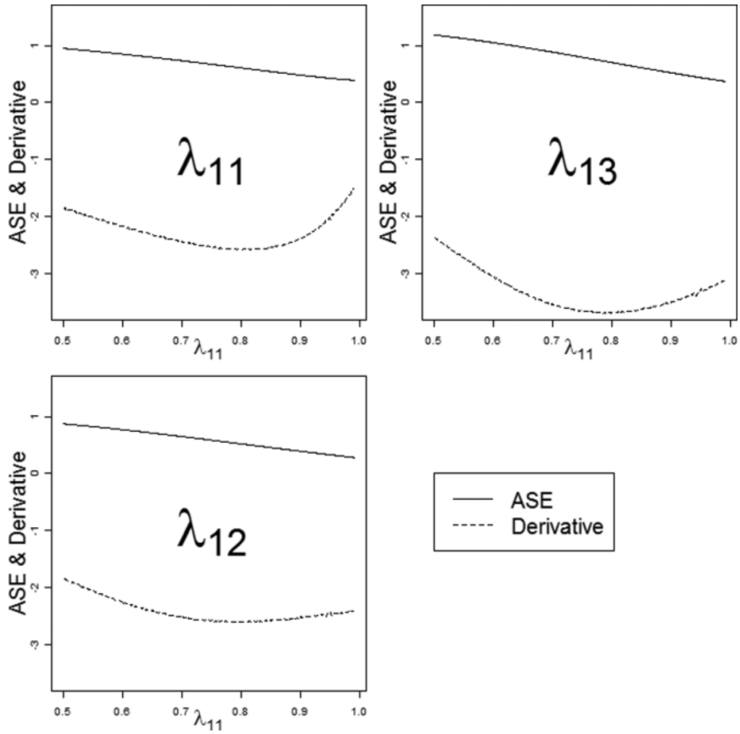

FIGURE 2. *Asymptotic standard errors and their derivatives for* λ_{11}, λ_{12}, *and* λ_{13}

Model II

Figure 3 displays the averages of asymptotic standard errors for rotated factor loadings under Model II. We can examine the three questions raised at the beginning of the section. First, the influences of different choices of factor rotation criteria on asymptotic standard errors are minimal when the factor loading matrix is slightly more complex than an independent cluster pattern. The differences among the four rotation criteria are more detectable than those of Model I, but the overall results are still very close. Second, factor loading patterns affect asymptotic standard errors. Asymptotic standard errors for λ_{11}, λ_{82}, λ_{13}, and λ_{83} are larger than those of other factor loadings. These four loadings are nonzero loadings for the two nonmarker variables MV1 and MV8. If a manifest variable has multiple nonzero loadings, such nonzero loadings tend to have larger asymptotic standard errors. "Zero" loadings of such nonmarker variables did have larger asymptotic standard errors, however. Factors 1 and 2 are strong factors, which are more clearly defined than Factor 3. The marker variables (MV2, MV3, MV4, MV5, MV6, and MV7) of the two factors have smaller asymptotic standard errors regardless of nonzero

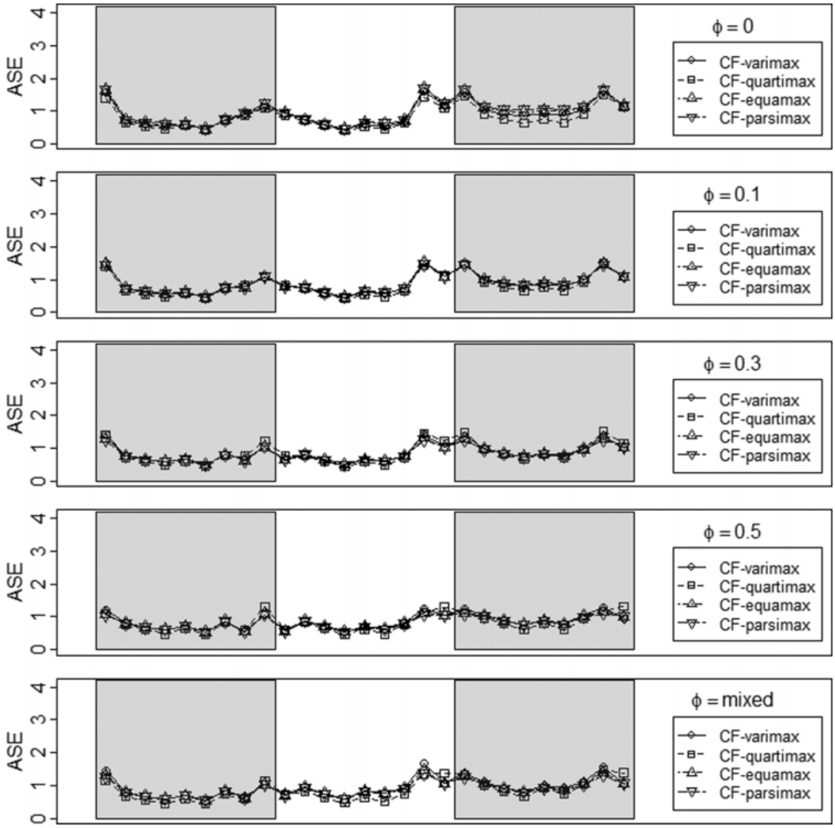

FIGURE 3. *Asymptotic standard errors for maximum likelihood rotated factor loadings of Model II. The factor loadings on Factors 1–3 are in the left shaded area ($\lambda_{11}, \lambda_{21}, \ldots, \lambda_{91}$), the middle unshaded area ($\lambda_{12}, \lambda_{22}, \ldots, \lambda_{92}$), and the right shaded area ($\lambda_{13}, \lambda_{23}, \ldots, \lambda_{93}$), respectively.*

loadings or "zero" loadings. Factor loadings of the marker variable (MV9) for the weak factor (F3) have larger asymptotic standard errors than those of the marker variables for F1 and F2. The prediction that high communalities result in smaller asymptotic standard errors for factor loadings is partially supported. The beneficial effect of high communalities on reducing asymptotic standard errors is applicable only to marker variables. The effect of magnitude is more complex. Asymptotic standard errors for large loadings and small loadings were more or less the same for marker variables, but asymptotic standard errors for large loadings are larger than asymptotic standard errors for small loadings for nonmarker variables.

Finally, we answer the question of whether different levels of factor correlation affect asymptotic standard errors. Three observations can be made:

(1) Asymptotic standard errors for factor loadings of nonmarker variables are larger in low factor correlation conditions, (2) asymptotic standard errors for factor loadings of marker variables for strong factors are larger in high factor correlation conditions, and (3) asymptotic standard errors for factor loadings of the marker variable for the weak factor remain more or less the same in different factor correlation conditions. The results of the interaction are in agreement with a conclusion made by Yuan et al. (2010) in the context of CFA. The influence of factor correlations on standard errors is highly complex. However, the influence of factor correlations is less substantial than the influence of factor loading patterns.

Model III

The factor F3 is even weaker in Model III than in the other two models. It now does not have any marker variables. Figure 4 displays the average asymptotic standard errors for rotated factor loadings under Model III. The three questions raised at the beginning of the section are more difficult to answer for Model III. An inspection of Figure 4 shows two differences between CF-quartimax asymptotic standard errors and the other three types of asymptotic standard errors. First, CF-quartimax asymptotic standard errors are larger than the other three types of asymptotic standard errors for nonzero loadings of the three nonmarker variables (MV1, MV8, and MV9) at the factor correlations of .3, .5, and the mixed level. Second, CF-quartimax asymptotic standard errors are slightly larger than the other three types of asymptotic standard errors for λ_{53}, λ_{63}, and λ_{73}. These three factor loadings are "zero" loadings of marker variables of the strong factor F2. Interestingly, CF-quartimax asymptotic standard errors and the other three types of asymptotic standard errors are similar for the three "zero" loadings associated with F1. There are two nonmarker variables MV8 and MV9 that load on both F2 and F3. There is only one nonmarker variable (MV1) that loads on both F1 and F3.

Because of the differences between CF-quartimax and the other three rotation methods, we answer the questions of whether factor loading patterns and factor correlations affect asymptotic standard errors for CF-quartimax and the other three rotation methods separately. With regard to CF-varimax, CF-parsimax, and CF-equamax, asymptotic standard errors for λ_{11}, λ_{82}, λ_{92}, λ_{13}, λ_{83}, and λ_{93} are larger than those of other parameters at the factor correlation levels of .0 and .1. These loadings are nonzero loadings of nonmarker variables.

For CF-quartimax rotation, asymptotic standard errors for λ_{82}, λ_{92}, λ_{83}, and λ_{93} increase substantially when the factor correlations increase from the levels of .0 and .1 to the levels of .3, .5, and mixed. These four loadings are nonzero loadings of the nonmarker variables (MV8 and MV9). A similar trend also occurred for two nonzero loadings (λ_{11} and λ_{12}) of the other nonmarker variable (MV1). The strong factor F2 shares two manifest variables (MV8 and

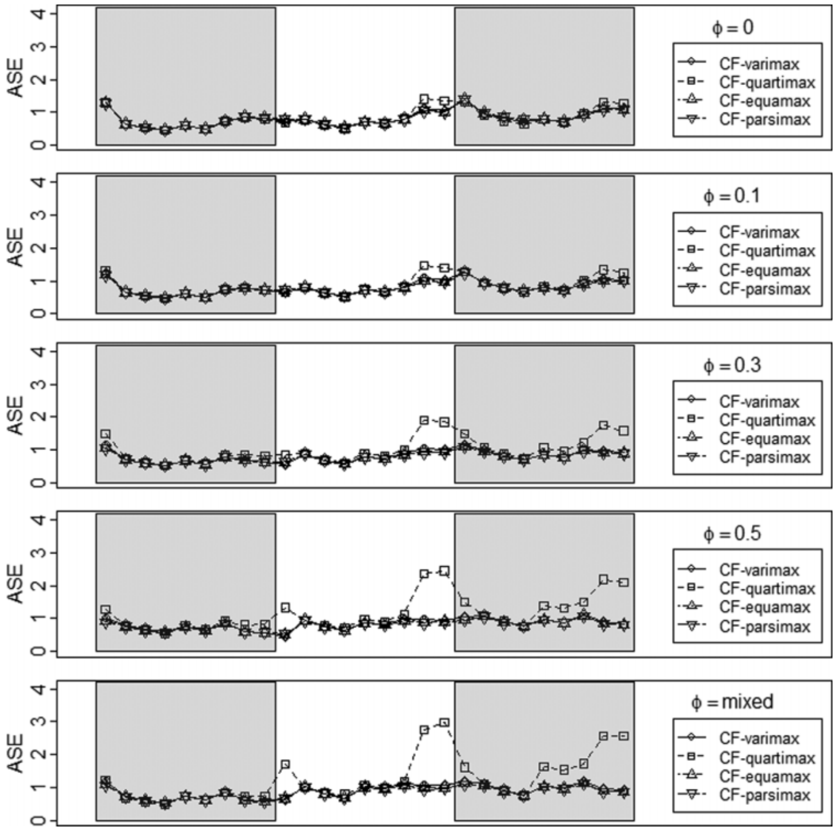

FIGURE 4. *Asymptotic standard errors for maximum likelihood rotated factor loadings of Model III. The factor loadings on Factors 1–3 are in the left shaded area, the middle unshaded area, and the right shaded area, respectively.*

MV9) with the weak factor F3, and the strong factor F1 shares only one manifest variable (MV1) with the weak factor F3. Standard errors for λ_{53}, λ_{63}, and λ_{73} are larger than standard errors for other factor loadings associated with marker variables at the factor correlation levels of .3, .5, and mixed. These three factor loadings correspond to "zero" factor loadings in the column of the weak factor F3. They also correspond to "zero" loadings of the marker variables for the factor F2.

Although we observe a substantial correlation and type interaction in both Model II and Model III, this result may not be generalizable to other factor loading patterns and values. In addition, the contribution of correlations is much more substantial than contributions of the manifest variable types and factor loading magnitudes.

Standard Errors and κ

To further understand how factor rotation affects asymptotic standard errors, we conducted another numerical exploration in which the CF family of rotation criteria were employed. In this numerical exploration, we manipulated the rotation parameter κ to include a full range of values between 0 and 1. The data are four manifest variable correlation matrices under Model III with the mixed level of factor correlations. We chose Model III because the four rotation methods produced different asymptotic standard errors in Model III. The four rotation methods CF-varimax, CF-quartimax, CF-equamax, and CF-parsimax are members of the CF family. For a nine-manifest variable three-factor EFA model, the κ for CF-quartimax, CF-varimax, CF-equamax, and CF-parsimax are 0, 1/9, 1/6, and 1/5, respectively.

Figure 5 displays how κ affects three rotated factor loadings and their asymptotic standard errors. The three factor loadings are a large loading of a marker variable of a strong factor (λ_{41}), a large loading of a nonmarker variable of a strong factor (λ_{92}), and a large loading of a nonmarker variable of a weak factor λ_{93}. These three-factor loadings were chosen to represent all large factor loadings. Results of large factor loadings of MV2, MV3, MV5, MV6, and MV7 are similar to λ_{41}, and results of large factor loadings of MV1 and MV8 are similar to λ_{92} and λ_{93}.

Three observations can be made about Figure 5. First, the CF rotation methods behave erratically when κ is close to 1. This erratic region varies slightly in different replications. Thus, we should avoid large κ values if the CF family is used. The κ values of CF-quartimax, CF-varimax, CF-equamax, and CF-parsimax are usually far away from the erratic region. The following two observations are made on the smooth regions of κ. The second observation is that asymptotic standard errors for λ_{92} and λ_{93} are highly inflated at a very small κ in Replications 1, 2, and 3. Table 4 presents rotated factor loadings and their asymptotic standard errors for $\kappa = 0.012$ in Replication #1. The rotated factor loadings are in agreement with Thurstone's simple structure. The values of these rotated factor loadings are close to the values displayed in Table 3. The asymptotic standard errors are much inflated, however. The inflation of asymptotic standard errors is particularly substantial for the three nonmarker variables (MV1, MV8, and MV9). The third observation is that the asymptotic standard errors for factor loadings are close to 1 in a large part of the smooth region of κ. Note that the asymptotic standard error $(\sqrt{n} \times \sigma_n)$ for a product moment correlation is approximately 1.

Concluding Remarks

We reported a surprising EFA result for a personality study: oblique CF-varimax rotation and oblique CF-quartimax rotation produced similar factor loadings and factor correlations but substantially different standard error estimates. We investigated the phenomenon by comparing oblique CF-varimax,

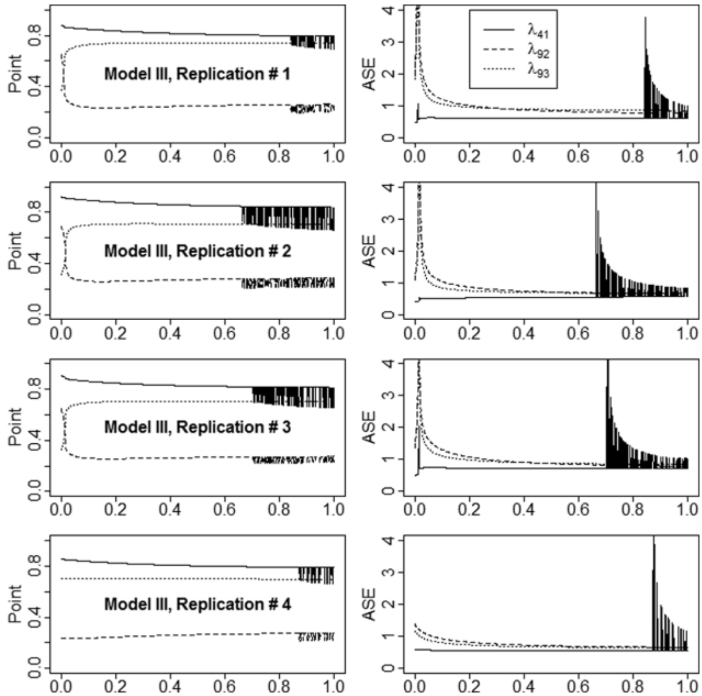

FIGURE 5. *Rotated Crawford–Ferguson factor loadings with different* κ *and their standard errors. Rotated factor loadings are shown in the left four plots. Their asymptotic standard errors* ($\mathrm{ASE} = \sqrt{n} \times \sigma_n$) *are shown in the right four plots. The two plots in the first row correspond to results from Replication #1. The plots in the second, third, and fourth rows correspond to results from Replications #2, #3, and #4, respectively.*

CF-quartimax, CF-equamax, and CF-parsimax rotation criteria on three EFA models of different levels of factorial complexity. When the factor structure has an independent cluster pattern, all rotation methods provide similar results. When the factor structure is less clear, different rotations can produce different asymptotic standard errors. We also investigated how changing κ in CF criteria affects asymptotic standard errors for factor loadings in complex factorial structures. The influences of κ include (a) CF rotation behaves erratically for large values of κ and (b) asymptotic standard errors can be highly inflated at small values of κ.

Jennrich (1973a) reported two phenomena with regard to asymptotic standard errors and factor rotation. The Wexler phenomenon refers to the observation that asymptotic standard errors for rotated factor loadings are much smaller than asymptotic standard errors for unrotated factor loadings. The anti-Wexler phenomenon refers to the observation that asymptotic standard errors for rotated

TABLE 4.
Oblique Crawford–Ferguson (κ = 0.12) Rotated Factor Loadings of Model III, Replication #1.

	F1	F2	F3
MV1	0.57 (10.43)	−0.10 (14.59)	0.50 (14.63)
MV2	0.69 (1.04)	0.03 (2.90)	−0.02 (2.86)
MV3	0.82 (0.73)	0.06 (3.81)	−0.03 (2.85)
MV4	0.87 (1.07)	0.00 (3.89)	−0.03 (2.85)
MV5	0.01 (2.83)	0.79 (5.19)	0.01 (7.91)
MV6	0.01 (2.89)	0.92 (6.63)	0.03 (9.76)
MV7	0.05 (2.81)	0.72 (4.17)	−0.01 (70.1)
MV8	−0.02 (7.65)	0.49 (21.73)	0.52 (19.44)
MV9	0.00 (9.24)	0.39 (23.47)	0.59 (20.50)

Note. Asymptotic standard errors ($\sqrt{n} \times \sigma$) are shown in parentheses. In comparison, the upper limit for the asymptotic standard error for a product moment correlation is 1.

factor loadings are much larger than asymptotic standard errors for unrotated factor loadings. The phenomenon observed in the empirical study is neither a Wexler phenomenon nor an anti-Wexler phenomenon, because it compares asymptotic standard errors for rotated factor loadings and factor correlations produced by different oblique rotation methods. If one regards factor rotation as a way to identify the EFA model, the Wexler phenomenon, the anti-Wexler phenomenon, and the phenomenon reported in the present article are unified by the same mathematical foundation. Although we can identify an EFA model using different factor rotation criteria, some rotation criteria result in more stable factor loadings and factor correlations than other rotation criteria.

We considered several factor loading patterns in the numerical exploration. Most studies focus on just the independent cluster pattern. When the factor loading matrix is an independent cluster pattern in EFA (Model I), higher communalities lead to smaller asymptotic standard errors. This is in agreement with previous studies (MacCallum et al., 1999; MacCallum & Tucker, 1991) on influences of factor loading patterns on their standard errors. However, results of Models II and III suggest the influences of communalities on asymptotic standard errors are less obvious for manifest variables that load on multiple factors. In addition, almost all rotation methods work well if the factor loading matrix has an independent cluster pattern. Different factor rotation methods can produce substantially different asymptotic standard errors for more complex factor loading patterns, however. In Model III, a common factor does not have its own marker variables and all three of its manifest variables have large loadings on two factors. Asymptotic standard errors for factor loadings associated with these three manifest variables were greatly inflated when a small κ is specified for the CF family. It is beneficial to include a marker variable for each factor when designing an EFA study.

We suggest that factor analysts examine standard error estimates when they compare results from multiple rotation methods. For example, CF-varimax rotation is preferred over CF-quartimax rotation for the personality study reported earlier. Although the two rotation methods provide similar point estimates for rotated factor loadings and factor correlations, CF-varimax rotation produces much smaller standard error estimates for parameters related to the two factors social potency and interpersonal relatedness. Conceptually, these two factors would be more successfully retrieved by CF-varimax than by CF-quartimax. A related issue is that these two factors are not intrinsically less stable than "dependence" and accommodation. Different choices of rotation criteria could change stabilities of different factors. It is premature to assign substantive meanings to such differential stabilities of factors based on just one rotation method. The advantage of CF-varimax over CF-quartimax could be reversed in other studies, however. Therefore, it is important to consider multiple rotation methods in EFA. If two factor rotation methods produce similar rotated factor loadings and factor correlations but different standard error estimates, the method with smaller standard errors is preferred.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

Notes

1. Although maximum likelihood estimation is derived for covariance matrices, replacing covariances with correlations is allowed for exploratory factor analysis (Shapiro & Browne, 1990).
2. The number of free parameters in a $p \times m$ loading matrix is $pm - m\,(m - 1)/2$ in orthogonal rotation and $pm - m\,(m - 1)$ in oblique rotation.
3. Although computing asymptotic standard errors does not require sample size in theory, we carried out the computation using CEFA. An arbitrary sample size ($N = 200$) was specified for CEFA. Therefore, the standard errors output directly from CEFA are appropriate for the sample size of 200. To compute asymptotic standard errors, we multiply these standard errors by $\sqrt{199}$.
4. We also computed standard errors for the factor loading patterns exactly as presented in Table 3. Such standard errors are given in Figures 1–3.
5. The asymptotic standard errors are computed using the bordered information matrix method. We also computed standard errors using the delta method for the models displayed in Table 3. Numerical derivatives used in the delta method were approximated using the three-point method with $\varepsilon = 0.001$ (the default value in CEFA). The two methods produced essentially identical asymptotic

standard errors. The bordered information matrix method standard errors and the delta method standard errors agree to the second decimal place for all parameters. Small differences occur at the third decimal place for several parameters. Two examples of such comparisons are presented in Figures 1–3.

6. The delta method can be employed to obtain standard errors for standardized factor loadings from unstandardized factor loadings and their asymptotic covariance matrix. If an unstandardized factor loading increases, the corresponding manifest variable variance increases. Therefore, the unstandardized factor loading needs to be divided by a larger variance to produce the standardized factor loading. The delta method involves derivatives of such divisions, which will shrink the corresponding standard errors even further.

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Manuscript received December 12, 2014
Revision received July 30, 2015
Accepted August 4, 2015