

## Online appendix to accompany:

Preacher, K. J., & Hancock, G. R. (2015). Meaningful aspects of change as novel random coefficients: A general method for reparameterizing longitudinal models. *Psychological Methods, 20*, 84-101.

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## 1. Mplus syntax for a reparameterized inverse quadratic function

*The data may be obtained from Marie Davidian's website:*

<http://www4.stat.ncsu.edu/~davidian/data/theophylline.dat>

*The data file should be transformed to wide format prior to analysis.)*

```
TITLE: davidian theophylline data, inverse quadratic;
DATA: FILE IS theophylline_wide.dat; !data file in ascii format
VARIABLE: NAMES ARE subject dose wt t1-t11 conc1-conc11; !all 25 variables
!subject: subject ID
!dose: oral dose of theophylline in mg/kg
!wt: subject weight in kg
!t1-t11: time since administration of dose in hours
!conc1-conc11: theophylline concentration in mg/L
USEVARIABLES ARE conc1-conc11; !we plan to model 11 variables
CONSTRAINT ARE t1-t11; !individual-specific time scores for use in loadings
ANALYSIS: ESTIMATOR=ML; ITERATIONS=2000; CONVERGENCE=.001; !estimation options
!BOOTSTRAP=5000; !for bootstrap confidence intervals if desired
MODEL:

[conc1-conc11*](z1-z11); !mean trend (z's defined below)
conc1-conc11*.7(ve); !residual variances constrained equal
conc1-conc10 PWITH conc2-conc11*.2(ve2); !optional lag-1 residual covariances
b*.002(vb); c@0; m*.5(vm); !random coeff. variances (only for maximizer and theta1)
b WITH c@0; b WITH m*; c WITH m@0; !random coefficient covariances
[b@0 c@0 m@0]; !factor means constrained to zero
b BY conc1*(b1); b BY conc2-conc11*(b2-b11); !loadings for the theta1 parameter (b)
c BY conc1*(c1); c BY conc2-conc11*(c2-c11); !loadings for the theta2 parameter (c)
m BY conc1*(m1); m BY conc2-conc11*(m2-m11); !loadings for the maximizer parameter (m)
```

MODEL CONSTRAINT:

NEW(mb\*.07 mc\*.02 mm\*1.77); !starts for thetal, theta2, and maximizer parameters

!intercepts constrained equal to target function

z1=t1/(mb\*t1+mc\*(mm^2+t1^2));  
z2=t2/(mb\*t2+mc\*(mm^2+t2^2));  
z3=t3/(mb\*t3+mc\*(mm^2+t3^2));  
z4=t4/(mb\*t4+mc\*(mm^2+t4^2));  
z5=t5/(mb\*t5+mc\*(mm^2+t5^2));  
z6=t6/(mb\*t6+mc\*(mm^2+t6^2));  
z7=t7/(mb\*t7+mc\*(mm^2+t7^2));  
z8=t8/(mb\*t8+mc\*(mm^2+t8^2));  
z9=t9/(mb\*t9+mc\*(mm^2+t9^2));  
z10=t10/(mb\*t10+mc\*(mm^2+t10^2));  
z11=t11/(mb\*t11+mc\*(mm^2+t11^2));

!loadings for "thetal" factor, constrained equal to d(y)/d(mb)

b1=-1\*t1^2/(mb\*t1+mc\*(mm^2+t1^2))^2;  
b2=-1\*t2^2/(mb\*t2+mc\*(mm^2+t2^2))^2;  
b3=-1\*t3^2/(mb\*t3+mc\*(mm^2+t3^2))^2;  
b4=-1\*t4^2/(mb\*t4+mc\*(mm^2+t4^2))^2;  
b5=-1\*t5^2/(mb\*t5+mc\*(mm^2+t5^2))^2;  
b6=-1\*t6^2/(mb\*t6+mc\*(mm^2+t6^2))^2;  
b7=-1\*t7^2/(mb\*t7+mc\*(mm^2+t7^2))^2;  
b8=-1\*t8^2/(mb\*t8+mc\*(mm^2+t8^2))^2;  
b9=-1\*t9^2/(mb\*t9+mc\*(mm^2+t9^2))^2;  
b10=-1\*t10^2/(mb\*t10+mc\*(mm^2+t10^2))^2;  
b11=-1\*t11^2/(mb\*t11+mc\*(mm^2+t11^2))^2;

!loadings for "theta2" factor, constrained equal to d(y)/d(mc)

c1=-1\*t1\*(mm^2+t1^2)/(mb\*t1+mc\*(mm^2+t1^2))^2; !not strictly necessary  
c2=-1\*t2\*(mm^2+t2^2)/(mb\*t2+mc\*(mm^2+t2^2))^2; !here because this  
c3=-1\*t3\*(mm^2+t3^2)/(mb\*t3+mc\*(mm^2+t3^2))^2; !coefficient is fixed,  
c4=-1\*t4\*(mm^2+t4^2)/(mb\*t4+mc\*(mm^2+t4^2))^2; !but included for  
c5=-1\*t5\*(mm^2+t5^2)/(mb\*t5+mc\*(mm^2+t5^2))^2; !completeness.  
c6=-1\*t6\*(mm^2+t6^2)/(mb\*t6+mc\*(mm^2+t6^2))^2;  
c7=-1\*t7\*(mm^2+t7^2)/(mb\*t7+mc\*(mm^2+t7^2))^2;  
c8=-1\*t8\*(mm^2+t8^2)/(mb\*t8+mc\*(mm^2+t8^2))^2;  
c9=-1\*t9\*(mm^2+t9^2)/(mb\*t9+mc\*(mm^2+t9^2))^2;  
c10=-1\*t10\*(mm^2+t10^2)/(mb\*t10+mc\*(mm^2+t10^2))^2;  
c11=-1\*t11\*(mm^2+t11^2)/(mb\*t11+mc\*(mm^2+t11^2))^2;

!loadings for "maximizer" factor, constrained equal to d(y)/d(mm)

m1=-2\*mm\*mc\*t1/(mb\*t1+mc\*(mm^2+t1^2))^2;  
m2=-2\*mm\*mc\*t2/(mb\*t2+mc\*(mm^2+t2^2))^2;  
m3=-2\*mm\*mc\*t3/(mb\*t3+mc\*(mm^2+t3^2))^2;  
m4=-2\*mm\*mc\*t4/(mb\*t4+mc\*(mm^2+t4^2))^2;  
m5=-2\*mm\*mc\*t5/(mb\*t5+mc\*(mm^2+t5^2))^2;  
m6=-2\*mm\*mc\*t6/(mb\*t6+mc\*(mm^2+t6^2))^2;  
m7=-2\*mm\*mc\*t7/(mb\*t7+mc\*(mm^2+t7^2))^2;  
m8=-2\*mm\*mc\*t8/(mb\*t8+mc\*(mm^2+t8^2))^2;  
m9=-2\*mm\*mc\*t9/(mb\*t9+mc\*(mm^2+t9^2))^2;  
m10=-2\*mm\*mc\*t10/(mb\*t10+mc\*(mm^2+t10^2))^2;  
m11=-2\*mm\*mc\*t11/(mb\*t11+mc\*(mm^2+t11^2))^2;

OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);

## 2. Mplus syntax for an unconditional bilinear spline model

TITLE: zerbe glucose;

```

DATA: FILE IS glucose.dat; !data file in ascii format
VARIABLE: NAMES ARE obese ph1-ph8; !all 9 variables
!obese: obesity status, 0=control, 1=obese
!ph1-ph8: plasma phosphate concentration repeated measures
USEVARIABLES ARE ph1-ph8; !we plan to model 8 variables
!ANALYSIS: BOOTSTRAP IS 1000; !for bootstrap confidence intervals if desired
MODEL:

[ph1-ph8*](z1-z8); !mean trend (z's defined below)
ph1-ph8*(ve); !residual variances constrained equal
w1-w4*; !random coefficient variances
w1 WITH w2*; w1 WITH w3*; w1 WITH w4*; !random coefficient covariances
w2 WITH w3*; w2 WITH w4*; w3 WITH w4*; !random coefficient covariances
[w1@0 w2@0 w3@0 w4@0]; !factor means constrained to zero
w1 BY ph1-ph8@1; !loadings for 1st factor, constrained to 1
w2 BY ph1*(w21); w2 BY ph2-ph8(w22-w28); !loadings for 2nd factor, defined below
w3 BY ph1*(w31); w3 BY ph2-ph8(w32-w38); !loadings for 3rd factor, defined below
w4 BY ph1*(w41); w4 BY ph2-ph8(w42-w48); !loadings for knot factor, defined below

MODEL CONSTRAINT:

!starts for parameters of the target function
NEW(mw1*3.4 mw2*-.26 mw3*.54 mw4*1.6 b1-b4);

!intercepts constrained equal to target function
z1=mw1+mw2*0.0+mw3*sqrt((0.0-mw4)^2);
z2=mw1+mw2*0.5+mw3*sqrt((0.5-mw4)^2);
z3=mw1+mw2*1.0+mw3*sqrt((1.0-mw4)^2);
z4=mw1+mw2*1.5+mw3*sqrt((1.5-mw4)^2);
z5=mw1+mw2*2.0+mw3*sqrt((2.0-mw4)^2);
z6=mw1+mw2*3.0+mw3*sqrt((3.0-mw4)^2);
z7=mw1+mw2*4.0+mw3*sqrt((4.0-mw4)^2);
z8=mw1+mw2*5.0+mw3*sqrt((5.0-mw4)^2);

!loadings for 2nd and 3rd factors, constrained to d(y)/d(mw2) and d(y)/d(mw3)
w21=0.0; w31=sqrt((0.0-mw4)^2);
w22=0.5; w32=sqrt((0.5-mw4)^2);
w23=1.0; w33=sqrt((1.0-mw4)^2);
w24=1.5; w34=sqrt((1.5-mw4)^2);
w25=2.0; w35=sqrt((2.0-mw4)^2);
w26=3.0; w36=sqrt((3.0-mw4)^2);
w27=4.0; w37=sqrt((4.0-mw4)^2);
w28=5.0; w38=sqrt((5.0-mw4)^2);

!loadings for knot factor, constrained to d(y)/d(mw4)
w41=(mw3*(mw4-0.0))/sqrt((0.0-mw4)^2);
w42=(mw3*(mw4-0.5))/sqrt((0.5-mw4)^2);
w43=(mw3*(mw4-1.0))/sqrt((1.0-mw4)^2);
w44=(mw3*(mw4-1.5))/sqrt((1.5-mw4)^2);
w45=(mw3*(mw4-2.0))/sqrt((2.0-mw4)^2);
w46=(mw3*(mw4-3.0))/sqrt((3.0-mw4)^2);
w47=(mw3*(mw4-4.0))/sqrt((4.0-mw4)^2);
w48=(mw3*(mw4-5.0))/sqrt((5.0-mw4)^2);

!parameters of original parameterization
b1=mw1+mw3*mw4;
b2=mw2-mw3;
b3=mw1-mw3*mw4;
b4=mw2+mw3;

OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);

```

Zerbe's (1979) data (N = 33), obtained from Zerbe (1979, p. 219).

```
0 3.4 3.3 3.0 2.6 2.2 2.5 3.4 4.4
0 3.7 2.6 2.6 1.9 2.9 3.2 3.1 3.9
0 4.0 4.1 3.1 2.3 2.9 3.1 3.9 4.0
0 3.6 3.0 2.2 2.8 2.9 3.9 3.8 4.0
0 4.1 3.8 2.1 3.0 3.6 3.4 3.6 3.7
0 3.8 2.2 2.0 2.6 3.8 3.6 3.0 3.5
0 3.8 3.0 2.4 2.5 3.1 3.4 3.5 3.7
0 4.4 3.9 2.8 2.1 3.6 3.8 4.0 3.9
0 5.0 4.0 3.4 3.4 3.3 3.6 4.0 4.3
0 3.7 3.1 2.9 2.2 1.5 2.3 2.7 2.8
0 3.7 2.6 2.6 2.3 2.9 2.2 3.1 3.9
0 4.4 3.7 3.1 3.2 3.7 4.3 3.9 4.8
0 4.7 3.1 3.2 3.3 3.2 4.2 3.7 4.3
1 4.3 3.3 3.0 2.6 2.2 2.5 2.4 3.4
1 5.0 4.9 4.1 3.7 3.7 4.1 4.7 4.9
1 4.6 4.4 3.9 3.9 3.7 4.2 4.8 5.0
1 4.3 3.9 3.1 3.1 3.1 3.1 3.6 4.0
1 3.1 3.1 3.3 2.6 2.6 1.9 2.3 2.7
1 4.8 5.0 2.9 2.8 2.2 3.1 3.5 3.6
1 3.7 3.1 3.3 2.8 2.9 3.6 4.3 4.4
1 5.4 4.7 3.9 4.1 2.8 3.7 3.5 3.7
1 3.0 2.5 2.3 2.2 2.1 2.6 3.2 3.5
1 4.9 5.0 4.1 3.7 3.7 4.1 4.7 4.9
1 4.8 4.3 4.7 4.6 4.7 3.7 3.6 3.9
1 4.4 4.2 4.2 3.4 3.5 3.4 3.9 4.0
1 4.9 4.3 4.0 4.0 3.3 4.1 4.2 4.3
1 5.1 4.1 4.6 4.1 3.4 4.2 4.4 4.9
1 4.8 4.6 4.6 4.4 4.1 4.0 3.8 3.8
1 4.2 3.5 3.8 3.6 3.3 3.1 3.5 3.9
1 6.6 6.1 5.2 4.1 4.3 3.8 4.2 4.8
1 3.6 3.4 3.1 2.8 2.1 2.4 2.5 3.5
1 4.5 4.0 3.7 3.3 2.4 2.3 3.1 3.3
1 4.6 4.4 3.8 3.8 3.8 3.6 3.8 3.8
```

### 3. Mplus syntax for a conditional bilinear spline model

The data may be obtained from Zerbe (1979, p. 219).

```
TITLE: zerbe glucose;
DATA: FILE IS glucose.dat; !data file in ascii format
DEFINE: xobese=abs(obese-1); !1=control; 0=obese
VARIABLE: NAMES ARE obese ph1-ph8; !all 9 variables
         !obese: obesity status, 0=control, 1=obese
         !ph1-ph8: plasma phosphate concentration repeated measures
USEVARIABLES ARE ph1-ph8 xobese; !we plan to model 9 variables
!ANALYSIS: BOOTSTRAP IS 1000; !for bootstrap confidence intervals if desired
MODEL:

[ph1-ph8*](z1-z8); !mean trend (z's defined below)
ph1-ph8*(ve); !residual variances constrained equal
w1-w4*; !random coefficient residual variances
w1 WITH w2*; w1 WITH w3*; w1 WITH w4*; !random coefficient residual covariances
w2 WITH w3*; w2 WITH w4*; w3 WITH w4*; !random coefficient residual covariances
[w1@0 w2@0 w3@0 w4@0]; !factor intercepts constrained to zero
w1 BY ph1-ph8@1; !loadings for 1st factor, constrained to 1
w2 BY ph1*(w21); w2 BY ph2-ph8(w22-w28); !loadings for 2nd factor, defined below
w3 BY ph1*(w31); w3 BY ph2-ph8(w32-w38); !loadings for 3rd factor, defined below
w4 BY ph1*(w41); w4 BY ph2-ph8(w42-w48); !loadings for knot factor, defined below
```

```
w1-w4 ON xobese; !regress random coefficients on obese.
```

```
MODEL CONSTRAINT:
```

```
!starts for parameters of the target function  
NEW(mw1*3.5 mw2*-.21 mw3*.49 mw4*1.95 b1-b4);
```

```
!intercepts constrained equal to target function
```

```
z1=mw1+mw2*0.0+mw3*sqrt((0.0-mw4)^2);  
z2=mw1+mw2*0.5+mw3*sqrt((0.5-mw4)^2);  
z3=mw1+mw2*1.0+mw3*sqrt((1.0-mw4)^2);  
z4=mw1+mw2*1.5+mw3*sqrt((1.5-mw4)^2);  
z5=mw1+mw2*2.0+mw3*sqrt((2.0-mw4)^2);  
z6=mw1+mw2*3.0+mw3*sqrt((3.0-mw4)^2);  
z7=mw1+mw2*4.0+mw3*sqrt((4.0-mw4)^2);  
z8=mw1+mw2*5.0+mw3*sqrt((5.0-mw4)^2);
```

```
!loadings for 2nd and 3rd factors, constrained to d(y)/d(mw2) and d(y)/d(mw3)
```

```
w21=0.0; w31=sqrt((0.0-mw4)^2);  
w22=0.5; w32=sqrt((0.5-mw4)^2);  
w23=1.0; w33=sqrt((1.0-mw4)^2);  
w24=1.5; w34=sqrt((1.5-mw4)^2);  
w25=2.0; w35=sqrt((2.0-mw4)^2);  
w26=3.0; w36=sqrt((3.0-mw4)^2);  
w27=4.0; w37=sqrt((4.0-mw4)^2);  
w28=5.0; w38=sqrt((5.0-mw4)^2);
```

```
!loadings for knot factor, constrained to d(y)/d(mw4)
```

```
w41=(mw3*(mw4-0.0))/sqrt((0.0-mw4)^2);  
w42=(mw3*(mw4-0.5))/sqrt((0.5-mw4)^2);  
w43=(mw3*(mw4-1.0))/sqrt((1.0-mw4)^2);  
w44=(mw3*(mw4-1.5))/sqrt((1.5-mw4)^2);  
w45=(mw3*(mw4-2.0))/sqrt((2.0-mw4)^2);  
w46=(mw3*(mw4-3.0))/sqrt((3.0-mw4)^2);  
w47=(mw3*(mw4-4.0))/sqrt((4.0-mw4)^2);  
w48=(mw3*(mw4-5.0))/sqrt((5.0-mw4)^2);
```

```
!parameters of original parameterization
```

```
b1=mw1+mw3*mw4;  
b2=mw2-mw3;  
b3=mw1-mw3*mw4;  
b4=mw2+mw3;
```

```
OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);
```

#### 4. Mplus syntax for a negative exponential function with intercept, asymptote, and rate

*Chaiken's data are included in installations of LISREL:*

<http://www.ssicentral.com/>

```
TITLE: chaiken, negative exponential, original parameterization;  
DATA: FILE IS schaiken2.dat; !data file in ascii format  
VARIABLE: NAMES ARE v1-v12 q1-q12; !all 24 variables  
    !v1-v12: verbal skill acquisition repeated measures  
    !q1-q12: quantitative skill acquisition repeated measures  
ANALYSIS: ESTIMATOR IS ML; !use maximum likelihood estimation  
MODEL:
```

```
!verbal
```

```
[v1-v12*](tv1-tv12); !mean trend for verbal  
v1-v12*1.97(vv1); !residual variances constrained equal
```

```

v1-v11 PWITH v2-v12*(dv1); !lag-1 error covariance
v1-v10 PWITH v3-v12*(dv2); !lag-2, etc...
v1-v9 PWITH v4-v12*(dv3);
v1-v8 PWITH v5-v12*(dv4);
v1-v7 PWITH v6-v12*(dv5);
v1-v6 PWITH v7-v12*(dv6);
v1-v5 PWITH v8-v12*(dv7);
v1-v4 PWITH v9-v12*(dv8);
v1-v3 PWITH v10-v12*(dv9);
v1-v2 PWITH v11-v12*(dv10);
v1 WITH v12*(dv11);
[av@0 bv@0 cv@0]; !factor means constrained to zero
av*; bv*; cv*; !random coefficient variances
av WITH bv* cv*; bv WITH cv*; !random coefficient covariances
av BY v1*(av1); av BY v2-v12(av2-av12); !loadings for 1st factor
bv BY v1*(bv1); bv BY v2-v12(bv2-bv12); !loadings for 2nd factor
cv BY v1*(cv1); cv BY v2-v12(cv2-cv12); !loadings for 3rd factor

!quantitative
[q1-q12*](tq1-tq12); !mean trend for quantitative
q1-q12*1.21(vq1); !residual variances constrained equal
q1-q11 PWITH q2-q12*(dq1); !lag-1 error covariance
q1-q10 PWITH q3-q12*(dq2); !lat-2, etc...
q1-q9 PWITH q4-q12*(dq3);
q1-q8 PWITH q5-q12*(dq4);
q1-q7 PWITH q6-q12*(dq5);
q1-q6 PWITH q7-q12*(dq6);
q1-q5 PWITH q8-q12*(dq7);
q1-q4 PWITH q9-q12*(dq8);
q1-q3 PWITH q10-q12*(dq9);
q1-q2 PWITH q11-q12*(dq10);
q1 WITH q12*(dq11);
[aq@0 bq@0 cq@0]; !factor means constrained to zero
aq*; bq*; cq*; !random coefficient variances
aq WITH bq* cq*; bq WITH cq; !random coefficient covariances
aq BY q1*(aq1); aq BY q2-q12(aq2-aq12); !loadings for 1st factor
bq BY q1*(bq1); bq BY q2-q12(bq2-bq12); !loadings for 2nd factor
cq BY q1*(cq1); cq BY q2-q12(cq2-cq12); !loadings for 3rd factor

!cross-domain covariances
av WITH aq* bq*; bv WITH aq* bq*;

MODEL CONSTRAINT:

!starts for parameters of the target function and for lagged error covariances
NEW(mav*6.8 mbv*21 mcv*.7 maq*8.6 mbq*16.5 mcq*.7 rhov*.27 rhoq*.31);

!error covariances
dv1= vv1*rhov; dq1= vq1*rhoq;
dv2= vv1*rhov^2; dq2= vq1*rhoq^2;
dv3= vv1*rhov^3; dq3= vq1*rhoq^3;
dv4= vv1*rhov^4; dq4= vq1*rhoq^4;
dv5= vv1*rhov^5; dq5= vq1*rhoq^5;
dv6= vv1*rhov^6; dq6= vq1*rhoq^6;
dv7= vv1*rhov^7; dq7= vq1*rhoq^7;
dv8= vv1*rhov^8; dq8= vq1*rhoq^8;
dv9= vv1*rhov^9; dq9= vq1*rhoq^9;
dv10=vv1*rhov^10; dq10=vq1*rhoq^10;
dv11=vv1*rhov^11; dq11=vq1*rhoq^11;

!verbal and quantitative intercepts constrained equal to target function
tv1 =mav-(mav-mbv)*exp(-1*mcv*( 1-1)); tq1 =maq-(maq-mbq)*exp(-1*mcq*( 1-1));
tv2 =mav-(mav-mbv)*exp(-1*mcv*( 2-1)); tq2 =maq-(maq-mbq)*exp(-1*mcq*( 2-1));

```

```

tv3 =mav-(mav-mbv)*exp(-1*mcv*( 3-1)); tq3 =maq-(maq-mbq)*exp(-1*mcq*( 3-1));
tv4 =mav-(mav-mbv)*exp(-1*mcv*( 4-1)); tq4 =maq-(maq-mbq)*exp(-1*mcq*( 4-1));
tv5 =mav-(mav-mbv)*exp(-1*mcv*( 5-1)); tq5 =maq-(maq-mbq)*exp(-1*mcq*( 5-1));
tv6 =mav-(mav-mbv)*exp(-1*mcv*( 6-1)); tq6 =maq-(maq-mbq)*exp(-1*mcq*( 6-1));
tv7 =mav-(mav-mbv)*exp(-1*mcv*( 7-1)); tq7 =maq-(maq-mbq)*exp(-1*mcq*( 7-1));
tv8 =mav-(mav-mbv)*exp(-1*mcv*( 8-1)); tq8 =maq-(maq-mbq)*exp(-1*mcq*( 8-1));
tv9 =mav-(mav-mbv)*exp(-1*mcv*( 9-1)); tq9 =maq-(maq-mbq)*exp(-1*mcq*( 9-1));
tv10=mav-(mav-mbv)*exp(-1*mcv*(10-1)); tq10=maq-(maq-mbq)*exp(-1*mcq*(10-1));
tv11=mav-(mav-mbv)*exp(-1*mcv*(11-1)); tq11=maq-(maq-mbq)*exp(-1*mcq*(11-1));
tv12=mav-(mav-mbv)*exp(-1*mcv*(12-1)); tq12=maq-(maq-mbq)*exp(-1*mcq*(12-1));

```

```

!loadings on 1st factor, constrained to d(y)/d(mav) and d(y)/d(maq)

```

```

av1 =1-exp(-1*mcv*( 1-1)); aq1 =1-exp(-1*mcq*( 1-1));
av2 =1-exp(-1*mcv*( 2-1)); aq2 =1-exp(-1*mcq*( 2-1));
av3 =1-exp(-1*mcv*( 3-1)); aq3 =1-exp(-1*mcq*( 3-1));
av4 =1-exp(-1*mcv*( 4-1)); aq4 =1-exp(-1*mcq*( 4-1));
av5 =1-exp(-1*mcv*( 5-1)); aq5 =1-exp(-1*mcq*( 5-1));
av6 =1-exp(-1*mcv*( 6-1)); aq6 =1-exp(-1*mcq*( 6-1));
av7 =1-exp(-1*mcv*( 7-1)); aq7 =1-exp(-1*mcq*( 7-1));
av8 =1-exp(-1*mcv*( 8-1)); aq8 =1-exp(-1*mcq*( 8-1));
av9 =1-exp(-1*mcv*( 9-1)); aq9 =1-exp(-1*mcq*( 9-1));
av10=1-exp(-1*mcv*(10-1)); aq10=1-exp(-1*mcq*(10-1));
av11=1-exp(-1*mcv*(11-1)); aq11=1-exp(-1*mcq*(11-1));
av12=1-exp(-1*mcv*(12-1)); aq12=1-exp(-1*mcq*(12-1));

```

```

!loadings on 2nd and 3rd factors for verbal, constrained to d(y)/d(mbv) and d(y)/d(mcv)

```

```

bv1 =exp(-1*mcv*( 1-1)); cv1 =(mav-mbv)*( 1-1)*exp(-1*mcv*( 1-1));
bv2 =exp(-1*mcv*( 2-1)); cv2 =(mav-mbv)*( 2-1)*exp(-1*mcv*( 2-1));
bv3 =exp(-1*mcv*( 3-1)); cv3 =(mav-mbv)*( 3-1)*exp(-1*mcv*( 3-1));
bv4 =exp(-1*mcv*( 4-1)); cv4 =(mav-mbv)*( 4-1)*exp(-1*mcv*( 4-1));
bv5 =exp(-1*mcv*( 5-1)); cv5 =(mav-mbv)*( 5-1)*exp(-1*mcv*( 5-1));
bv6 =exp(-1*mcv*( 6-1)); cv6 =(mav-mbv)*( 6-1)*exp(-1*mcv*( 6-1));
bv7 =exp(-1*mcv*( 7-1)); cv7 =(mav-mbv)*( 7-1)*exp(-1*mcv*( 7-1));
bv8 =exp(-1*mcv*( 8-1)); cv8 =(mav-mbv)*( 8-1)*exp(-1*mcv*( 8-1));
bv9 =exp(-1*mcv*( 9-1)); cv9 =(mav-mbv)*( 9-1)*exp(-1*mcv*( 9-1));
bv10=exp(-1*mcv*(10-1)); cv10=(mav-mbv)*(10-1)*exp(-1*mcv*(10-1));
bv11=exp(-1*mcv*(11-1)); cv11=(mav-mbv)*(11-1)*exp(-1*mcv*(11-1));
bv12=exp(-1*mcv*(12-1)); cv12=(mav-mbv)*(12-1)*exp(-1*mcv*(12-1));

```

```

!loadings on 2nd and 3rd factors for quant., constrained to d(y)/d(mbv) and d(y)/d(mcv)

```

```

bq1 =exp(-1*mcq*( 1-1)); cq1 =(maq-mbq)*( 1-1)*exp(-1*mcq*( 1-1));
bq2 =exp(-1*mcq*( 2-1)); cq2 =(maq-mbq)*( 2-1)*exp(-1*mcq*( 2-1));
bq3 =exp(-1*mcq*( 3-1)); cq3 =(maq-mbq)*( 3-1)*exp(-1*mcq*( 3-1));
bq4 =exp(-1*mcq*( 4-1)); cq4 =(maq-mbq)*( 4-1)*exp(-1*mcq*( 4-1));
bq5 =exp(-1*mcq*( 5-1)); cq5 =(maq-mbq)*( 5-1)*exp(-1*mcq*( 5-1));
bq6 =exp(-1*mcq*( 6-1)); cq6 =(maq-mbq)*( 6-1)*exp(-1*mcq*( 6-1));
bq7 =exp(-1*mcq*( 7-1)); cq7 =(maq-mbq)*( 7-1)*exp(-1*mcq*( 7-1));
bq8 =exp(-1*mcq*( 8-1)); cq8 =(maq-mbq)*( 8-1)*exp(-1*mcq*( 8-1));
bq9 =exp(-1*mcq*( 9-1)); cq9 =(maq-mbq)*( 9-1)*exp(-1*mcq*( 9-1));
bq10=exp(-1*mcq*(10-1)); cq10=(maq-mbq)*(10-1)*exp(-1*mcq*(10-1));
bq11=exp(-1*mcq*(11-1)); cq11=(maq-mbq)*(11-1)*exp(-1*mcq*(11-1));
bq12=exp(-1*mcq*(12-1)); cq12=(maq-mbq)*(12-1)*exp(-1*mcq*(12-1));

```

```

OUTPUT: TECH1 TECH4;

```

## 5. Mplus syntax for a negative exponential function with intercept, half-life, and ARC

*Chaiken's data are included in installations of LISREL:*

<http://www.ssicentral.com/>

```

TITLE: chaiken, negative exponential with halflife and arc;

```

```

DATA: FILE IS schaik2.dat; !data file in ascii format
VARIABLE: NAMES ARE v1-v12 q1-q12; !all 24 variables
!v1-v12: verbal skill acquisition repeated measures
!q1-q12: quantitative skill acquisition repeated measures
ANALYSIS: ESTIMATOR IS ML; !use maximum likelihood estimation
MODEL:

!verbal
[v1-v12*](tv1-tv12); !mean trend for verbal
v1-v12*1.97(vv1); !residual variances constrained equal
v1-v11 PWITH v2-v12*(dv1); !lag-1 error covariance
v1-v10 PWITH v3-v12*(dv2); !lag-2, etc...
v1-v9 PWITH v4-v12*(dv3);
v1-v8 PWITH v5-v12*(dv4);
v1-v7 PWITH v6-v12*(dv5);
v1-v6 PWITH v7-v12*(dv6);
v1-v5 PWITH v8-v12*(dv7);
v1-v4 PWITH v9-v12*(dv8);
v1-v3 PWITH v10-v12*(dv9);
v1-v2 PWITH v11-v12*(dv10);
v1 WITH v12*(dv11);
[av@0 bv@0 cv@0]; !factor means constrained to zero
av*1.6; bv*92; cv*10; !random coefficient variances
av WITH bv*3.7 cv*; bv WITH cv*; !random coefficient covariances
av BY v1*(av1); av BY v2-v12(av2-av12); !loadings for 1st factor
bv BY v1*(bv1); bv BY v2-v12(bv2-bv12); !loadings for 2nd factor
cv BY v1*(cv1); cv BY v2-v12(cv2-cv12); !loadings for 3rd factor

!quantitative
[q1-q12*](tq1-tq12); !mean trend for quantitative
q1-q12*1.21(vq1); !residual variances constrained equal
q1-q11 PWITH q2-q12*(dq1); !lag-1 error covariance
q1-q10 PWITH q3-q12*(dq2); !lag-2, etc...
q1-q9 PWITH q4-q12*(dq3);
q1-q8 PWITH q5-q12*(dq4);
q1-q7 PWITH q6-q12*(dq5);
q1-q6 PWITH q7-q12*(dq6);
q1-q5 PWITH q8-q12*(dq7);
q1-q4 PWITH q9-q12*(dq8);
q1-q3 PWITH q10-q12*(dq9);
q1-q2 PWITH q11-q12*(dq10);
q1 WITH q12*(dq11);
[aq@0 bq@0 cq@0]; !factor means constrained to zero
aq*4; bq*35; cq*10; !random coefficient variances
aq WITH bq*7.4 cq*; bq WITH cq; !random coefficient covariances
aq BY q1*(aq1); aq BY q2-q12(aq2-aq12); !loadings for 1st factor
bq BY q1*(bq1); bq BY q2-q12(bq2-bq12); !loadings for 2nd factor
cq BY q1*(cq1); cq BY q2-q12(cq2-cq12); !loadings for 3rd factor

!cross-domain covariances
av WITH aq* bq* cq*; bv WITH aq* bq* cq*; cv WITH aq* bq* cq*;

MODEL CONSTRAINT:

!starts for parameters of the target function and for lagged error covariances
NEW(mav*1.7 mbv*-1.5 mcv*1.96 maq*4.2 mbq*-.9 mcq*1.99 rhov*.27 rhoq*.3 diff*0);

!error covariances
dv1= vv1*rhov; dq1= vq1*rhoq;
dv2= vv1*rhov^2; dq2= vq1*rhoq^2;
dv3= vv1*rhov^3; dq3= vq1*rhoq^3;
dv4= vv1*rhov^4; dq4= vq1*rhoq^4;
dv5= vv1*rhov^5; dq5= vq1*rhoq^5;

```



```

dv6= vv1*rhov^6; dq6= vq1*rhoq^6;
dv7= vv1*rhov^7; dq7= vq1*rhoq^7;
dv8= vv1*rhov^8; dq8= vq1*rhoq^8;
dv9= vv1*rhov^9; dq9= vq1*rhoq^9;
dv10=vv1*rhov^10; dq10=vq1*rhoq^10;
dv11=vv1*rhov^11; dq11=vq1*rhoq^11;

```

```

!verbal intercepts constrained equal to target function
tv1 =mav+(mbv*11*(.5^(( 1-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv2 =mav+(mbv*11*(.5^(( 2-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv3 =mav+(mbv*11*(.5^(( 3-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv4 =mav+(mbv*11*(.5^(( 4-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv5 =mav+(mbv*11*(.5^(( 5-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv6 =mav+(mbv*11*(.5^(( 6-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv7 =mav+(mbv*11*(.5^(( 7-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv8 =mav+(mbv*11*(.5^(( 8-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv9 =mav+(mbv*11*(.5^(( 9-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv10=mav+(mbv*11*(.5^((10-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv11=mav+(mbv*11*(.5^((11-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv12=mav+(mbv*11*(.5^((12-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);

```

```

!loadings on 1st factor, constrained to d(y)/d(mav) and d(y)/d(maq)
0=1-av1 ; 0=1-aq1 ;
0=1-av2 ; 0=1-aq2 ;
0=1-av3 ; 0=1-aq3 ;
0=1-av4 ; 0=1-aq4 ;
0=1-av5 ; 0=1-aq5 ;
0=1-av6 ; 0=1-aq6 ;
0=1-av7 ; 0=1-aq7 ;
0=1-av8 ; 0=1-aq8 ;
0=1-av9 ; 0=1-aq9 ;
0=1-av10; 0=1-aq10;
0=1-av11; 0=1-aq11;
0=1-av12; 0=1-aq12;

```

```

!loadings on 2nd factor for verbal, constrained to d(y)/d(mbv)
bv1 =(11*(.5^(( 1-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv2 =(11*(.5^(( 2-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv3 =(11*(.5^(( 3-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv4 =(11*(.5^(( 4-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv5 =(11*(.5^(( 5-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv6 =(11*(.5^(( 6-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv7 =(11*(.5^(( 7-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv8 =(11*(.5^(( 8-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv9 =(11*(.5^(( 9-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv10=(11*(.5^((10-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv11=(11*(.5^((11-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv12=(11*(.5^((12-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);

```

```

!loadings on 3rd factor for verbal, constrained to d(y)/d(mcv)
cv1 =(.5^( 1/(mcv-1)))*mcv*((.5^(1/(mcv-1)))*( 1-1)+(.5^(12/(mcv-1)))*(12- 1))*11*ln(.5))/
((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv2 =(.5^( 2/(mcv-1)))*mcv*((.5^(1/(mcv-1)))*( 2-1)+(.5^(12/(mcv-1)))*(12- 2))*11*ln(.5))/
((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv3 =(.5^( 3/(mcv-1)))*mcv*((.5^(1/(mcv-1)))*( 3-1)+(.5^(12/(mcv-1)))*(12- 3))*11*ln(.5))/
((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv4 =(.5^( 4/(mcv-1)))*mcv*((.5^(1/(mcv-1)))*( 4-1)+(.5^(12/(mcv-1)))*(12- 4))*11*ln(.5))/
((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv5 =(.5^( 5/(mcv-1)))*mcv*((.5^(1/(mcv-1)))*( 5-1)+(.5^(12/(mcv-1)))*(12- 5))*11*ln(.5))/
((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv6 =(.5^( 6/(mcv-1)))*mcv*((.5^(1/(mcv-1)))*( 6-1)+(.5^(12/(mcv-1)))*(12- 6))*11*ln(.5))/
((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv7 =(.5^( 7/(mcv-1)))*mcv*((.5^(1/(mcv-1)))*( 7-1)+(.5^(12/(mcv-1)))*(12- 7))*11*ln(.5))/

```

```

((mcv-1)^2)*((.5^(11/(mcv-1))-.5^(1/(mcv-1))))^2));
cv8 =(.5^( 8/(mcv-1))*mcy* (.5^(1/(mcv-1))*( 8-1)+.5^(12/(mcv-1))*(12- 8))*11*ln(.5))/
((mcv-1)^2)*((.5^(11/(mcv-1))-.5^(1/(mcv-1))))^2));
cv9 =(.5^( 9/(mcv-1))*mcy* (.5^(1/(mcv-1))*( 9-1)+.5^(12/(mcv-1))*(12- 9))*11*ln(.5))/
((mcv-1)^2)*((.5^(11/(mcv-1))-.5^(1/(mcv-1))))^2));
cv10=(.5^(10/(mcv-1))*mcy* (.5^(1/(mcv-1))*(10-1)+.5^(12/(mcv-1))*(12-10))*11*ln(.5))/
((mcv-1)^2)*((.5^(11/(mcv-1))-.5^(1/(mcv-1))))^2));
cv11=(.5^(11/(mcv-1))*mcy* (.5^(1/(mcv-1))*(11-1)+.5^(12/(mcv-1))*(12-11))*11*ln(.5))/
((mcv-1)^2)*((.5^(11/(mcv-1))-.5^(1/(mcv-1))))^2));
cv12=(.5^(12/(mcv-1))*mcy* (.5^(1/(mcv-1))*(12-1)+.5^(12/(mcv-1))*(12-12))*11*ln(.5))/
((mcv-1)^2)*((.5^(11/(mcv-1))-.5^(1/(mcv-1))))^2));

```

**!quantitative intercepts constrained equal to target function**

```

tq1 =maq+(mbq*11*(.5^(( 1-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq2 =maq+(mbq*11*(.5^(( 2-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq3 =maq+(mbq*11*(.5^(( 3-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq4 =maq+(mbq*11*(.5^(( 4-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq5 =maq+(mbq*11*(.5^(( 5-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq6 =maq+(mbq*11*(.5^(( 6-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq7 =maq+(mbq*11*(.5^(( 7-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq8 =maq+(mbq*11*(.5^(( 8-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq9 =maq+(mbq*11*(.5^(( 9-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq10=maq+(mbq*11*(.5^((10-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq11=maq+(mbq*11*(.5^((11-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq12=maq+(mbq*11*(.5^((12-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);

```

**!loadings on 2nd factor for quantitative, constrained to d(y)/d(mbv)**

```

bq1 = (11*(.5^(( 1-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq2 = (11*(.5^(( 2-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq3 = (11*(.5^(( 3-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq4 = (11*(.5^(( 4-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq5 = (11*(.5^(( 5-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq6 = (11*(.5^(( 6-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq7 = (11*(.5^(( 7-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq8 = (11*(.5^(( 8-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq9 = (11*(.5^(( 9-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq10=(11*(.5^((10-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq11=(11*(.5^((11-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq12=(11*(.5^((12-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);

```

**!loadings on 3rd factor for quantitative, constrained to d(y)/d(mcv)**

```

cq1 =(.5^( 1/(mcq-1))*mcq* (.5^(1/(mcq-1))*( 1-1)+.5^(12/(mcq-1))*(12- 1))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));
cq2 =(.5^( 2/(mcq-1))*mcq* (.5^(1/(mcq-1))*( 2-1)+.5^(12/(mcq-1))*(12- 2))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));
cq3 =(.5^( 3/(mcq-1))*mcq* (.5^(1/(mcq-1))*( 3-1)+.5^(12/(mcq-1))*(12- 3))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));
cq4 =(.5^( 4/(mcq-1))*mcq* (.5^(1/(mcq-1))*( 4-1)+.5^(12/(mcq-1))*(12- 4))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));
cq5 =(.5^( 5/(mcq-1))*mcq* (.5^(1/(mcq-1))*( 5-1)+.5^(12/(mcq-1))*(12- 5))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));
cq6 =(.5^( 6/(mcq-1))*mcq* (.5^(1/(mcq-1))*( 6-1)+.5^(12/(mcq-1))*(12- 6))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));
cq7 =(.5^( 7/(mcq-1))*mcq* (.5^(1/(mcq-1))*( 7-1)+.5^(12/(mcq-1))*(12- 7))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));
cq8 =(.5^( 8/(mcq-1))*mcq* (.5^(1/(mcq-1))*( 8-1)+.5^(12/(mcq-1))*(12- 8))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));
cq9 =(.5^( 9/(mcq-1))*mcq* (.5^(1/(mcq-1))*( 9-1)+.5^(12/(mcq-1))*(12- 9))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));
cq10=(.5^(10/(mcq-1))*mcq* (.5^(1/(mcq-1))*(10-1)+.5^(12/(mcq-1))*(12-10))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));
cq11=(.5^(11/(mcq-1))*mcq* (.5^(1/(mcq-1))*(11-1)+.5^(12/(mcq-1))*(12-11))*11*ln(.5))/
((mcq-1)^2)*((.5^(11/(mcq-1))-.5^(1/(mcq-1))))^2));

```

```
cq12=(.5^(12/(mcq-1))*mcq*(.5^(1/(mcq-1)))*(12-1)+(.5^(12/(mcq-1)))*(12-12))*11*ln(.5)/
((mcq-1)^2)*((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2);
```

```
diff=mcv-mcq; !difference in the ARC parameters for verbal and quantitative
```

```
OUTPUT: TECH1 TECH4;
```

## 6. Mathematica code for obtaining partial derivatives of an exponential function

```
y = f[t_] = Exp[a + b * x]
```

```
 $e^{a+bx}$ 
```

```
yp1 = D[f[t], a]
```

```
 $e^{a+bx}$ 
```

```
yp2 = D[f[t], b]
```

```
 $e^{a+bx} x$ 
```

```
Lambda = {{yp1, yp2}}
```

```
 $\{ \{ e^{a+bx}, e^{a+bx} x \} \}$ 
```



## 7. Mplus syntax for a quadratic function fit to Kanfer & Ackerman's data

*Traditional structured latent curve:*

```
TITLE: Cudeck and du Toit (2002) quadratic curve;
DATA: FILE IS kanfer.dat; !data file in ascii format
TYPE IS COVARIANCE MEANS; !input data are covariances and means
NOBSERVATIONS IS 140; !sample size is 140
VARIABLE: NAMES ARE y1-y9; !all 9 variables
!y1-y9:
ANALYSIS: ESTIMATOR=ML; !use maximum likelihood estimation
ITERATIONS=5000; !use 5000 iterations
MODEL: y1-y9*9.9(v); [y1-y9@0]; !residual covariances constrained equal, intercepts = 0
[a0*16 ay*39](ma0 may); [ax@0]; !intercept and maximum means estimated, maximizer = 0
a0*123; ax*18; ay*53; !random coefficient variances
a0 WITH ax*-.8 ay*35; ax WITH ay*8.9; !random coefficient covariances
a0 BY y1-y9*(a01-a09); ax BY y1-y9*(ax1-ax9); ay BY y1-y9*(ay1-ay9); !loadings

MODEL CONSTRAINT: NEW(max*8.8); !define maximizer mean
```

```
!loadings on maximum, intercept, and maximizer factors, constrained to
!d(y)/d(may), d(y)/d(ma0), and d(y)/d(max)
ay1=1-((0/max)-1)^2; a01=((0/max)-1)^2; ax1=2*(may-ma0)*(0-max)*0/max^3;
ay2=1-((1/max)-1)^2; a02=((1/max)-1)^2; ax2=2*(may-ma0)*(1-max)*1/max^3;
ay3=1-((2/max)-1)^2; a03=((2/max)-1)^2; ax3=2*(may-ma0)*(2-max)*2/max^3;
ay4=1-((3/max)-1)^2; a04=((3/max)-1)^2; ax4=2*(may-ma0)*(3-max)*3/max^3;
ay5=1-((4/max)-1)^2; a05=((4/max)-1)^2; ax5=2*(may-ma0)*(4-max)*4/max^3;
ay6=1-((5/max)-1)^2; a06=((5/max)-1)^2; ax6=2*(may-ma0)*(5-max)*5/max^3;
ay7=1-((6/max)-1)^2; a07=((6/max)-1)^2; ax7=2*(may-ma0)*(6-max)*6/max^3;
ay8=1-((7/max)-1)^2; a08=((7/max)-1)^2; ax8=2*(may-ma0)*(7-max)*7/max^3;
```

```
ay9=1-((8/max)-1)^2; a09=((8/max)-1)^2; ax9=2*(may-ma0)*(8-max)*8/max^3;
```

```
OUTPUT: TECH1 TECH4;
```

### *Modified structured latent curve:*

```
TITLE: Cudeck and du Toit (2002) quadratic curve;
DATA: FILE IS kanfer.dat; !data file in ascii format
TYPE IS COVARIANCE MEANS; !input data are covariances and means
NOBSEVATIONS IS 140; !sample size is 140
VARIABLE: NAMES ARE y1-y9; !all 9 variables
ANALYSIS: ESTIMATOR=ML; !use maximum likelihood estimation
ITERATIONS=5000; !use 5000 iterations
MODEL: [y1-y9*](my1-my9); !intercepts constrained equal to target function
y1-y9*9.9(v); !residual covariances constrained equal
[a0@0 ax@0 ay@0]; !factor means constrained to zero
a0*123; ax*18; ay*53; !random coefficient variances
a0 WITH ax*-.8 ay*35; ax WITH ay*8.9; !random coefficient covariances
a0 BY y1-y9*(a01-a09); ax BY y1-y9*(ax1-ax9); ay BY y1-y9*(ay1-ay9); !loadings
```

```
MODEL CONSTRAINT:
```

```
NEW(ma0*16 max*8.8 may*39); !define mean intercept, maximizer, and maximum
```

```
!intercepts constrained equal to target function
```

```
my1=may-(may-ma0)*((0/max)-1)^2;
my2=may-(may-ma0)*((1/max)-1)^2;
my3=may-(may-ma0)*((2/max)-1)^2;
my4=may-(may-ma0)*((3/max)-1)^2;
my5=may-(may-ma0)*((4/max)-1)^2;
my6=may-(may-ma0)*((5/max)-1)^2;
my7=may-(may-ma0)*((6/max)-1)^2;
my8=may-(may-ma0)*((7/max)-1)^2;
my9=may-(may-ma0)*((8/max)-1)^2;
```

```
!loadings on maximum, intercept, and maximizer factors, constrained to
!d(y)/d(may), d(y)/d(ma0), and d(y)/d(max)
```

```
ay1=1-((0/max)-1)^2; a01=((0/max)-1)^2; ax1=2*(may-ma0)*(0-max)*0/max^3;
ay2=1-((1/max)-1)^2; a02=((1/max)-1)^2; ax2=2*(may-ma0)*(1-max)*1/max^3;
ay3=1-((2/max)-1)^2; a03=((2/max)-1)^2; ax3=2*(may-ma0)*(2-max)*2/max^3;
ay4=1-((3/max)-1)^2; a04=((3/max)-1)^2; ax4=2*(may-ma0)*(3-max)*3/max^3;
ay5=1-((4/max)-1)^2; a05=((4/max)-1)^2; ax5=2*(may-ma0)*(4-max)*4/max^3;
ay6=1-((5/max)-1)^2; a06=((5/max)-1)^2; ax6=2*(may-ma0)*(5-max)*5/max^3;
ay7=1-((6/max)-1)^2; a07=((6/max)-1)^2; ax7=2*(may-ma0)*(6-max)*6/max^3;
ay8=1-((7/max)-1)^2; a08=((7/max)-1)^2; ax8=2*(may-ma0)*(7-max)*7/max^3;
ay9=1-((8/max)-1)^2; a09=((8/max)-1)^2; ax9=2*(may-ma0)*(8-max)*8/max^3;
```

```
OUTPUT: TECH1 TECH4;
```

### *Kanfer & Ackerman's (1989) data (N = 140), a mean vector and covariance matrix:*

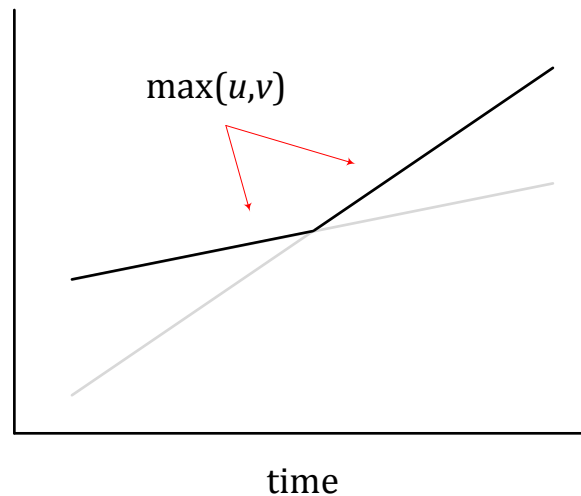
```
20.121 25.521 29.321 32.400 34.186 35.600 37.729 39.029 38.786
93.650
79.637 93.564
72.890 87.718 95.732
62.623 76.941 81.886 86.054
53.342 63.803 69.612 70.611 73.666
43.820 50.644 54.443 56.446 58.353 59.154
35.183 41.099 46.016 45.494 49.900 46.163 54.169
39.839 44.099 46.105 44.074 48.345 45.847 49.329 60.528
35.497 39.169 42.262 39.429 42.954 42.950 47.120 54.570 66.183
```

## 8. Derivation of Equation 11: Reparameterization of a bilinear spline target function

A common expression for the bilinear spline model is:

$$y = \begin{cases} \theta_1 + \theta_2 t & t \leq \theta_k \\ \theta_3 + \theta_4 t & t > \theta_k \end{cases}$$

where  $\theta_1$  and  $\theta_2$  are the intercept and slope for the first segment,  $\theta_3$  and  $\theta_4$  are the intercept and slope for the second segment, and  $\theta_k$  is the knot. This expression is not convenient for linearization, so it needs to be changed. To perform the first step of the reparameterization, it is first necessary to decide whether the slope of the second segment is larger or smaller than the slope of the first segment. In most applications, this choice will be clear by inspection. In cases for which the slope of the second segment is greater than the slope of the first segment (as in our example), a bilinear spline model can be understood as the maximum of the two functions at any point  $t$ . Visually, at the knot point the linear equation for the second segment "takes over" where the equation for the first segment leaves off, so it is clear why the maximum would be chosen in this instance (if the slope of the second segment is expected to be less than that of the first, we would use the minimum rather than the maximum):



Harring, Cudeck, and du Toit (2006, Appx. C) and Kohli and Harring (2013, Appx. A) provide a convenient expression for the minimum of two functions:

$$\min(u, v) = \frac{1}{2} \left( u + v - \sqrt{(u - v)^2} \right),$$

from which we may derive a convenient expression for the maximum:

$$\max(u,v) = \frac{1}{2} \left( u + v + \sqrt{(u-v)^2} \right).$$

In words, the maximum of two functions  $u$  and  $v$  is the sum of the two functions plus the absolute value of their difference, all divided by 2. The first step of the reparameterization consists of substituting the earlier expressions for each segment into this  $\max(\cdot)$  function:

$$\begin{aligned} \min(u,v) &= \frac{1}{2} \left( u + v + \sqrt{(u-v)^2} \right) \\ &= \frac{1}{2} \left( \theta_1 + \theta_2 t + \theta_3 + \theta_4 t + \sqrt{(\theta_1 + \theta_2 t - (\theta_3 + \theta_4 t))^2} \right) \\ &= \frac{1}{2} \left( (\theta_1 + \theta_3) + (\theta_2 + \theta_4)t + \sqrt{[(\theta_2 - \theta_4)t + (\theta_1 - \theta_3)]^2} \right) \end{aligned}$$

Typically, the two functions join at the knot, in which case there are effectively four parameters, not five. The second step in this reparameterization involves eliminating one of the parameters by replacing it with an equivalent expression involving some of the other parameters. Returning to the original expressions for each segment, we observe that where  $t = \theta_k$ ,

$$\begin{aligned} \theta_1 + \theta_2 \theta_k &= \theta_3 + \theta_4 \theta_k \\ \theta_1 - \theta_3 &= (\theta_4 - \theta_2) \theta_k \end{aligned}$$

Therefore, we can substitute  $(\theta_4 - \theta_2) \theta_k$  for  $(\theta_1 - \theta_3)$  in the  $\max(\cdot)$  function, yielding:

$$\begin{aligned} \max(u,v) &= \frac{1}{2} \left( (\theta_1 + \theta_3) + (\theta_2 + \theta_4)t + \sqrt{[(\theta_2 - \theta_4)t + (\theta_1 - \theta_3)]^2} \right) \\ &= \frac{1}{2} \left( (\theta_1 + \theta_3) + (\theta_2 + \theta_4)t + \sqrt{[(\theta_2 - \theta_4)t + (\theta_4 - \theta_2)\theta_k]^2} \right) \\ &= \frac{1}{2} \left( (\theta_1 + \theta_3) + (\theta_2 + \theta_4)t + \sqrt{[(\theta_4 - \theta_2)\theta_k - (\theta_4 - \theta_2)t]^2} \right) \\ &= \frac{1}{2} \left( (\theta_1 + \theta_3) + (\theta_2 + \theta_4)t + \sqrt{[(\theta_4 - \theta_2)(\theta_k - t)]^2} \right) \\ &= \frac{1}{2} \left( (\theta_1 + \theta_3) + (\theta_2 + \theta_4)t + \sqrt{(\theta_4 - \theta_2)^2 (\theta_k - t)^2} \right) \\ &= \frac{1}{2} \left( (\theta_1 + \theta_3) + (\theta_2 + \theta_4)t + (\theta_4 - \theta_2) \sqrt{(t - \theta_k)^2} \right) \end{aligned}$$

In the final step, bringing  $(\theta_4 - \theta_2)$  outside the radical is permissible because we started with the assumption that  $\theta_4 > \theta_2$ , implying that  $(\theta_4 - \theta_2)$  is a positive square root. Finally, given that we are

emphasizing the importance of  $\theta_\kappa$  and do not care much about the other parts of the equation, we can make the following substitutions with no loss:

$$\omega_1 = \frac{\theta_1 + \theta_3}{2} \quad \omega_2 = \frac{\theta_2 + \theta_4}{2} \quad \omega_3 = \frac{\theta_4 - \theta_2}{2}$$

so that

$$y = \omega_1 + \omega_2 t + \omega_3 \sqrt{(t - \theta_\kappa)^2}.$$

## 9. Annotated output: inverse quadratic function

*Only the most relevant output has been included below.*

```
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10/02/2014 3:18 PM
```

INPUT INSTRUCTIONS

```
TITLE: davidian theophylline data, inverse quadratic;
DATA: FILE IS theophylline_wide.dat; !data file in ascii format
VARIABLE: NAMES ARE subject dose wt t1-t11 conc1-conc11; !all 25 variables
!subject: subject ID
!dose: oral dose of theophylline in mg/kg
!wt: subject weight in kg
!t1-t11: time since administration of dose in hours
!conc1-conc11: theophylline concentration in mg/L
USEVARIABLES ARE conc1-conc11; !we plan to model 11 variables
CONSTRAINT ARE t1-t11; !individual-specific time scores for use in loadings
ANALYSIS: ESTIMATOR=ML; ITERATIONS=2000; CONVERGENCE=.001; !estimation options
!BOOTSTRAP=5000; !for bootstrap confidence intervals if desired
MODEL:

[conc1-conc11*](z1-z11); !mean trend (z's defined below)
conc1-conc11*.7(ve); !residual variances constrained equal
conc1-conc10 PWITH conc2-conc11*.2(ve2); !optional lag-1 residual covariances
b*.002(vb); c@0; m*.5(vm); !random coeff. variances (only for maximizer and theta1)
b WITH c@0; b WITH m*; c WITH m@0; !random coefficient covariances
[b@0 c@0 m@0]; !factor means constrained to zero
b BY conc1*(b1); b BY conc2-conc11*(b2-b11); !loadings for the theta1 parameter (b)
c BY conc1*(c1); c BY conc2-conc11*(c2-c11); !loadings for the theta2 parameter (c)
m BY conc1*(m1); m BY conc2-conc11*(m2-m11); !loadings for the maximizer parameter (m)

MODEL CONSTRAINT:

NEW(mb*.07 mc*.02 mm*1.77); !starts for theta1, theta2, and maximizer parameters

!intercepts constrained equal to target function
z1=t1/(mb*t1+mc*(mm^2+t1^2));
z2=t2/(mb*t2+mc*(mm^2+t2^2));
z3=t3/(mb*t3+mc*(mm^2+t3^2));
z4=t4/(mb*t4+mc*(mm^2+t4^2));
z5=t5/(mb*t5+mc*(mm^2+t5^2));
z6=t6/(mb*t6+mc*(mm^2+t6^2));
z7=t7/(mb*t7+mc*(mm^2+t7^2));
```

```
z8=t8/(mb*t8+mc*(mm^2+t8^2));
z9=t9/(mb*t9+mc*(mm^2+t9^2));
z10=t10/(mb*t10+mc*(mm^2+t10^2));
z11=t11/(mb*t11+mc*(mm^2+t11^2));
```

```
!loadings for "theta1" factor, constrained equal to d(y)/d(mb)
```

```
b1=-1*t1^2/(mb*t1+mc*(mm^2+t1^2))^2;
b2=-1*t2^2/(mb*t2+mc*(mm^2+t2^2))^2;
b3=-1*t3^2/(mb*t3+mc*(mm^2+t3^2))^2;
b4=-1*t4^2/(mb*t4+mc*(mm^2+t4^2))^2;
b5=-1*t5^2/(mb*t5+mc*(mm^2+t5^2))^2;
b6=-1*t6^2/(mb*t6+mc*(mm^2+t6^2))^2;
b7=-1*t7^2/(mb*t7+mc*(mm^2+t7^2))^2;
b8=-1*t8^2/(mb*t8+mc*(mm^2+t8^2))^2;
b9=-1*t9^2/(mb*t9+mc*(mm^2+t9^2))^2;
b10=-1*t10^2/(mb*t10+mc*(mm^2+t10^2))^2;
b11=-1*t11^2/(mb*t11+mc*(mm^2+t11^2))^2;
```

```
!loadings for "theta2" factor, constrained equal to d(y)/d(mc)
```

```
c1=-1*t1*(mm^2+t1^2)/(mb*t1+mc*(mm^2+t1^2))^2; !not strictly necessary
c2=-1*t2*(mm^2+t2^2)/(mb*t2+mc*(mm^2+t2^2))^2; !here because this
c3=-1*t3*(mm^2+t3^2)/(mb*t3+mc*(mm^2+t3^2))^2; !coefficient is fixed,
c4=-1*t4*(mm^2+t4^2)/(mb*t4+mc*(mm^2+t4^2))^2; !but included for
c5=-1*t5*(mm^2+t5^2)/(mb*t5+mc*(mm^2+t5^2))^2; !completeness.
c6=-1*t6*(mm^2+t6^2)/(mb*t6+mc*(mm^2+t6^2))^2;
c7=-1*t7*(mm^2+t7^2)/(mb*t7+mc*(mm^2+t7^2))^2;
c8=-1*t8*(mm^2+t8^2)/(mb*t8+mc*(mm^2+t8^2))^2;
c9=-1*t9*(mm^2+t9^2)/(mb*t9+mc*(mm^2+t9^2))^2;
c10=-1*t10*(mm^2+t10^2)/(mb*t10+mc*(mm^2+t10^2))^2;
c11=-1*t11*(mm^2+t11^2)/(mb*t11+mc*(mm^2+t11^2))^2;
```

```
!loadings for "maximizer" factor, constrained equal to d(y)/d(mm)
```

```
m1=-2*mm*mc*t1/(mb*t1+mc*(mm^2+t1^2))^2;
m2=-2*mm*mc*t2/(mb*t2+mc*(mm^2+t2^2))^2;
m3=-2*mm*mc*t3/(mb*t3+mc*(mm^2+t3^2))^2;
m4=-2*mm*mc*t4/(mb*t4+mc*(mm^2+t4^2))^2;
m5=-2*mm*mc*t5/(mb*t5+mc*(mm^2+t5^2))^2;
m6=-2*mm*mc*t6/(mb*t6+mc*(mm^2+t6^2))^2;
m7=-2*mm*mc*t7/(mb*t7+mc*(mm^2+t7^2))^2;
m8=-2*mm*mc*t8/(mb*t8+mc*(mm^2+t8^2))^2;
m9=-2*mm*mc*t9/(mb*t9+mc*(mm^2+t9^2))^2;
m10=-2*mm*mc*t10/(mb*t10+mc*(mm^2+t10^2))^2;
m11=-2*mm*mc*t11/(mb*t11+mc*(mm^2+t11^2))^2;
```

```
OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);
```

```
INPUT READING TERMINATED NORMALLY
```

```
dauidian theophylline data, inverse quadratic;
```

#### SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	12
Number of dependent variables	11
Number of independent variables	0
Number of continuous latent variables	3

```
Observed dependent variables
```





	CONC10	999.000	0.000	999.000	999.000	
	CONC11	999.000	0.000	999.000	999.000	
M	BY					
	CONC1	999.000	0.000	999.000	999.000	
	CONC2	999.000	0.000	999.000	999.000	
	CONC3	999.000	0.000	999.000	999.000	
	CONC4	999.000	0.000	999.000	999.000	
	CONC5	999.000	0.000	999.000	999.000	
	CONC6	999.000	0.000	999.000	999.000	
	CONC7	999.000	0.000	999.000	999.000	
	CONC8	999.000	0.000	999.000	999.000	
	CONC9	999.000	0.000	999.000	999.000	
	CONC10	999.000	0.000	999.000	999.000	
	CONC11	999.000	0.000	999.000	999.000	
B	WITH					
	C	0.000	0.000	999.000	999.000	
	M	-0.026	0.016	-1.572	0.116	!COVARIANCE OF MAXIMIZER !AND THETA1 RANDOM !COEFFICIENTS
C	WITH					
	M	0.000	0.000	999.000	999.000	
CONC1	WITH					
	CONC2	0.226	0.077	2.943	0.003	!LAG-1 ERROR COVARIANCE
CONC2	WITH					!
	CONC3	0.226	0.077	2.943	0.003	!
CONC3	WITH					!
	CONC4	0.226	0.077	2.943	0.003	!
CONC4	WITH					!
	CONC5	0.226	0.077	2.943	0.003	!
CONC5	WITH					!
	CONC6	0.226	0.077	2.943	0.003	!
CONC6	WITH					!
	CONC7	0.226	0.077	2.943	0.003	!
CONC7	WITH					!
	CONC8	0.226	0.077	2.943	0.003	!
CONC8	WITH					!
	CONC9	0.226	0.077	2.943	0.003	!
CONC9	WITH					!
	CONC10	0.226	0.077	2.943	0.003	!
CONC10	WITH					!
	CONC11	0.226	0.077	2.943	0.003	!
Means						
	B	0.000	0.000	999.000	999.000	!FACTOR MEANS FIXED = 0
	C	0.000	0.000	999.000	999.000	!
	M	0.000	0.000	999.000	999.000	!
Intercepts						
	CONC1	999.000	0.000	999.000	999.000	
	CONC2	999.000	0.000	999.000	999.000	
	CONC3	999.000	0.000	999.000	999.000	
	CONC4	999.000	0.000	999.000	999.000	

CONC5	999.000	0.000	999.000	999.000
CONC6	999.000	0.000	999.000	999.000
CONC7	999.000	0.000	999.000	999.000
CONC8	999.000	0.000	999.000	999.000
CONC9	999.000	0.000	999.000	999.000
CONC10	999.000	0.000	999.000	999.000
CONC11	999.000	0.000	999.000	999.000

Variances

B	0.002	0.001	1.559	0.119	!VARIANCES OF THETA1,
C	0.000	0.000	999.000	999.000	!THETA2, AND MAXIMIZER
M	0.522	0.291	1.796	0.073	!COEFFICIENTS

Residual Variances

CONC1	0.731	0.119	6.119	0.000	!RESIDUAL VARIANCE, HELD
CONC2	0.731	0.119	6.119	0.000	!EQUAL OVER TIME
CONC3	0.731	0.119	6.119	0.000	!
CONC4	0.731	0.119	6.119	0.000	!
CONC5	0.731	0.119	6.119	0.000	!
CONC6	0.731	0.119	6.119	0.000	!
CONC7	0.731	0.119	6.119	0.000	!
CONC8	0.731	0.119	6.119	0.000	!
CONC9	0.731	0.119	6.119	0.000	!
CONC10	0.731	0.119	6.119	0.000	!
CONC11	0.731	0.119	6.119	0.000	!

New/Additional Parameters

MB	0.069	0.013	5.258	0.000	!MEAN THETA1 ESTIMATE
MC	0.017	0.001	14.672	0.000	!MEAN THETA2 ESTIMATE
MM	1.768	0.189	9.342	0.000	!MEAN MAXIMIZER ESTIMATE

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.496E-07  
 (ratio of smallest to largest eigenvalue)

## 10. Annotated output: unconditional bilinear spline model

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 06/10/2014 5:36 PM

INPUT INSTRUCTIONS

```

TITLE: zerbe glucose;
DATA: FILE IS glucose.dat; !data file in ascii format
VARIABLE: NAMES ARE obese ph1-ph8; !all 9 variables
USEVARIABLES ARE ph1-ph8; !we plan to model 8 variables
!obese: obesity status, 0=control, 1=obese
!ph1-ph8: plasma phosphate concentration repeated measures
!ANALYSIS: BOOTSTRAP IS 1000; !for bootstrap confidence intervals if desired
MODEL:

[ph1-ph8*](z1-z8); !mean trend (z's defined below)
ph1-ph8*(ve); !residual variances constrained equal
w1-w4*; !random coefficient variances
w1 WITH w2*; w1 WITH w3*; w1 WITH w4*; !random coefficient covariances
w2 WITH w3*; w2 WITH w4*; w3 WITH w4*; !random coefficient covariances
[w1@0 w2@0 w3@0 w4@0]; !factor means constrained to zero
w1 BY ph1-ph8@1; !loadings for 1st factor, constrained to 1
w2 BY ph1*(w21); w2 BY ph2-ph8(w22-w28); !loadings for 2nd factor, defined below
w3 BY ph1*(w31); w3 BY ph2-ph8(w32-w38); !loadings for 3rd factor, defined below

```

w4 BY ph1\*(w41); w4 BY ph2-ph8(w42-w48); !loadings for knot factor, defined below

MODEL CONSTRAINT:

!starts for parameters of the target function  
NEW(mw1\*3.4 mw2\*-.26 mw3\*.54 mw4\*1.6 b1-b4);

!intercepts constrained equal to target function  
z1=mw1+mw2\*0.0+mw3\*sqrt((0.0-mw4)^2);  
z2=mw1+mw2\*0.5+mw3\*sqrt((0.5-mw4)^2);  
z3=mw1+mw2\*1.0+mw3\*sqrt((1.0-mw4)^2);  
z4=mw1+mw2\*1.5+mw3\*sqrt((1.5-mw4)^2);  
z5=mw1+mw2\*2.0+mw3\*sqrt((2.0-mw4)^2);  
z6=mw1+mw2\*3.0+mw3\*sqrt((3.0-mw4)^2);  
z7=mw1+mw2\*4.0+mw3\*sqrt((4.0-mw4)^2);  
z8=mw1+mw2\*5.0+mw3\*sqrt((5.0-mw4)^2);

!loadings for 2nd and 3rd factors, constrained to d(y)/d(mw2) and d(y)/d(mw3)  
w21=0.0; w31=sqrt((0.0-mw4)^2);  
w22=0.5; w32=sqrt((0.5-mw4)^2);  
w23=1.0; w33=sqrt((1.0-mw4)^2);  
w24=1.5; w34=sqrt((1.5-mw4)^2);  
w25=2.0; w35=sqrt((2.0-mw4)^2);  
w26=3.0; w36=sqrt((3.0-mw4)^2);  
w27=4.0; w37=sqrt((4.0-mw4)^2);  
w28=5.0; w38=sqrt((5.0-mw4)^2);

!loadings for knot factor, constrained to d(y)/d(mw4)  
w41=(mw3\*(mw4-0.0))/sqrt((0.0-mw4)^2);  
w42=(mw3\*(mw4-0.5))/sqrt((0.5-mw4)^2);  
w43=(mw3\*(mw4-1.0))/sqrt((1.0-mw4)^2);  
w44=(mw3\*(mw4-1.5))/sqrt((1.5-mw4)^2);  
w45=(mw3\*(mw4-2.0))/sqrt((2.0-mw4)^2);  
w46=(mw3\*(mw4-3.0))/sqrt((3.0-mw4)^2);  
w47=(mw3\*(mw4-4.0))/sqrt((4.0-mw4)^2);  
w48=(mw3\*(mw4-5.0))/sqrt((5.0-mw4)^2);

!parameters of original parameterization  
b1=mw1+mw3\*mw4;  
b2=mw2-mw3;  
b3=mw1-mw3\*mw4;  
b4=mw2+mw3;

OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);

INPUT READING TERMINATED NORMALLY

zerbe glucose;

SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	33
Number of dependent variables	8
Number of independent variables	0
Number of continuous latent variables	4

Observed dependent variables

Continuous					
PH1	PH2	PH3	PH4	PH5	PH6
PH7	PH8				

Continuous latent variables

W1 W2 W3 W4

Estimator ML  
Information matrix OBSERVED  
Maximum number of iterations 1000  
Convergence criterion 0.500D-04  
Maximum number of steepest descent iterations 20

Input data file(s)  
glucose.dat

Input data format FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters 15

Loglikelihood

H0 Value -176.244  
H1 Value -142.162

Information Criteria

Akaike (AIC) 382.487  
Bayesian (BIC) 404.935  
Sample-Size Adjusted BIC 358.147  
(n\* = (n + 2) / 24)

Chi-Square Test of Model Fit

Value 68.163  
Degrees of Freedom 29  
P-Value 0.0001

RMSEA (Root Mean Square Error Of Approximation)

Estimate 0.202  
90 Percent C.I. 0.140 0.265  
Probability RMSEA <= .05 0.000

CFI/TLI

CFI 0.843  
TLI 0.849

Chi-Square Test of Model Fit for the Baseline Model

Value 277.636  
Degrees of Freedom 28  
P-Value 0.0000

SRMR (Standardized Root Mean Square Residual)

Value 0.212

MODEL RESULTS

Two-Tailed

		Estimate	S.E.	Est./S.E.	P-Value	
W1	BY					
	PH1	1.000	0.000	999.000	999.000	
	PH2	1.000	0.000	999.000	999.000	
	PH3	1.000	0.000	999.000	999.000	
	PH4	1.000	0.000	999.000	999.000	
	PH5	1.000	0.000	999.000	999.000	
	PH6	1.000	0.000	999.000	999.000	
	PH7	1.000	0.000	999.000	999.000	
	PH8	1.000	0.000	999.000	999.000	
W2	BY					
	PH1	0.000	0.000	999.000	999.000	
	PH2	0.500	0.000	999.000	999.000	
	PH3	1.000	0.000	999.000	999.000	
	PH4	1.500	0.000	999.000	999.000	
	PH5	2.000	0.000	999.000	999.000	
	PH6	3.000	0.000	999.000	999.000	
	PH7	4.000	0.000	999.000	999.000	
	PH8	5.000	0.000	999.000	999.000	
W3	BY					
	PH1	1.588	0.112	14.181	0.000	
	PH2	1.088	0.112	9.715	0.000	
	PH3	0.588	0.112	5.248	0.000	
	PH4	0.088	0.112	0.782	0.434	
	PH5	0.412	0.112	3.684	0.000	
	PH6	1.412	0.112	12.616	0.000	
	PH7	2.412	0.112	21.549	0.000	
	PH8	3.412	0.112	30.481	0.000	
W4	BY					
	PH1	0.539	0.040	13.436	0.000	
	PH2	0.539	0.040	13.436	0.000	
	PH3	0.539	0.040	13.436	0.000	
	PH4	0.539	0.040	13.436	0.000	
	PH5	-0.539	0.040	-13.436	0.000	
	PH6	-0.539	0.040	-13.436	0.000	
	PH7	-0.539	0.040	-13.436	0.000	
	PH8	-0.539	0.040	-13.436	0.000	
W1	WITH					!COVARIANCES OF RANDOM
W2		-0.030	0.028	-1.064	0.287	!COEFFICIENTS
W3		-0.024	0.030	-0.810	0.418	!
W4		-0.008	0.085	-0.089	0.929	!
W2	WITH					!
W3		-0.009	0.009	-1.028	0.304	!
W4		0.018	0.026	0.701	0.484	!
W3	WITH					!
W4		0.001	0.027	0.036	0.971	!
Means						
W1		0.000	0.000	999.000	999.000	!FACTOR MEANS FIXED = 0
W2		0.000	0.000	999.000	999.000	!
W3		0.000	0.000	999.000	999.000	!
W4		0.000	0.000	999.000	999.000	!
Intercepts						
PH1		4.256	0.134	31.879	0.000	
PH2		3.858	0.124	31.115	0.000	

PH3	3.459	0.123	28.182	0.000
PH4	3.061	0.130	23.537	0.000
PH5	3.107	0.120	25.928	0.000
PH6	3.388	0.104	32.705	0.000
PH7	3.668	0.098	37.354	0.000
PH8	3.949	0.105	37.477	0.000

Variances

W1	0.457	0.135	3.397	0.001	!VARIANCES OF RANDOM
W2	0.013	0.010	1.291	0.197	!COEFFICIENTS
W3	0.027	0.013	2.029	0.042	!
W4	0.254	0.110	2.304	0.021	!

Residual Variances

PH1	0.103	0.013	8.124	0.000	!RESIDUAL VARIANCE, HELD EQUAL
PH2	0.103	0.013	8.124	0.000	!OVER TIME
PH3	0.103	0.013	8.124	0.000	!
PH4	0.103	0.013	8.124	0.000	!
PH5	0.103	0.013	8.124	0.000	!
PH6	0.103	0.013	8.124	0.000	!
PH7	0.103	0.013	8.124	0.000	!
PH8	0.103	0.013	8.124	0.000	!

New/Additional Parameters

MW1	3.400	0.128	26.470	0.000	!MEAN OF 1ST GROWTH PARAMETER
MW2	-0.258	0.034	-7.526	0.000	!MEAN OF 2ND GROWTH PARAMETER
MW3	0.539	0.040	13.436	0.000	!MEAN OF 3RD GROWTH PARAMETER
MW4	1.588	0.112	14.181	0.000	!MEAN OF KNOT PARAMETER
B1	4.256	0.134	31.879	0.000	!INTERCEPT FOR FIRST SEGMENT
B2	-0.797	0.066	-12.158	0.000	!SLOPE FOR FIRST SEGMENT
B3	2.545	0.171	14.881	0.000	!INTERCEPT FOR SECOND SEGMENT
B4	0.281	0.036	7.876	0.000	!SLOPE FOR SECOND SEGMENT

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.260E-05  
 (ratio of smallest to largest eigenvalue)

## 11. Annotated output: conditional bilinear spline model

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INPUT INSTRUCTIONS

```

TITLE: zerbe glucose;
DATA: FILE IS glucose.dat; !data file in ascii format
DEFINE: xobese=abs(obese-1); !1=control; 0=obese
VARIABLE: NAMES ARE obese ph1-ph8; !all 9 variables
!obese: obesity status, 0=control, 1=obese
!ph1-ph8: plasma phosphate concentration repeated measures
USEVARIABLES ARE ph1-ph8 xobese; !we plan to model 9 variables
!ANALYSIS: BOOTSTRAP IS 1000; !for bootstrap confidence intervals if desired
MODEL:

[ph1-ph8*](z1-z8); !mean trend (z's defined below)
ph1-ph8*(ve); !residual variances constrained equal
w1-w4*; !random coefficient residual variances
w1 WITH w2*; w1 WITH w3*; w1 WITH w4*; !random coefficient residual covariances
w2 WITH w3*; w2 WITH w4*; w3 WITH w4*; !random coefficient residual covariances
[w1@0 w2@0 w3@0 w4@0]; !factor intercepts constrained to zero

```

```

w1 BY ph1-ph8@1; !loadings for 1st factor, constrained to 1
w2 BY ph1*(w21); w2 BY ph2-ph8(w22-w28); !loadings for 2nd factor, defined below
w3 BY ph1*(w31); w3 BY ph2-ph8(w32-w38); !loadings for 3rd factor, defined below
w4 BY ph1*(w41); w4 BY ph2-ph8(w42-w48); !loadings for knot factor, defined below
w1-w4 ON xobese; !regress random coefficients on obese

```

```

MODEL CONSTRAINT:

```

```

!starts for parameters of the target function
NEW(mw1*3.5 mw2*-.21 mw3*.49 mw4*1.95 b1-b4);

```

```

!intercepts constrained equal to target function

```

```

z1=mw1+mw2*0.0+mw3*sqrt((0.0-mw4)^2);
z2=mw1+mw2*0.5+mw3*sqrt((0.5-mw4)^2);
z3=mw1+mw2*1.0+mw3*sqrt((1.0-mw4)^2);
z4=mw1+mw2*1.5+mw3*sqrt((1.5-mw4)^2);
z5=mw1+mw2*2.0+mw3*sqrt((2.0-mw4)^2);
z6=mw1+mw2*3.0+mw3*sqrt((3.0-mw4)^2);
z7=mw1+mw2*4.0+mw3*sqrt((4.0-mw4)^2);
z8=mw1+mw2*5.0+mw3*sqrt((5.0-mw4)^2);

```

```

!loadings for 2nd and 3rd factors, constrained to d(y)/d(mw2) and d(y)/d(mw3)

```

```

w21=0.0; w31=sqrt((0.0-mw4)^2);
w22=0.5; w32=sqrt((0.5-mw4)^2);
w23=1.0; w33=sqrt((1.0-mw4)^2);
w24=1.5; w34=sqrt((1.5-mw4)^2);
w25=2.0; w35=sqrt((2.0-mw4)^2);
w26=3.0; w36=sqrt((3.0-mw4)^2);
w27=4.0; w37=sqrt((4.0-mw4)^2);
w28=5.0; w38=sqrt((5.0-mw4)^2);

```

```

!loadings for knot factor, constrained to d(y)/d(mw4)

```

```

w41=(mw3*(mw4-0.0))/sqrt((0.0-mw4)^2);
w42=(mw3*(mw4-0.5))/sqrt((0.5-mw4)^2);
w43=(mw3*(mw4-1.0))/sqrt((1.0-mw4)^2);
w44=(mw3*(mw4-1.5))/sqrt((1.5-mw4)^2);
w45=(mw3*(mw4-2.0))/sqrt((2.0-mw4)^2);
w46=(mw3*(mw4-3.0))/sqrt((3.0-mw4)^2);
w47=(mw3*(mw4-4.0))/sqrt((4.0-mw4)^2);
w48=(mw3*(mw4-5.0))/sqrt((5.0-mw4)^2);

```

```

!parameters of original parameterization

```

```

b1=mw1+mw3*mw4;
b2=mw2-mw3;
b3=mw1-mw3*mw4;
b4=mw2+mw3;

```

```

OUTPUT: TECH1; !CINTERVAL(BCBOOTSTRAP);

```

```

INPUT READING TERMINATED NORMALLY

```

```

zerbe glucose;

```

```

SUMMARY OF ANALYSIS

```

Number of groups	1
Number of observations	33
Number of dependent variables	8
Number of independent variables	1
Number of continuous latent variables	4

```

Observed dependent variables

```





SRMR (Standardized Root Mean Square Residual)

Value 0.192

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
W1	BY				
	PH1	1.000	0.000	999.000	999.000
	PH2	1.000	0.000	999.000	999.000
	PH3	1.000	0.000	999.000	999.000
	PH4	1.000	0.000	999.000	999.000
	PH5	1.000	0.000	999.000	999.000
	PH6	1.000	0.000	999.000	999.000
	PH7	1.000	0.000	999.000	999.000
	PH8	1.000	0.000	999.000	999.000
W2	BY				
	PH1	0.000	0.000	999.000	999.000
	PH2	0.500	0.000	999.000	999.000
	PH3	1.000	0.000	999.000	999.000
	PH4	1.500	0.000	999.000	999.000
	PH5	2.000	0.000	999.000	999.000
	PH6	3.000	0.000	999.000	999.000
	PH7	4.000	0.000	999.000	999.000
	PH8	5.000	0.000	999.000	999.000
W3	BY				
	PH1	1.951	0.123	15.799	0.000
	PH2	1.451	0.123	11.749	0.000
	PH3	0.951	0.123	7.699	0.000
	PH4	0.451	0.123	3.649	0.000
	PH5	0.049	0.123	0.401	0.689
	PH6	1.049	0.123	8.501	0.000
	PH7	2.049	0.123	16.600	0.000
	PH8	3.049	0.123	24.700	0.000
W4	BY				
	PH1	0.490	0.050	9.860	0.000
	PH2	0.490	0.050	9.860	0.000
	PH3	0.490	0.050	9.860	0.000
	PH4	0.490	0.050	9.860	0.000
	PH5	-0.490	0.050	-9.860	0.000
	PH6	-0.490	0.050	-9.860	0.000
	PH7	-0.490	0.050	-9.860	0.000
	PH8	-0.490	0.050	-9.860	0.000
W1	ON				!REGRESSION OF 1ST RANDOM
	XOBESE	-0.387	0.254	-1.521	0.128 !COEFFICIENT ON OBESITY
W2	ON				!REGRESSION OF 2ND RANDOM
	XOBESE	-0.117	0.067	-1.750	0.080 !COEFFICIENT ON OBESITY
W3	ON				!REGRESSION OF 3RD RANDOM
	XOBESE	0.124	0.079	1.560	0.119 !COEFFICIENT ON OBESITY
W4	ON				!REGRESSION OF RANDOM KNOT
	XOBESE	-1.013	0.253	-4.004	0.000 !COEFFICIENT ON OBESITY
W1	WITH				!COVARIANCES OF RANDOM
	W2	-0.041	0.027	-1.499	0.134 !COEFFICIENT RESIDUALS

W3		-0.013	0.028	-0.464	0.643	!
W4		-0.084	0.077	-1.093	0.274	!
						!
W2	WITH					!
W3		-0.006	0.008	-0.714	0.475	!
W4		-0.001	0.023	-0.061	0.951	!
						!
W3	WITH					!
W4		0.011	0.023	0.468	0.639	!
Intercepts						
PH1		4.509	0.156	28.814	0.000	
PH2		4.158	0.136	30.638	0.000	
PH3		3.807	0.125	30.533	0.000	
PH4		3.456	0.126	27.431	0.000	
PH5		3.154	0.153	20.563	0.000	
PH6		3.432	0.132	25.906	0.000	
PH7		3.711	0.126	29.539	0.000	
PH8		3.989	0.135	29.566	0.000	
W1		0.000	0.000	999.000	999.000	!FACTOR INTERCEPTS FIXED = 0
W2		0.000	0.000	999.000	999.000	!
W3		0.000	0.000	999.000	999.000	!
W4		0.000	0.000	999.000	999.000	!
Residual Variances						
PH1		0.103	0.013	8.124	0.000	!RESIDUAL VARIANCE, HELD EQUAL
PH2		0.103	0.013	8.124	0.000	!OVER TIME
PH3		0.103	0.013	8.124	0.000	!
PH4		0.103	0.013	8.124	0.000	!
PH5		0.103	0.013	8.124	0.000	!
PH6		0.103	0.013	8.124	0.000	!
PH7		0.103	0.013	8.124	0.000	!
PH8		0.103	0.013	8.124	0.000	!
W1		0.421	0.126	3.349	0.001	!RESIDUAL VARIANCES OF RANDOM
W2		0.010	0.009	1.043	0.297	!COEFFICIENTS
W3		0.024	0.013	1.879	0.060	!
W4		0.076	0.082	0.929	0.353	!
New/Additional Parameters						
MW1		3.553	0.160	22.272	0.000	!CONDITIONAL MEANS OF ALL 4
MW2		-0.212	0.042	-5.026	0.000	!GROWTH PARAMETERS
MW3		0.490	0.050	9.860	0.000	!
MW4		1.951	0.123	15.799	0.000	!
B1		4.509	0.156	28.814	0.000	!CONDITIONAL MEANS OF ORIGINAL
B2		-0.702	0.080	-8.778	0.000	!INTERCEPTS AND SLOPES FOR
B3		2.596	0.219	11.845	0.000	!FIRST AND SECOND SEGMENTS
B4		0.278	0.046	6.080	0.000	!

#### QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.308E-05  
 (ratio of smallest to largest eigenvalue)

## 12. Annotated output: negative exponential function with intercept, asymptote, and rate

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#### INPUT INSTRUCTIONS

TITLE: chaiken, negative exponential, original parameterization;

```

DATA: FILE IS schaik2.dat; !data file in ascii format
VARIABLE: NAMES ARE v1-v12 q1-q12; !all 24 variables
!v1-v12: verbal skill acquisition repeated measures
!q1-q12: quantitative skill acquisition repeated measures
ANALYSIS: ESTIMATOR IS ML; !use maximum likelihood estimation
MODEL:

!verbal
[v1-v12*](tv1-tv12); !mean trend for verbal
v1-v12*1.97(vv1); !residual variances constrained equal
v1-v11 PWITH v2-v12*(dv1); !lag-1 error covariance
v1-v10 PWITH v3-v12*(dv2); !lag-2, etc...
v1-v9 PWITH v4-v12*(dv3);
v1-v8 PWITH v5-v12*(dv4);
v1-v7 PWITH v6-v12*(dv5);
v1-v6 PWITH v7-v12*(dv6);
v1-v5 PWITH v8-v12*(dv7);
v1-v4 PWITH v9-v12*(dv8);
v1-v3 PWITH v10-v12*(dv9);
v1-v2 PWITH v11-v12*(dv10);
v1 WITH v12*(dv11);
[av@0 bv@0 cv@0]; !factor means constrained to zero
av*; bv*; cv*; !random coefficient variances
av WITH bv* cv*; bv WITH cv*; !random coefficient covariances
av BY v1*(av1); av BY v2-v12(av2-av12); !loadings for 1st factor
bv BY v1*(bv1); bv BY v2-v12(bv2-bv12); !loadings for 2nd factor
cv BY v1*(cv1); cv BY v2-v12(cv2-cv12); !loadings for 3rd factor

!quantitative
[q1-q12*](tq1-tq12); !mean trend for quantitative
q1-q12*1.21(vq1); !residual variances constrained equal
q1-q11 PWITH q2-q12*(dq1); !lag-1 error covariance
q1-q10 PWITH q3-q12*(dq2); !lag-2, etc...
q1-q9 PWITH q4-q12*(dq3);
q1-q8 PWITH q5-q12*(dq4);
q1-q7 PWITH q6-q12*(dq5);
q1-q6 PWITH q7-q12*(dq6);
q1-q5 PWITH q8-q12*(dq7);
q1-q4 PWITH q9-q12*(dq8);
q1-q3 PWITH q10-q12*(dq9);
q1-q2 PWITH q11-q12*(dq10);
q1 WITH q12*(dq11);
[aq@0 bq@0 cq@0]; !factor means constrained to zero
aq*; bq*; cq*; !random coefficient variances
aq WITH bq* cq*; bq WITH cq; !random coefficient covariances
aq BY q1*(aq1); aq BY q2-q12(aq2-aq12); !loadings for 1st factor
bq BY q1*(bq1); bq BY q2-q12(bq2-bq12); !loadings for 2nd factor
cq BY q1*(cq1); cq BY q2-q12(cq2-cq12); !loadings for 3rd factor

!cross-domain covariances
av WITH aq* bq*; bv WITH aq* bq*;

MODEL CONSTRAINT:

!starts for parameters of the target function and for lagged error covariances
NEW(mav*6.8 mbv*21 mcv*.7 maq*8.6 mbq*16.5 mcq*.7 rhov*.27 rhoq*.31);

!error covariances
dv1= vv1*rhov; dq1= vq1*rhoq;
dv2= vv1*rhov^2; dq2= vq1*rhoq^2;
dv3= vv1*rhov^3; dq3= vq1*rhoq^3;
dv4= vv1*rhov^4; dq4= vq1*rhoq^4;
dv5= vv1*rhov^5; dq5= vq1*rhoq^5;

```

```
dv6= vv1*rhov^6; dq6= vq1*rhoq^6;
dv7= vv1*rhov^7; dq7= vq1*rhoq^7;
dv8= vv1*rhov^8; dq8= vq1*rhoq^8;
dv9= vv1*rhov^9; dq9= vq1*rhoq^9;
dv10=vv1*rhov^10; dq10=vq1*rhoq^10;
dv11=vv1*rhov^11; dq11=vq1*rhoq^11;
```

```
!verbal and quantitative intercepts constrained equal to target function
tv1 =mav-(mav-mbv)*exp(-1*mcv*( 1-1)); tq1 =maq-(maq-mbq)*exp(-1*mcq*( 1-1));
tv2 =mav-(mav-mbv)*exp(-1*mcv*( 2-1)); tq2 =maq-(maq-mbq)*exp(-1*mcq*( 2-1));
tv3 =mav-(mav-mbv)*exp(-1*mcv*( 3-1)); tq3 =maq-(maq-mbq)*exp(-1*mcq*( 3-1));
tv4 =mav-(mav-mbv)*exp(-1*mcv*( 4-1)); tq4 =maq-(maq-mbq)*exp(-1*mcq*( 4-1));
tv5 =mav-(mav-mbv)*exp(-1*mcv*( 5-1)); tq5 =maq-(maq-mbq)*exp(-1*mcq*( 5-1));
tv6 =mav-(mav-mbv)*exp(-1*mcv*( 6-1)); tq6 =maq-(maq-mbq)*exp(-1*mcq*( 6-1));
tv7 =mav-(mav-mbv)*exp(-1*mcv*( 7-1)); tq7 =maq-(maq-mbq)*exp(-1*mcq*( 7-1));
tv8 =mav-(mav-mbv)*exp(-1*mcv*( 8-1)); tq8 =maq-(maq-mbq)*exp(-1*mcq*( 8-1));
tv9 =mav-(mav-mbv)*exp(-1*mcv*( 9-1)); tq9 =maq-(maq-mbq)*exp(-1*mcq*( 9-1));
tv10=mav-(mav-mbv)*exp(-1*mcv*(10-1)); tq10=maq-(maq-mbq)*exp(-1*mcq*(10-1));
tv11=mav-(mav-mbv)*exp(-1*mcv*(11-1)); tq11=maq-(maq-mbq)*exp(-1*mcq*(11-1));
tv12=mav-(mav-mbv)*exp(-1*mcv*(12-1)); tq12=maq-(maq-mbq)*exp(-1*mcq*(12-1));
```

```
!loadings on 1st factor, constrained to d(y)/d(mav) and d(y)/d(maq)
av1 =1-exp(-1*mcv*( 1-1)); aq1 =1-exp(-1*mcq*( 1-1));
av2 =1-exp(-1*mcv*( 2-1)); aq2 =1-exp(-1*mcq*( 2-1));
av3 =1-exp(-1*mcv*( 3-1)); aq3 =1-exp(-1*mcq*( 3-1));
av4 =1-exp(-1*mcv*( 4-1)); aq4 =1-exp(-1*mcq*( 4-1));
av5 =1-exp(-1*mcv*( 5-1)); aq5 =1-exp(-1*mcq*( 5-1));
av6 =1-exp(-1*mcv*( 6-1)); aq6 =1-exp(-1*mcq*( 6-1));
av7 =1-exp(-1*mcv*( 7-1)); aq7 =1-exp(-1*mcq*( 7-1));
av8 =1-exp(-1*mcv*( 8-1)); aq8 =1-exp(-1*mcq*( 8-1));
av9 =1-exp(-1*mcv*( 9-1)); aq9 =1-exp(-1*mcq*( 9-1));
av10=1-exp(-1*mcv*(10-1)); aq10=1-exp(-1*mcq*(10-1));
av11=1-exp(-1*mcv*(11-1)); aq11=1-exp(-1*mcq*(11-1));
av12=1-exp(-1*mcv*(12-1)); aq12=1-exp(-1*mcq*(12-1));
```

```
!loadings on 2nd and 3rd factors for verbal, constrained to d(y)/d(mbv) and d(y)/d(mcv)
bv1 =exp(-1*mcv*( 1-1)); cv1 =(mav-mbv)*( 1-1)*exp(-1*mcv*( 1-1));
bv2 =exp(-1*mcv*( 2-1)); cv2 =(mav-mbv)*( 2-1)*exp(-1*mcv*( 2-1));
bv3 =exp(-1*mcv*( 3-1)); cv3 =(mav-mbv)*( 3-1)*exp(-1*mcv*( 3-1));
bv4 =exp(-1*mcv*( 4-1)); cv4 =(mav-mbv)*( 4-1)*exp(-1*mcv*( 4-1));
bv5 =exp(-1*mcv*( 5-1)); cv5 =(mav-mbv)*( 5-1)*exp(-1*mcv*( 5-1));
bv6 =exp(-1*mcv*( 6-1)); cv6 =(mav-mbv)*( 6-1)*exp(-1*mcv*( 6-1));
bv7 =exp(-1*mcv*( 7-1)); cv7 =(mav-mbv)*( 7-1)*exp(-1*mcv*( 7-1));
bv8 =exp(-1*mcv*( 8-1)); cv8 =(mav-mbv)*( 8-1)*exp(-1*mcv*( 8-1));
bv9 =exp(-1*mcv*( 9-1)); cv9 =(mav-mbv)*( 9-1)*exp(-1*mcv*( 9-1));
bv10=exp(-1*mcv*(10-1)); cv10=(mav-mbv)*(10-1)*exp(-1*mcv*(10-1));
bv11=exp(-1*mcv*(11-1)); cv11=(mav-mbv)*(11-1)*exp(-1*mcv*(11-1));
bv12=exp(-1*mcv*(12-1)); cv12=(mav-mbv)*(12-1)*exp(-1*mcv*(12-1));
```

```
!loadings on 2nd and 3rd factors for quant., constrained to d(y)/d(mbv) and d(y)/d(mcv)
bq1 =exp(-1*mcq*( 1-1)); cq1 =(maq-mbq)*( 1-1)*exp(-1*mcq*( 1-1));
bq2 =exp(-1*mcq*( 2-1)); cq2 =(maq-mbq)*( 2-1)*exp(-1*mcq*( 2-1));
bq3 =exp(-1*mcq*( 3-1)); cq3 =(maq-mbq)*( 3-1)*exp(-1*mcq*( 3-1));
bq4 =exp(-1*mcq*( 4-1)); cq4 =(maq-mbq)*( 4-1)*exp(-1*mcq*( 4-1));
bq5 =exp(-1*mcq*( 5-1)); cq5 =(maq-mbq)*( 5-1)*exp(-1*mcq*( 5-1));
bq6 =exp(-1*mcq*( 6-1)); cq6 =(maq-mbq)*( 6-1)*exp(-1*mcq*( 6-1));
bq7 =exp(-1*mcq*( 7-1)); cq7 =(maq-mbq)*( 7-1)*exp(-1*mcq*( 7-1));
bq8 =exp(-1*mcq*( 8-1)); cq8 =(maq-mbq)*( 8-1)*exp(-1*mcq*( 8-1));
bq9 =exp(-1*mcq*( 9-1)); cq9 =(maq-mbq)*( 9-1)*exp(-1*mcq*( 9-1));
bq10=exp(-1*mcq*(10-1)); cq10=(maq-mbq)*(10-1)*exp(-1*mcq*(10-1));
bq11=exp(-1*mcq*(11-1)); cq11=(maq-mbq)*(11-1)*exp(-1*mcq*(11-1));
bq12=exp(-1*mcq*(12-1)); cq12=(maq-mbq)*(12-1)*exp(-1*mcq*(12-1));
```

OUTPUT: TECH1 TECH4;

INPUT READING TERMINATED NORMALLY

chaiken, negative exponential, original parameterization;

SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	228
Number of dependent variables	24
Number of independent variables	0
Number of continuous latent variables	6

Observed dependent variables

Continuous					
V1	V2	V3	V4	V5	V6
V7	V8	V9	V10	V11	V12
Q1	Q2	Q3	Q4	Q5	Q6
Q7	Q8	Q9	Q10	Q11	Q12

Continuous latent variables

AV	BV	CV	AQ	BQ	CQ
----	----	----	----	----	----

Estimator	ML
Information matrix	OBSERVED
Maximum number of iterations	1000
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20

Input data file(s)  
schaiken2.dat

Input data format FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters	31
---------------------------	----

Loglikelihood

H0 Value	-10314.946
H1 Value	-9097.350

Information Criteria

Akaike (AIC)	20691.891
Bayesian (BIC)	20798.201
Sample-Size Adjusted BIC	20699.952
(n* = (n + 2) / 24)	

Chi-Square Test of Model Fit

Value	2435.191
Degrees of Freedom	293
P-Value	0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.179	
90 Percent C.I.	0.173	0.186
Probability RMSEA <= .05	0.000	

CFI/TLI

CFI	0.730
TLI	0.745

Chi-Square Test of Model Fit for the Baseline Model

Value	8204.815
Degrees of Freedom	276
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.218
-------	-------

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
AV	BY				
	V1	0.000	0.000	999.000	999.000
	V2	0.514	0.010	53.981	0.000
	V3	0.763	0.009	82.480	0.000
	V4	0.885	0.007	131.042	0.000
	V5	0.944	0.004	215.555	0.000
	V6	0.973	0.003	365.328	0.000
	V7	0.987	0.002	634.904	0.000
	V8	0.994	0.001	1126.553	0.000
	V9	0.997	0.000	2033.343	0.000
	V10	0.998	0.000	3721.911	0.000
	V11	0.999	0.000	6892.175	0.000
	V12	1.000	0.000	12886.546	0.000
BV	BY				
	V1	1.000	0.000	999.000	999.000
	V2	0.486	0.010	51.121	0.000
	V3	0.237	0.009	25.561	0.000
	V4	0.115	0.007	17.040	0.000
	V5	0.056	0.004	12.780	0.000
	V6	0.027	0.003	10.224	0.000
	V7	0.013	0.002	8.520	0.000
	V8	0.006	0.001	7.303	0.000
	V9	0.003	0.000	6.390	0.000
	V10	0.002	0.000	5.680	0.000
	V11	0.001	0.000	5.112	0.000
	V12	0.000	0.000	4.647	0.000
CV	BY				
	V1	0.000	0.000	999.000	999.000
	V2	-7.020	0.352	-19.949	0.000
	V3	-6.829	0.422	-16.191	0.000
	V4	-4.982	0.382	-13.039	0.000
	V5	-3.231	0.302	-10.712	0.000
	V6	-1.965	0.218	-9.012	0.000
	V7	-1.147	0.148	-7.744	0.000
	V8	-0.651	0.096	-6.772	0.000
	V9	-0.362	0.060	-6.009	0.000
	V10	-0.198	0.037	-5.395	0.000

V11	-0.107	0.022	-4.892	0.000
V12	-0.057	0.013	-4.474	0.000

AQ	BY				
Q1		0.000	0.000	999.000	999.000
Q2		0.502	0.012	43.058	0.000
Q3		0.752	0.012	64.796	0.000
Q4		0.877	0.009	101.169	0.000
Q5		0.939	0.006	163.257	0.000
Q6		0.970	0.004	271.095	0.000
Q7		0.985	0.002	461.203	0.000
Q8		0.992	0.001	800.636	0.000
Q9		0.996	0.001	1413.322	0.000
Q10		0.998	0.000	2529.599	0.000
Q11		0.999	0.000	4579.770	0.000
Q12		1.000	0.000	8371.340	0.000

BQ	BY				
Q1		1.000	0.000	999.000	999.000
Q2		0.498	0.012	42.642	0.000
Q3		0.248	0.012	21.321	0.000
Q4		0.123	0.009	14.214	0.000
Q5		0.061	0.006	10.661	0.000
Q6		0.030	0.004	8.528	0.000
Q7		0.015	0.002	7.107	0.000
Q8		0.008	0.001	6.092	0.000
Q9		0.004	0.001	5.330	0.000
Q10		0.002	0.000	4.738	0.000
Q11		0.001	0.000	4.264	0.000
Q12		0.000	0.000	3.877	0.000

CQ	BY				
Q1		0.000	0.000	999.000	999.000
Q2		-3.913	0.187	-20.931	0.000
Q3		-3.894	0.240	-16.220	0.000
Q4		-2.906	0.233	-12.481	0.000
Q5		-1.928	0.194	-9.932	0.000
Q6		-1.199	0.147	-8.177	0.000
Q7		-0.716	0.103	-6.922	0.000
Q8		-0.416	0.069	-5.988	0.000
Q9		-0.236	0.045	-5.270	0.000
Q10		-0.132	0.028	-4.702	0.000
Q11		-0.073	0.017	-4.243	0.000
Q12		-0.040	0.010	-3.864	0.000

AV	WITH					!COVARIANCES OF VERBAL (V) AND
BV		3.783	0.976	3.875	0.000	!QUANTITATIVE (Q) RANDOM
CV		-0.147	0.059	-2.479	0.013	!COEFFICIENTS
AQ		1.664	0.235	7.097	0.000	!
BQ		2.051	0.606	3.384	0.001	!
BV	WITH					!
CV		-0.468	0.391	-1.196	0.232	!
AQ		8.242	1.468	5.616	0.000	!
BQ		23.425	4.200	5.578	0.000	!
AQ	WITH					!
BQ		7.414	0.985	7.530	0.000	!
CQ		-0.126	0.073	-1.733	0.083	!
CV		-0.205	0.082	-2.484	0.013	!
BQ	WITH					!
CQ		0.540	0.204	2.647	0.008	!



CV		-0.392	0.232	-1.688	0.091	!
						!
CQ	WITH					!
AV		-0.072	0.049	-1.484	0.138	!
BV		0.371	0.318	1.166	0.243	!
CV		0.036	0.018	1.956	0.050	!
						!
V1	WITH					!
V2		0.544	0.092	5.905	0.000	!
V3		0.150	0.045	3.337	0.001	!
V4		0.041	0.018	2.321	0.020	!
V5		0.011	0.006	1.779	0.075	!
V6		0.003	0.002	1.442	0.149	!
V7		0.001	0.001	1.212	0.226	!
V8		0.000	0.000	1.045	0.296	!
V9		0.000	0.000	0.919	0.358	!
V10		0.000	0.000	0.820	0.412	!
V11		0.000	0.000	0.740	0.459	!
V12		0.000	0.000	0.674	0.500	!
						!
V2	WITH					!
V3		0.544	0.092	5.905	0.000	!
V4		0.150	0.045	3.337	0.001	!
V5		0.041	0.018	2.321	0.020	!
V6		0.011	0.006	1.779	0.075	!
V7		0.003	0.002	1.442	0.149	!
V8		0.001	0.001	1.212	0.226	!
V9		0.000	0.000	1.045	0.296	!
V10		0.000	0.000	0.919	0.358	!
V11		0.000	0.000	0.820	0.412	!
V12		0.000	0.000	0.740	0.459	!
						!
V3	WITH					!
V4		0.544	0.092	5.905	0.000	!
V5		0.150	0.045	3.337	0.001	!
V6		0.041	0.018	2.321	0.020	!
V7		0.011	0.006	1.779	0.075	!
V8		0.003	0.002	1.442	0.149	!
V9		0.001	0.001	1.212	0.226	!
V10		0.000	0.000	1.045	0.296	!
V11		0.000	0.000	0.919	0.358	!
V12		0.000	0.000	0.820	0.412	!
						!
V4	WITH					!
V5		0.544	0.092	5.905	0.000	!
V6		0.150	0.045	3.337	0.001	!
V7		0.041	0.018	2.321	0.020	!
V8		0.011	0.006	1.779	0.075	!
V9		0.003	0.002	1.442	0.149	!
V10		0.001	0.001	1.212	0.226	!
V11		0.000	0.000	1.045	0.296	!
V12		0.000	0.000	0.919	0.358	!
						!
V5	WITH					!
V6		0.544	0.092	5.905	0.000	!
V7		0.150	0.045	3.337	0.001	!
V8		0.041	0.018	2.321	0.020	!
V9		0.011	0.006	1.779	0.075	!
V10		0.003	0.002	1.442	0.149	!
V11		0.001	0.001	1.212	0.226	!
V12		0.000	0.000	1.045	0.296	!
						!
V6	WITH					!

V7		0.544	0.092	5.905	0.000	!
V8		0.150	0.045	3.337	0.001	!
V9		0.041	0.018	2.321	0.020	!
V10		0.011	0.006	1.779	0.075	!
V11		0.003	0.002	1.442	0.149	!
V12		0.001	0.001	1.212	0.226	!
V7	WITH					!
V8		0.544	0.092	5.905	0.000	!
V9		0.150	0.045	3.337	0.001	!
V10		0.041	0.018	2.321	0.020	!
V11		0.011	0.006	1.779	0.075	!
V12		0.003	0.002	1.442	0.149	!
V8	WITH					!
V9		0.544	0.092	5.905	0.000	!
V10		0.150	0.045	3.337	0.001	!
V11		0.041	0.018	2.321	0.020	!
V12		0.011	0.006	1.779	0.075	!
V9	WITH					!
V10		0.544	0.092	5.905	0.000	!
V11		0.150	0.045	3.337	0.001	!
V12		0.041	0.018	2.321	0.020	!
V10	WITH					!
V11		0.544	0.092	5.905	0.000	!
V12		0.150	0.045	3.337	0.001	!
V11	WITH					!
V12		0.544	0.092	5.905	0.000	!
Q1	WITH					!
Q2		0.345	0.053	6.528	0.000	!
Q3		0.107	0.028	3.787	0.000	!
Q4		0.033	0.012	2.662	0.008	!
Q5		0.010	0.005	2.051	0.040	!
Q6		0.003	0.002	1.668	0.095	!
Q7		0.001	0.001	1.405	0.160	!
Q8		0.000	0.000	1.214	0.225	!
Q9		0.000	0.000	1.068	0.285	!
Q10		0.000	0.000	0.954	0.340	!
Q11		0.000	0.000	0.862	0.389	!
Q12		0.000	0.000	0.786	0.432	!
Q2	WITH					!
Q3		0.345	0.053	6.528	0.000	!
Q4		0.107	0.028	3.787	0.000	!
Q5		0.033	0.012	2.662	0.008	!
Q6		0.010	0.005	2.051	0.040	!
Q7		0.003	0.002	1.668	0.095	!
Q8		0.001	0.001	1.405	0.160	!
Q9		0.000	0.000	1.214	0.225	!
Q10		0.000	0.000	1.068	0.285	!
Q11		0.000	0.000	0.954	0.340	!
Q12		0.000	0.000	0.862	0.389	!
Q3	WITH					!
Q4		0.345	0.053	6.528	0.000	!
Q5		0.107	0.028	3.787	0.000	!
Q6		0.033	0.012	2.662	0.008	!
Q7		0.010	0.005	2.051	0.040	!
Q8		0.003	0.002	1.668	0.095	!

!AUTOREGRESSIVE RESIDUAL  
!COVARIANCE STRUCTURE FOR  
!QUANTITATIVE REPEATED MEASURES

Q9		0.001	0.001	1.405	0.160	!
Q10		0.000	0.000	1.214	0.225	!
Q11		0.000	0.000	1.068	0.285	!
Q12		0.000	0.000	0.954	0.340	!
Q4	WITH					!
Q5		0.345	0.053	6.528	0.000	!
Q6		0.107	0.028	3.787	0.000	!
Q7		0.033	0.012	2.662	0.008	!
Q8		0.010	0.005	2.051	0.040	!
Q9		0.003	0.002	1.668	0.095	!
Q10		0.001	0.001	1.405	0.160	!
Q11		0.000	0.000	1.214	0.225	!
Q12		0.000	0.000	1.068	0.285	!
Q5	WITH					!
Q6		0.345	0.053	6.528	0.000	!
Q7		0.107	0.028	3.787	0.000	!
Q8		0.033	0.012	2.662	0.008	!
Q9		0.010	0.005	2.051	0.040	!
Q10		0.003	0.002	1.668	0.095	!
Q11		0.001	0.001	1.405	0.160	!
Q12		0.000	0.000	1.214	0.225	!
Q6	WITH					!
Q7		0.345	0.053	6.528	0.000	!
Q8		0.107	0.028	3.787	0.000	!
Q9		0.033	0.012	2.662	0.008	!
Q10		0.010	0.005	2.051	0.040	!
Q11		0.003	0.002	1.668	0.095	!
Q12		0.001	0.001	1.405	0.160	!
Q7	WITH					!
Q8		0.345	0.053	6.528	0.000	!
Q9		0.107	0.028	3.787	0.000	!
Q10		0.033	0.012	2.662	0.008	!
Q11		0.010	0.005	2.051	0.040	!
Q12		0.003	0.002	1.668	0.095	!
Q8	WITH					!
Q9		0.345	0.053	6.528	0.000	!
Q10		0.107	0.028	3.787	0.000	!
Q11		0.033	0.012	2.662	0.008	!
Q12		0.010	0.005	2.051	0.040	!
Q9	WITH					!
Q10		0.345	0.053	6.528	0.000	!
Q11		0.107	0.028	3.787	0.000	!
Q12		0.033	0.012	2.662	0.008	!
Q10	WITH					!
Q11		0.345	0.053	6.528	0.000	!
Q12		0.107	0.028	3.787	0.000	!
Q11	WITH					!
Q12		0.345	0.053	6.528	0.000	!
Means						
AV		0.000	0.000	999.000	999.000	!FACTOR MEANS FIXED = 0
BV		0.000	0.000	999.000	999.000	!
CV		0.000	0.000	999.000	999.000	!
AQ		0.000	0.000	999.000	999.000	!
BQ		0.000	0.000	999.000	999.000	!

CQ	0.000	0.000	999.000	999.000	!
Intercepts					
V1	21.278	0.661	32.192	0.000	
V2	13.865	0.374	37.074	0.000	
V3	10.260	0.238	43.076	0.000	
V4	8.506	0.164	51.793	0.000	
V5	7.653	0.126	60.760	0.000	
V6	7.238	0.109	66.405	0.000	
V7	7.036	0.103	68.566	0.000	
V8	6.938	0.101	69.035	0.000	
V9	6.890	0.100	68.996	0.000	
V10	6.867	0.100	68.879	0.000	
V11	6.856	0.100	68.789	0.000	
V12	6.850	0.100	68.733	0.000	
Q1	16.492	0.404	40.778	0.000	
Q2	12.541	0.261	47.962	0.000	
Q3	10.575	0.202	52.297	0.000	
Q4	9.597	0.171	56.083	0.000	
Q5	9.111	0.155	58.658	0.000	
Q6	8.868	0.148	59.868	0.000	
Q7	8.748	0.145	60.270	0.000	
Q8	8.688	0.144	60.347	0.000	
Q9	8.658	0.144	60.330	0.000	
Q10	8.643	0.143	60.299	0.000	
Q11	8.636	0.143	60.275	0.000	
Q12	8.632	0.143	60.261	0.000	
Variances					
AV	1.652	0.207	7.981	0.000	!VARIANCES OF RANDOM
BV	92.144	8.816	10.452	0.000	!COEFFICIENTS
CV	0.272	0.044	6.232	0.000	!
AQ	4.187	0.423	9.908	0.000	!
BQ	35.772	3.456	10.351	0.000	!
CQ	0.179	0.030	6.058	0.000	!
Residual Variances					
V1	1.974	0.092	21.355	0.000	!VERBAL RESIDUAL VARIANCE
V2	1.974	0.092	21.355	0.000	!HELD EQUAL OVER TIME
V3	1.974	0.092	21.355	0.000	!
V4	1.974	0.092	21.355	0.000	!
V5	1.974	0.092	21.355	0.000	!
V6	1.974	0.092	21.355	0.000	!
V7	1.974	0.092	21.355	0.000	!
V8	1.974	0.092	21.355	0.000	!
V9	1.974	0.092	21.355	0.000	!
V10	1.974	0.092	21.355	0.000	!
V11	1.974	0.092	21.355	0.000	!
V12	1.974	0.092	21.355	0.000	!
Q1	1.112	0.055	20.403	0.000	!QUANTITATIVE RESIDUAL VARIANCE
Q2	1.112	0.055	20.403	0.000	!HELD EQUAL OVER TIME
Q3	1.112	0.055	20.403	0.000	!
Q4	1.112	0.055	20.403	0.000	!
Q5	1.112	0.055	20.403	0.000	!
Q6	1.112	0.055	20.403	0.000	!
Q7	1.112	0.055	20.403	0.000	!
Q8	1.112	0.055	20.403	0.000	!
Q9	1.112	0.055	20.403	0.000	!
Q10	1.112	0.055	20.403	0.000	!
Q11	1.112	0.055	20.403	0.000	!
Q12	1.112	0.055	20.403	0.000	!

New/Additional Parameters

MAV	6.845	0.100	68.667	0.000	!VERBAL MEAN ASYMPTOTE
MBV	21.278	0.661	32.192	0.000	!VERBAL MEAN INITIAL VALUE
MCV	0.721	0.020	36.845	0.000	!VERBAL MEAN RATE PARAMETER
MAQ	8.629	0.143	60.242	0.000	!QUANT. MEAN ASYMPTOTE
MBQ	16.492	0.404	40.778	0.000	!QUANT. MEAN INITIAL VALUE
MCQ	0.698	0.023	29.765	0.000	!QUANT. MEAN RATE PARAMETER
RHOV	0.276	0.036	7.597	0.000	!AUTOREGRESSIVE PARAMETER (V)
RHOQ	0.310	0.035	8.910	0.000	!AUTOREGRESSIVE PARAMETER (Q)

#### QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.168E-03  
 (ratio of smallest to largest eigenvalue)

### 13. Annotated output: negative exponential function with intercept, half-life, and ARC

Mplus VERSION 7.2  
 MUTHEN & MUTHEN  
 06/10/2014 5:55 PM

#### INPUT INSTRUCTIONS

```

TITLE: chaiken, negative exponential with halflife and arc;
DATA: FILE IS schaik2.dat; !data file in ascii format
VARIABLE: NAMES ARE v1-v12 q1-q12; !all 24 variables
!v1-v12: verbal skill acquisition repeated measures
!q1-q12: quantitative skill acquisition repeated measures
ANALYSIS: ESTIMATOR IS ML; !use maximum likelihood estimation
MODEL:

!verbal
[v1-v12*](tv1-tv12); !mean trend for verbal
v1-v12*1.97(vv1); !residual variances constrained equal
v1-v11 PWITH v2-v12*(dv1); !lag-1 error covariance
v1-v10 PWITH v3-v12*(dv2); !lag-2, etc...
v1-v9 PWITH v4-v12*(dv3);
v1-v8 PWITH v5-v12*(dv4);
v1-v7 PWITH v6-v12*(dv5);
v1-v6 PWITH v7-v12*(dv6);
v1-v5 PWITH v8-v12*(dv7);
v1-v4 PWITH v9-v12*(dv8);
v1-v3 PWITH v10-v12*(dv9);
v1-v2 PWITH v11-v12*(dv10);
v1 WITH v12*(dv11);
[av@0 bv@0 cv@0]; !factor means constrained to zero
av*1.6; bv*92; cv*10; !random coefficient variances
av WITH bv*3.7 cv*; bv WITH cv*; !random coefficient covariances
av BY v1*(av1); av BY v2-v12(av2-av12); !loadings for 1st factor
bv BY v1*(bv1); bv BY v2-v12(bv2-bv12); !loadings for 2nd factor
cv BY v1*(cv1); cv BY v2-v12(cv2-cv12); !loadings for 3rd factor

!quantitative
[q1-q12*](tq1-tq12); !mean trend for quantitative
q1-q12*1.21(vq1); !residual variances constrained equal
q1-q11 PWITH q2-q12*(dq1); !lag-1 error covariance
q1-q10 PWITH q3-q12*(dq2); !lag-2, etc...
q1-q9 PWITH q4-q12*(dq3);
q1-q8 PWITH q5-q12*(dq4);
q1-q7 PWITH q6-q12*(dq5);
q1-q6 PWITH q7-q12*(dq6);
q1-q5 PWITH q8-q12*(dq7);

```

```

q1-q4 PWITH q9-q12*(dq8);
q1-q3 PWITH q10-q12*(dq9);
q1-q2 PWITH q11-q12*(dq10);
q1 WITH q12*(dq11);
[eq@0 bq@0 cq@0]; !factor means constrained to zero
aq*4; bq*35; cq*10; !random coefficient variances
aq WITH bq*7.4 cq*; bq WITH cq; !random coefficient covariances
aq BY q1*(aq1); aq BY q2-q12(aq2-aq12); !loadings for 1st factor
bq BY q1*(bq1); bq BY q2-q12(bq2-bq12); !loadings for 2nd factor
cq BY q1*(cq1); cq BY q2-q12(cq2-cq12); !loadings for 3rd factor

!cross-domain covariances
av WITH aq* bq* cq*; bv WITH aq* bq* cq*; cv WITH aq* bq* cq*;

MODEL CONSTRAINT:

!starts for parameters of the target function and for lagged error covariances
NEW(mav*1.7 mbv*-1.5 mcv*1.96 maq*4.2 mbq*-.9 mcq*1.99 rhov*.27 rhoq*.3 diff*0);

!error covariances
dv1= vv1*rhov; dq1= vq1*rhoq;
dv2= vv1*rhov^2; dq2= vq1*rhoq^2;
dv3= vv1*rhov^3; dq3= vq1*rhoq^3;
dv4= vv1*rhov^4; dq4= vq1*rhoq^4;
dv5= vv1*rhov^5; dq5= vq1*rhoq^5;
dv6= vv1*rhov^6; dq6= vq1*rhoq^6;
dv7= vv1*rhov^7; dq7= vq1*rhoq^7;
dv8= vv1*rhov^8; dq8= vq1*rhoq^8;
dv9= vv1*rhov^9; dq9= vq1*rhoq^9;
dv10=vv1*rhov^10; dq10=vq1*rhoq^10;
dv11=vv1*rhov^11; dq11=vq1*rhoq^11;

!verbal intercepts constrained equal to target function
tv1 =mav+(mbv*11*(.5^(( 1-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv2 =mav+(mbv*11*(.5^(( 2-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv3 =mav+(mbv*11*(.5^(( 3-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv4 =mav+(mbv*11*(.5^(( 4-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv5 =mav+(mbv*11*(.5^(( 5-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv6 =mav+(mbv*11*(.5^(( 6-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv7 =mav+(mbv*11*(.5^(( 7-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv8 =mav+(mbv*11*(.5^(( 8-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv9 =mav+(mbv*11*(.5^(( 9-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv10=mav+(mbv*11*(.5^((10-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv11=mav+(mbv*11*(.5^((11-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
tv12=mav+(mbv*11*(.5^((12-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);

!loadings on 1st factor, constrained to d(y)/d(mav) and d(y)/d(maq)
0=1-av1 ; 0=1-aq1 ;
0=1-av2 ; 0=1-aq2 ;
0=1-av3 ; 0=1-aq3 ;
0=1-av4 ; 0=1-aq4 ;
0=1-av5 ; 0=1-aq5 ;
0=1-av6 ; 0=1-aq6 ;
0=1-av7 ; 0=1-aq7 ;
0=1-av8 ; 0=1-aq8 ;
0=1-av9 ; 0=1-aq9 ;
0=1-av10; 0=1-aq10;
0=1-av11; 0=1-aq11;
0=1-av12; 0=1-aq12;

!loadings on 2nd factor for verbal, constrained to d(y)/d(mbv)
bv1 = (11*(.5^(( 1-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);
bv2 = (11*(.5^(( 2-1)/(mcv-1))))/((.5^(11/(mcv-1)))-1);

```

```

bv3 =(11*(.5^(( 3-1)/(mcv-1)))/((.5^(11/(mcv-1)))-1);
bv4 =(11*(.5^(( 4-1)/(mcv-1)))/((.5^(11/(mcv-1)))-1);
bv5 =(11*(.5^(( 5-1)/(mcv-1)))/((.5^(11/(mcv-1)))-1);
bv6 =(11*(.5^(( 6-1)/(mcv-1)))/((.5^(11/(mcv-1)))-1);
bv7 =(11*(.5^(( 7-1)/(mcv-1)))/((.5^(11/(mcv-1)))-1);
bv8 =(11*(.5^(( 8-1)/(mcv-1)))/((.5^(11/(mcv-1)))-1);
bv9 =(11*(.5^(( 9-1)/(mcv-1)))/((.5^(11/(mcv-1)))-1);
bv10=(11*(.5^((10-1)/(mcv-1)))/((.5^(11/(mcv-1)))-1);
bv11=(11*(.5^((11-1)/(mcv-1)))/((.5^(11/(mcv-1)))-1);
bv12=(11*(.5^((12-1)/(mcv-1)))/((.5^(11/(mcv-1)))-1);

```

!loadings on 3rd factor for verbal, constrained to d(y)/d(mcv)

```

cv1 =(.5^( 1/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 1-1)+(.5^(12/(mcv-1)))*(12- 1))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv2 =(.5^( 2/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 2-1)+(.5^(12/(mcv-1)))*(12- 2))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv3 =(.5^( 3/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 3-1)+(.5^(12/(mcv-1)))*(12- 3))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv4 =(.5^( 4/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 4-1)+(.5^(12/(mcv-1)))*(12- 4))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv5 =(.5^( 5/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 5-1)+(.5^(12/(mcv-1)))*(12- 5))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv6 =(.5^( 6/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 6-1)+(.5^(12/(mcv-1)))*(12- 6))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv7 =(.5^( 7/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 7-1)+(.5^(12/(mcv-1)))*(12- 7))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv8 =(.5^( 8/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 8-1)+(.5^(12/(mcv-1)))*(12- 8))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv9 =(.5^( 9/(mcv-1))*mcv*((.5^(1/(mcv-1)))*( 9-1)+(.5^(12/(mcv-1)))*(12- 9))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv10=(.5^(10/(mcv-1))*mcv*((.5^(1/(mcv-1)))*(10-1)+(.5^(12/(mcv-1)))*(12-10))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv11=(.5^(11/(mcv-1))*mcv*((.5^(1/(mcv-1)))*(11-1)+(.5^(12/(mcv-1)))*(12-11))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));
cv12=(.5^(12/(mcv-1))*mcv*((.5^(1/(mcv-1)))*(12-1)+(.5^(12/(mcv-1)))*(12-12))*11*ln(.5))/
(((mcv-1)^2)*(((.5^(11/(mcv-1)))-(.5^(1/(mcv-1))))^2));

```

!quantitative intercepts constrained equal to target function

```

tq1 =maq+(mbq*11*(.5^(( 1-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq2 =maq+(mbq*11*(.5^(( 2-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq3 =maq+(mbq*11*(.5^(( 3-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq4 =maq+(mbq*11*(.5^(( 4-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq5 =maq+(mbq*11*(.5^(( 5-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq6 =maq+(mbq*11*(.5^(( 6-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq7 =maq+(mbq*11*(.5^(( 7-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq8 =maq+(mbq*11*(.5^(( 8-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq9 =maq+(mbq*11*(.5^(( 9-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq10=maq+(mbq*11*(.5^((10-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq11=maq+(mbq*11*(.5^((11-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
tq12=maq+(mbq*11*(.5^((12-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);

```

!loadings on 2nd factor for quantitative, constrained to d(y)/d(mbv)

```

bq1 =(11*(.5^(( 1-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq2 =(11*(.5^(( 2-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq3 =(11*(.5^(( 3-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq4 =(11*(.5^(( 4-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq5 =(11*(.5^(( 5-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq6 =(11*(.5^(( 6-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq7 =(11*(.5^(( 7-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq8 =(11*(.5^(( 8-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq9 =(11*(.5^(( 9-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq10=(11*(.5^((10-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
bq11=(11*(.5^((11-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);

```

```
bq12=(11*(.5^(12-1)/(mcq-1)))/((.5^(11/(mcq-1)))-1);
```

```
!loadings on 3rd factor for quantitative, constrained to d(y)/d(mcv)
```

```
cq1 =(.5^( 1/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 1-1)+(.5^(12/(mcq-1)))*(12- 1))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq2 =(.5^( 2/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 2-1)+(.5^(12/(mcq-1)))*(12- 2))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq3 =(.5^( 3/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 3-1)+(.5^(12/(mcq-1)))*(12- 3))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq4 =(.5^( 4/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 4-1)+(.5^(12/(mcq-1)))*(12- 4))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq5 =(.5^( 5/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 5-1)+(.5^(12/(mcq-1)))*(12- 5))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq6 =(.5^( 6/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 6-1)+(.5^(12/(mcq-1)))*(12- 6))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq7 =(.5^( 7/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 7-1)+(.5^(12/(mcq-1)))*(12- 7))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq8 =(.5^( 8/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 8-1)+(.5^(12/(mcq-1)))*(12- 8))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq9 =(.5^( 9/(mcq-1))*mcq*((.5^(1/(mcq-1)))*( 9-1)+(.5^(12/(mcq-1)))*(12- 9))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq10=(.5^(10/(mcq-1))*mcq*((.5^(1/(mcq-1)))*(10-1)+(.5^(12/(mcq-1)))*(12-10))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq11=(.5^(11/(mcq-1))*mcq*((.5^(1/(mcq-1)))*(11-1)+(.5^(12/(mcq-1)))*(12-11))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
cq12=(.5^(12/(mcq-1))*mcq*((.5^(1/(mcq-1)))*(12-1)+(.5^(12/(mcq-1)))*(12-12))*11*ln(.5))/  
(((mcq-1)^2)*(((.5^(11/(mcq-1)))-(.5^(1/(mcq-1))))^2));
```

```
diff=mcv-mcq; !difference in the ARC parameters for verbal and quantitative
```

```
OUTPUT: TECH1 TECH4;
```

```
INPUT READING TERMINATED NORMALLY
```

```
chaiken, negative exponential with halflife and arc;
```

#### SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	228
Number of dependent variables	24
Number of independent variables	0
Number of continuous latent variables	6

#### Observed dependent variables

Continuous					
V1	V2	V3	V4	V5	V6
V7	V8	V9	V10	V11	V12
Q1	Q2	Q3	Q4	Q5	Q6
Q7	Q8	Q9	Q10	Q11	Q12

#### Continuous latent variables

AV	BV	CV	AQ	BQ	CQ
----	----	----	----	----	----

Estimator	ML
Information matrix	OBSERVED
Maximum number of iterations	1000
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20

```
Input data file(s)
```



schaiken2.dat

Input data format FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters 31

Loglikelihood

H0 Value -10314.946  
H1 Value -9097.350

Information Criteria

Akaike (AIC) 20691.891  
Bayesian (BIC) 20798.201  
Sample-Size Adjusted BIC 20699.952  
(n\* = (n + 2) / 24)

Chi-Square Test of Model Fit

Value 2435.191  
Degrees of Freedom 293  
P-Value 0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate 0.179  
90 Percent C.I. 0.173 0.186  
Probability RMSEA <= .05 0.000

CFI/TLI

CFI 0.730  
TLI 0.745

Chi-Square Test of Model Fit for the Baseline Model

Value 8204.815  
Degrees of Freedom 276  
P-Value 0.0000

SRMR (Standardized Root Mean Square Residual)

Value 0.218

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
AV	BY				
	V1	1.000	0.000	999.000	999.000
	V2	1.000	0.000	999.000	999.000
	V3	1.000	0.000	999.000	999.000
	V4	1.000	0.000	999.000	999.000
	V5	1.000	0.000	999.000	999.000
	V6	1.000	0.000	999.000	999.000
	V7	1.000	0.000	999.000	999.000
	V8	1.000	0.000	999.000	999.000

V9		1.000	0.000	999.000	999.000
V10		1.000	0.000	999.000	999.000
V11		1.000	0.000	999.000	999.000
V12		1.000	0.000	999.000	999.000
BV	BY				
V1		-11.004	0.001	*****	0.000
V2		-5.352	0.105	-50.919	0.000
V3		-2.603	0.102	-25.510	0.000
V4		-1.266	0.074	-17.018	0.000
V5		-0.616	0.048	-12.768	0.000
V6		-0.300	0.029	-10.216	0.000
V7		-0.146	0.017	-8.515	0.000
V8		-0.071	0.010	-7.299	0.000
V9		-0.034	0.005	-6.387	0.000
V10		-0.017	0.003	-5.678	0.000
V11		-0.008	0.002	-5.110	0.000
V12		-0.004	0.001	-4.646	0.000
CV	BY				
V1		-0.064	0.011	-5.731	0.000
V2		-7.906	0.161	-49.135	0.000
V3		-7.676	0.009	-822.989	0.000
V4		-5.596	0.102	-54.932	0.000
V5		-3.628	0.137	-26.529	0.000
V6		-2.206	0.126	-17.478	0.000
V7		-1.287	0.099	-13.030	0.000
V8		-0.730	0.070	-10.386	0.000
V9		-0.406	0.047	-8.633	0.000
V10		-0.222	0.030	-7.386	0.000
V11		-0.120	0.019	-6.454	0.000
V12		-0.064	0.011	-5.731	0.000
AQ	BY				
Q1		1.000	0.000	999.000	999.000
Q2		1.000	0.000	999.000	999.000
Q3		1.000	0.000	999.000	999.000
Q4		1.000	0.000	999.000	999.000
Q5		1.000	0.000	999.000	999.000
Q6		1.000	0.000	999.000	999.000
Q7		1.000	0.000	999.000	999.000
Q8		1.000	0.000	999.000	999.000
Q9		1.000	0.000	999.000	999.000
Q10		1.000	0.000	999.000	999.000
Q11		1.000	0.000	999.000	999.000
Q12		1.000	0.000	999.000	999.000
BQ	BY				
Q1		-11.005	0.001	-8371.135	0.000
Q2		-5.476	0.129	-42.427	0.000
Q3		-2.725	0.128	-21.267	0.000
Q4		-1.356	0.096	-14.190	0.000
Q5		-0.675	0.063	-10.647	0.000
Q6		-0.336	0.039	-8.520	0.000
Q7		-0.167	0.024	-7.101	0.000
Q8		-0.083	0.014	-6.087	0.000
Q9		-0.041	0.008	-5.327	0.000
Q10		-0.021	0.004	-4.735	0.000
Q11		-0.010	0.002	-4.262	0.000
Q12		-0.005	0.001	-3.875	0.000
CQ	BY				
Q1		-0.079	0.016	-4.809	0.000

Q2		-7.717	0.196	-39.405	0.000
Q3		-7.661	0.020	-387.668	0.000
Q4		-5.713	0.118	-48.419	0.000
Q5		-3.788	0.167	-22.730	0.000
Q6		-2.356	0.159	-14.841	0.000
Q7		-1.406	0.128	-11.015	0.000
Q8		-0.816	0.093	-8.756	0.000
Q9		-0.464	0.064	-7.266	0.000
Q10		-0.260	0.042	-6.209	0.000
Q11		-0.144	0.026	-5.420	0.000
Q12		-0.079	0.016	-4.809	0.000
AV	WITH				!COVARIANCES OF VERBAL (V) AND
BV		-0.193	0.084	-2.303	0.021 !QUANTITATIVE (Q) RANDOM
CV		-0.131	0.051	-2.592	0.010 !COEFFICIENTS
AQ		1.664	0.235	7.097	0.000 !
BQ		-0.035	0.046	-0.765	0.444 !
CQ		-0.037	0.025	-1.494	0.135 !
					!
BV	WITH				!
CV		0.025	0.030	0.834	0.404 !
AQ		-0.597	0.126	-4.718	0.000 !
BQ		0.122	0.028	4.403	0.000 !
CQ		-0.021	0.014	-1.434	0.152 !
					!
AQ	WITH				!
BQ		-0.293	0.068	-4.288	0.000 !
CQ		-0.064	0.036	-1.780	0.075 !
					!
BQ	WITH				!
CQ		-0.031	0.008	-3.696	0.000 !
					!
CV	WITH				!
AQ		-0.183	0.071	-2.558	0.011 !
BQ		0.015	0.016	0.966	0.334 !
CQ		0.016	0.008	2.008	0.045 !
					!
V1	WITH				!AUTOREGRESSIVE RESIDUAL
V2		0.544	0.092	5.905	0.000 !COVARIANCE STRUCTURE FOR
V3		0.150	0.045	3.337	0.001 !VERBAL REPEATED MEASURES
V4		0.041	0.018	2.321	0.020 !
V5		0.011	0.006	1.779	0.075 !
V6		0.003	0.002	1.442	0.149 !
V7		0.001	0.001	1.212	0.226 !
V8		0.000	0.000	1.045	0.296 !
V9		0.000	0.000	0.919	0.358 !
V10		0.000	0.000	0.820	0.412 !
V11		0.000	0.000	0.740	0.459 !
V12		0.000	0.000	0.674	0.500 !
					!
V2	WITH				!
V3		0.544	0.092	5.905	0.000 !
V4		0.150	0.045	3.337	0.001 !
V5		0.041	0.018	2.321	0.020 !
V6		0.011	0.006	1.779	0.075 !
V7		0.003	0.002	1.442	0.149 !
V8		0.001	0.001	1.212	0.226 !
V9		0.000	0.000	1.045	0.296 !
V10		0.000	0.000	0.919	0.358 !
V11		0.000	0.000	0.820	0.412 !
V12		0.000	0.000	0.740	0.459 !
					!
V3	WITH				!

V4		0.544	0.092	5.905	0.000	!
V5		0.150	0.045	3.337	0.001	!
V6		0.041	0.018	2.321	0.020	!
V7		0.011	0.006	1.779	0.075	!
V8		0.003	0.002	1.442	0.149	!
V9		0.001	0.001	1.212	0.226	!
V10		0.000	0.000	1.045	0.296	!
V11		0.000	0.000	0.919	0.358	!
V12		0.000	0.000	0.820	0.412	!

V4	WITH					!
V5		0.544	0.092	5.905	0.000	!
V6		0.150	0.045	3.337	0.001	!
V7		0.041	0.018	2.321	0.020	!
V8		0.011	0.006	1.779	0.075	!
V9		0.003	0.002	1.442	0.149	!
V10		0.001	0.001	1.212	0.226	!
V11		0.000	0.000	1.045	0.296	!
V12		0.000	0.000	0.919	0.358	!

V5	WITH					!
V6		0.544	0.092	5.905	0.000	!
V7		0.150	0.045	3.337	0.001	!
V8		0.041	0.018	2.321	0.020	!
V9		0.011	0.006	1.779	0.075	!
V10		0.003	0.002	1.442	0.149	!
V11		0.001	0.001	1.212	0.226	!
V12		0.000	0.000	1.045	0.296	!

V6	WITH					!
V7		0.544	0.092	5.905	0.000	!
V8		0.150	0.045	3.337	0.001	!
V9		0.041	0.018	2.321	0.020	!
V10		0.011	0.006	1.779	0.075	!
V11		0.003	0.002	1.442	0.149	!
V12		0.001	0.001	1.212	0.226	!

V7	WITH					!
V8		0.544	0.092	5.905	0.000	!
V9		0.150	0.045	3.337	0.001	!
V10		0.041	0.018	2.321	0.020	!
V11		0.011	0.006	1.779	0.075	!
V12		0.003	0.002	1.442	0.149	!

V8	WITH					!
V9		0.544	0.092	5.905	0.000	!
V10		0.150	0.045	3.337	0.001	!
V11		0.041	0.018	2.321	0.020	!
V12		0.011	0.006	1.779	0.075	!

V9	WITH					!
V10		0.544	0.092	5.905	0.000	!
V11		0.150	0.045	3.337	0.001	!
V12		0.041	0.018	2.321	0.020	!

V10	WITH					!
V11		0.544	0.092	5.905	0.000	!
V12		0.150	0.045	3.337	0.001	!

V11	WITH					!
V12		0.544	0.092	5.905	0.000	!

Q1	WITH					!AUTOREGRESSIVE RESIDUAL
----	------	--	--	--	--	--------------------------

Q2		0.345	0.053	6.528	0.000	!COVARIANCE STRUCTURE FOR
Q3		0.107	0.028	3.787	0.000	!QUANTITATIVE REPEATED MEASURES
Q4		0.033	0.012	2.662	0.008	!
Q5		0.010	0.005	2.051	0.040	!
Q6		0.003	0.002	1.668	0.095	!
Q7		0.001	0.001	1.405	0.160	!
Q8		0.000	0.000	1.214	0.225	!
Q9		0.000	0.000	1.068	0.285	!
Q10		0.000	0.000	0.954	0.340	!
Q11		0.000	0.000	0.862	0.389	!
Q12		0.000	0.000	0.786	0.432	!
Q2	WITH					!
Q3		0.345	0.053	6.528	0.000	!
Q4		0.107	0.028	3.787	0.000	!
Q5		0.033	0.012	2.662	0.008	!
Q6		0.010	0.005	2.051	0.040	!
Q7		0.003	0.002	1.668	0.095	!
Q8		0.001	0.001	1.405	0.160	!
Q9		0.000	0.000	1.214	0.225	!
Q10		0.000	0.000	1.068	0.285	!
Q11		0.000	0.000	0.954	0.340	!
Q12		0.000	0.000	0.862	0.389	!
Q3	WITH					!
Q4		0.345	0.053	6.528	0.000	!
Q5		0.107	0.028	3.787	0.000	!
Q6		0.033	0.012	2.662	0.008	!
Q7		0.010	0.005	2.051	0.040	!
Q8		0.003	0.002	1.668	0.095	!
Q9		0.001	0.001	1.405	0.160	!
Q10		0.000	0.000	1.214	0.225	!
Q11		0.000	0.000	1.068	0.285	!
Q12		0.000	0.000	0.954	0.340	!
Q4	WITH					!
Q5		0.345	0.053	6.528	0.000	!
Q6		0.107	0.028	3.787	0.000	!
Q7		0.033	0.012	2.662	0.008	!
Q8		0.010	0.005	2.051	0.040	!
Q9		0.003	0.002	1.668	0.095	!
Q10		0.001	0.001	1.405	0.160	!
Q11		0.000	0.000	1.214	0.225	!
Q12		0.000	0.000	1.068	0.285	!
Q5	WITH					!
Q6		0.345	0.053	6.528	0.000	!
Q7		0.107	0.028	3.787	0.000	!
Q8		0.033	0.012	2.662	0.008	!
Q9		0.010	0.005	2.051	0.040	!
Q10		0.003	0.002	1.668	0.095	!
Q11		0.001	0.001	1.405	0.160	!
Q12		0.000	0.000	1.214	0.225	!
Q6	WITH					!
Q7		0.345	0.053	6.528	0.000	!
Q8		0.107	0.028	3.787	0.000	!
Q9		0.033	0.012	2.662	0.008	!
Q10		0.010	0.005	2.051	0.040	!
Q11		0.003	0.002	1.668	0.095	!
Q12		0.001	0.001	1.405	0.160	!
Q7	WITH					!

Q8		0.345	0.053	6.528	0.000	!
Q9		0.107	0.028	3.787	0.000	!
Q10		0.033	0.012	2.662	0.008	!
Q11		0.010	0.005	2.051	0.040	!
Q12		0.003	0.002	1.668	0.095	!
Q8	WITH					!
Q9		0.345	0.053	6.528	0.000	!
Q10		0.107	0.028	3.787	0.000	!
Q11		0.033	0.012	2.662	0.008	!
Q12		0.010	0.005	2.051	0.040	!
Q9	WITH					!
Q10		0.345	0.053	6.528	0.000	!
Q11		0.107	0.028	3.787	0.000	!
Q12		0.033	0.012	2.662	0.008	!
Q10	WITH					!
Q11		0.345	0.053	6.528	0.000	!
Q12		0.107	0.028	3.787	0.000	!
Q11	WITH					!
Q12		0.345	0.053	6.528	0.000	!
Means						
AV		0.000	0.000	999.000	999.000	!FACTOR MEANS FIXED = 0
BV		0.000	0.000	999.000	999.000	!
CV		0.000	0.000	999.000	999.000	!
AQ		0.000	0.000	999.000	999.000	!
BQ		0.000	0.000	999.000	999.000	!
CQ		0.000	0.000	999.000	999.000	!
Intercepts						
V1		21.278	0.661	32.192	0.000	
V2		13.865	0.374	37.074	0.000	
V3		10.260	0.238	43.076	0.000	
V4		8.506	0.164	51.793	0.000	
V5		7.653	0.126	60.759	0.000	
V6		7.238	0.109	66.405	0.000	
V7		7.036	0.103	68.566	0.000	
V8		6.938	0.101	69.035	0.000	
V9		6.890	0.100	68.996	0.000	
V10		6.867	0.100	68.879	0.000	
V11		6.856	0.100	68.788	0.000	
V12		6.850	0.100	68.732	0.000	
Q1		16.492	0.404	40.779	0.000	
Q2		12.541	0.261	47.963	0.000	
Q3		10.576	0.202	52.297	0.000	
Q4		9.597	0.171	56.084	0.000	
Q5		9.111	0.155	58.658	0.000	
Q6		8.868	0.148	59.868	0.000	
Q7		8.748	0.145	60.270	0.000	
Q8		8.688	0.144	60.347	0.000	
Q9		8.658	0.144	60.330	0.000	
Q10		8.643	0.143	60.299	0.000	
Q11		8.636	0.143	60.275	0.000	
Q12		8.632	0.143	60.261	0.000	
Variances						
AV		1.652	0.207	7.981	0.000	!VARIANCES OF RANDOM
BV		0.712	0.069	10.383	0.000	!COEFFICIENTS
CV		0.216	0.023	9.563	0.000	!
AQ		4.187	0.423	9.908	0.000	!

BQ	0.208	0.021	10.104	0.000 !
CQ	0.046	0.006	7.922	0.000 !

Residual Variances

V1	1.974	0.092	21.355	0.000 !	!VERBAL RESIDUAL VARIANCE
V2	1.974	0.092	21.355	0.000 !	!HELD EQUAL OVER TIME
V3	1.974	0.092	21.355	0.000 !	
V4	1.974	0.092	21.355	0.000 !	
V5	1.974	0.092	21.355	0.000 !	
V6	1.974	0.092	21.355	0.000 !	
V7	1.974	0.092	21.355	0.000 !	
V8	1.974	0.092	21.355	0.000 !	
V9	1.974	0.092	21.355	0.000 !	
V10	1.974	0.092	21.355	0.000 !	
V11	1.974	0.092	21.355	0.000 !	
V12	1.974	0.092	21.355	0.000 !	
Q1	1.112	0.055	20.403	0.000 !	!QUANTITATIVE RESIDUAL VARIANCE
Q2	1.112	0.055	20.403	0.000 !	!HELD EQUAL OVER TIME
Q3	1.112	0.055	20.403	0.000 !	
Q4	1.112	0.055	20.403	0.000 !	
Q5	1.112	0.055	20.403	0.000 !	
Q6	1.112	0.055	20.403	0.000 !	
Q7	1.112	0.055	20.403	0.000 !	
Q8	1.112	0.055	20.403	0.000 !	
Q9	1.112	0.055	20.403	0.000 !	
Q10	1.112	0.055	20.403	0.000 !	
Q11	1.112	0.055	20.403	0.000 !	
Q12	1.112	0.055	20.403	0.000 !	

New/Additional Parameters

MAV	6.845	0.100	68.666	0.000 !	!VERBAL MEAN ASYMPTOTE
MBV	-1.312	0.058	-22.498	0.000 !	!VERBAL MEAN ARC PARAMETER
MCV	1.962	0.026	75.155	0.000 !	!VERBAL MEAN HALF-LIFE
MAQ	8.629	0.143	60.242	0.000 !	!QUANT. MEAN ASYMPTOTE
MBQ	-0.715	0.031	-23.159	0.000 !	!QUANT. MEAN ARC PARAMETER
MCQ	1.993	0.033	59.738	0.000 !	!QUANT. MEAN HALF-LIFE
RHOV	0.276	0.036	7.597	0.000 !	!AUTOREGRESSIVE PARAMETER (V)
RHOQ	0.310	0.035	8.910	0.000 !	!AUTOREGRESSIVE PARAMETER (Q)
DIFF	-0.031	0.042	-0.749	0.454 !	!DIFFERENCE IN HALF-LIFES

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.214E-04  
 (ratio of smallest to largest eigenvalue)

## 14. Annotated output: quadratic function fit to Kanfer & Ackerman's data (traditional)

Mplus VERSION 7.2  
 MUTHEN & MUTHEN  
 06/10/2014 11:00 PM

INPUT INSTRUCTIONS

```
TITLE: Cudeck and du Toit (2002) quadratic curve;
DATA: FILE IS kanfer.dat; !data file in ascii format
TYPE IS COVARIANCE MEANS; !input data are covariances and means
NOBSERVATIONS IS 140; !sample size is 140
VARIABLE: NAMES ARE y1-y9; !all 9 variables
ANALYSIS: ESTIMATOR=ML; !use maximum likelihood estimation
ITERATIONS=5000; !use 5000 iterations
MODEL: y1-y9*9.9(v); [y1-y9@0]; !residual covariances constrained equal, intercepts = 0
[a0*16 ay*39](ma0 may); [ax@0]; !intercept and maximum means estimated, maximizer = 0
```

a0\*123; ax\*18; ay\*53; !random coefficient variances  
a0 WITH ax\*-.8 ay\*35; ax WITH ay\*8.9; !random coefficient covariances  
a0 BY y1-y9\*(a01-a09); ax BY y1-y9\*(ax1-ax9); ay BY y1-y9\*(ay1-ay9); !loadings

MODEL CONSTRAINT: NEW(max\*8.8); !define maximizer mean

!loadings on maximum, intercept, and maximizer factors, constrained to  
!d(y)/d(may), d(y)/d(ma0), and d(y)/d(max)  
ay1=1-((0/max)-1)^2; a01=((0/max)-1)^2; ax1=2\*(may-ma0)\*(0-max)\*0/max^3;  
ay2=1-((1/max)-1)^2; a02=((1/max)-1)^2; ax2=2\*(may-ma0)\*(1-max)\*1/max^3;  
ay3=1-((2/max)-1)^2; a03=((2/max)-1)^2; ax3=2\*(may-ma0)\*(2-max)\*2/max^3;  
ay4=1-((3/max)-1)^2; a04=((3/max)-1)^2; ax4=2\*(may-ma0)\*(3-max)\*3/max^3;  
ay5=1-((4/max)-1)^2; a05=((4/max)-1)^2; ax5=2\*(may-ma0)\*(4-max)\*4/max^3;  
ay6=1-((5/max)-1)^2; a06=((5/max)-1)^2; ax6=2\*(may-ma0)\*(5-max)\*5/max^3;  
ay7=1-((6/max)-1)^2; a07=((6/max)-1)^2; ax7=2\*(may-ma0)\*(6-max)\*6/max^3;  
ay8=1-((7/max)-1)^2; a08=((7/max)-1)^2; ax8=2\*(may-ma0)\*(7-max)\*7/max^3;  
ay9=1-((8/max)-1)^2; a09=((8/max)-1)^2; ax9=2\*(may-ma0)\*(8-max)\*8/max^3;

OUTPUT: TECH1 TECH4;

INPUT READING TERMINATED NORMALLY

Cudeck and du Toit (2002) quadratic curve;

SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	140
Number of dependent variables	9
Number of independent variables	0
Number of continuous latent variables	3

Observed dependent variables

Continuous					
Y1	Y2	Y3	Y4	Y5	Y6
Y7	Y8	Y9			

Continuous latent variables

A0	AX	AY
----	----	----

Estimator	ML
Information matrix	EXPECTED
Maximum number of iterations	5000
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20

Input data file(s)

kanfer.dat

Input data format FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters	10
---------------------------	----

Loglikelihood

H0 Value	-3743.236
H1 Value	-3645.210



Information Criteria

Akaike (AIC)	7506.473
Bayesian (BIC)	7535.889
Sample-Size Adjusted BIC	7504.251
(n* = (n + 2) / 24)	

Chi-Square Test of Model Fit

Value	196.054
Degrees of Freedom	44
P-Value	0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.157	
90 Percent C.I.	0.135	0.180
Probability RMSEA <= .05	0.000	

CFI/TLI

CFI	0.909
TLI	0.925

Chi-Square Test of Model Fit for the Baseline Model

Value	1702.962
Degrees of Freedom	36
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.076
-------	-------

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
A0	BY				
	Y1	1.000	0.000	999.000	999.000
	Y2	0.760	0.012	64.221	0.000
	Y3	0.552	0.020	27.380	0.000
	Y4	0.378	0.025	15.099	0.000
	Y5	0.237	0.026	8.959	0.000
	Y6	0.128	0.024	5.275	0.000
	Y7	0.053	0.019	2.819	0.005
	Y8	0.010	0.010	1.064	0.287
	Y9	0.001	0.003	0.252	0.801
AX	BY				
	Y1	0.000	0.000	999.000	999.000
	Y2	-0.519	0.046	11.153	0.000
	Y3	-0.884	0.071	12.430	0.000
	Y4	-1.097	0.075	14.701	0.000
	Y5	-1.157	0.060	19.389	0.000
	Y6	-1.065	0.044	24.352	0.000
	Y7	-0.819	0.081	10.172	0.000
	Y8	-0.421	0.162	2.603	0.009
	Y9	0.130	0.270	-0.482	0.630
AY	BY				

Y1		0.000	0.000	999.000	999.000	
Y2		0.240	0.012	20.318	0.000	
Y3		0.448	0.020	22.194	0.000	
Y4		0.622	0.025	24.853	0.000	
Y5		0.763	0.026	28.917	0.000	
Y6		0.872	0.024	35.897	0.000	
Y7		0.947	0.019	50.687	0.000	
Y8		0.990	0.010	103.048	0.000	
Y9		0.999	0.003	336.999	0.000	
A0	WITH					!COVARIANCES OF RANDOM
AX		-8.248	4.168	-1.979	0.048	!COEFFICIENTS
AY		34.623	7.489	4.623	0.000	!
						!
AX	WITH					!
AY		8.864	6.611	1.341	0.180	!
Means						
A0		20.732	0.846	24.510	0.000	!MEAN INTERCEPT (ESTIMATED)
AX		0.000	0.000	999.000	999.000	!MEAN MAXIMIZER (FIXED = 0)
AY		38.772	0.655	59.185	0.000	!MEAN MAXIMUM (ESTIMATED)
Intercepts						
Y1		0.000	0.000	999.000	999.000	!REPEATED MEASURE INTERCEPTS
Y2		0.000	0.000	999.000	999.000	!ALL FIXED = 0
Y3		0.000	0.000	999.000	999.000	!
Y4		0.000	0.000	999.000	999.000	!
Y5		0.000	0.000	999.000	999.000	!
Y6		0.000	0.000	999.000	999.000	!
Y7		0.000	0.000	999.000	999.000	!
Y8		0.000	0.000	999.000	999.000	!
Y9		0.000	0.000	999.000	999.000	!
Variances						
A0		93.610	11.976	7.816	0.000	!VARIANCE OF INTERCEPT FACTOR
AX		18.019	6.969	2.585	0.010	!VARIANCE OF MAXIMIZER FACTOR
AY		54.708	8.395	6.517	0.000	!VARIANCE OF MAXIMUM FACTOR
Residual Variances						
Y1		9.921	0.484	20.494	0.000	!RESIDUAL VARIANCE HELD EQUAL
Y2		9.921	0.484	20.494	0.000	!OVER TIME
Y3		9.921	0.484	20.494	0.000	!
Y4		9.921	0.484	20.494	0.000	!
Y5		9.921	0.484	20.494	0.000	!
Y6		9.921	0.484	20.494	0.000	!
Y7		9.921	0.484	20.494	0.000	!
Y8		9.921	0.484	20.494	0.000	!
Y9		9.921	0.484	20.494	0.000	!
New/Additional Parameters						
MAX		7.787	0.412	18.924	0.000	!MEAN MAXIMIZER

#### QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.200E-03  
 (ratio of smallest to largest eigenvalue)

## 15. Annotated output: quadratic function fit to Kanfer & Ackerman's data (modified)

06/10/2014 11:00 PM

INPUT INSTRUCTIONS

TITLE: Cudeck and du Toit (2002) quadratic curve;  
DATA: FILE IS kanfer.dat; !data file in ascii format  
TYPE IS COVARIANCE MEANS; !input data are covariances and means  
NOBSERVATIONS IS 140; !sample size is 140  
VARIABLE: NAMES ARE y1-y9; !all 9 variables  
ANALYSIS: ESTIMATOR=ML; !use maximum likelihood estimation  
ITERATIONS=5000; !use 5000 iterations  
MODEL: [y1-y9\*] (my1-my9); !intercepts constrained equal to target function  
y1-y9\*9.9(v); !residual covariances constrained equal  
[a0@0 ax@0 ay@0]; !factor means constrained to zero  
a0\*123; ax\*18; ay\*53; !random coefficient variances  
a0 WITH ax\*-.8 ay\*35; ax WITH ay\*8.9; !random coefficient covariances  
a0 BY y1-y9\*(a01-a09); ax BY y1-y9\*(ax1-ax9); ay BY y1-y9\*(ay1-ay9); !loadings

MODEL CONSTRAINT:

NEW(ma0\*16 max\*8.8 may\*39); !define mean intercept, maximizer, and maximum

!intercepts constrained equal to target function

my1=may-(may-ma0)\*((0/max)-1)^2;  
my2=may-(may-ma0)\*((1/max)-1)^2;  
my3=may-(may-ma0)\*((2/max)-1)^2;  
my4=may-(may-ma0)\*((3/max)-1)^2;  
my5=may-(may-ma0)\*((4/max)-1)^2;  
my6=may-(may-ma0)\*((5/max)-1)^2;  
my7=may-(may-ma0)\*((6/max)-1)^2;  
my8=may-(may-ma0)\*((7/max)-1)^2;  
my9=may-(may-ma0)\*((8/max)-1)^2;

!loadings on maximum, intercept, and maximizer factors, constrained to  
!d(y)/d(may), d(y)/d(ma0), and d(y)/d(max)

ay1=1-((0/max)-1)^2; a01=((0/max)-1)^2; ax1=2\*(may-ma0)\*(0-max)\*0/max^3;  
ay2=1-((1/max)-1)^2; a02=((1/max)-1)^2; ax2=2\*(may-ma0)\*(1-max)\*1/max^3;  
ay3=1-((2/max)-1)^2; a03=((2/max)-1)^2; ax3=2\*(may-ma0)\*(2-max)\*2/max^3;  
ay4=1-((3/max)-1)^2; a04=((3/max)-1)^2; ax4=2\*(may-ma0)\*(3-max)\*3/max^3;  
ay5=1-((4/max)-1)^2; a05=((4/max)-1)^2; ax5=2\*(may-ma0)\*(4-max)\*4/max^3;  
ay6=1-((5/max)-1)^2; a06=((5/max)-1)^2; ax6=2\*(may-ma0)\*(5-max)\*5/max^3;  
ay7=1-((6/max)-1)^2; a07=((6/max)-1)^2; ax7=2\*(may-ma0)\*(6-max)\*6/max^3;  
ay8=1-((7/max)-1)^2; a08=((7/max)-1)^2; ax8=2\*(may-ma0)\*(7-max)\*7/max^3;  
ay9=1-((8/max)-1)^2; a09=((8/max)-1)^2; ax9=2\*(may-ma0)\*(8-max)\*8/max^3;

OUTPUT: TECH1 TECH4;

INPUT READING TERMINATED NORMALLY

Cudeck and du Toit (2002) quadratic curve;

SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	140
Number of dependent variables	9
Number of independent variables	0
Number of continuous latent variables	3

Observed dependent variables

Continuous					
Y1	Y2	Y3	Y4	Y5	Y6

Y7	Y8	Y9	
Continuous latent variables			
A0	AX	AY	
Estimator			ML
Information matrix			EXPECTED
Maximum number of iterations			5000
Convergence criterion			0.500D-04
Maximum number of steepest descent iterations			20

Input data file(s)  
kanfer.dat

Input data format FREE

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters 10

Loglikelihood

H0 Value	-3743.236
H1 Value	-3645.210

Information Criteria

Akaike (AIC)	7506.473
Bayesian (BIC)	7535.889
Sample-Size Adjusted BIC	7504.251
(n* = (n + 2) / 24)	

Chi-Square Test of Model Fit

Value	196.054
Degrees of Freedom	44
P-Value	0.0000

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.157	
90 Percent C.I.	0.135	0.180
Probability RMSEA <= .05	0.000	

CFI/TLI

CFI	0.909
TLI	0.925

Chi-Square Test of Model Fit for the Baseline Model

Value	1702.962
Degrees of Freedom	36
P-Value	0.0000

SRMR (Standardized Root Mean Square Residual)

Value	0.076
-------	-------

MODEL RESULTS

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	
A0	BY					
	Y1	1.000	0.000	999.000	999.000	
	Y2	0.760	0.012	64.222	0.000	
	Y3	0.552	0.020	27.380	0.000	
	Y4	0.378	0.025	15.099	0.000	
	Y5	0.237	0.026	8.959	0.000	
	Y6	0.128	0.024	5.275	0.000	
	Y7	0.053	0.019	2.819	0.005	
	Y8	0.010	0.010	1.064	0.287	
	Y9	0.001	0.003	0.252	0.801	
AX	BY					
	Y1	0.000	0.000	999.000	999.000	
	Y2	-0.519	0.046	-11.154	0.000	
	Y3	-0.884	0.071	-12.430	0.000	
	Y4	-1.097	0.075	-14.701	0.000	
	Y5	-1.157	0.060	-19.389	0.000	
	Y6	-1.065	0.044	-24.352	0.000	
	Y7	-0.819	0.081	-10.172	0.000	
	Y8	-0.421	0.162	-2.603	0.009	
	Y9	0.130	0.270	0.482	0.630	
AY	BY					
	Y1	0.000	0.000	999.000	999.000	
	Y2	0.240	0.012	20.318	0.000	
	Y3	0.448	0.020	22.194	0.000	
	Y4	0.622	0.025	24.854	0.000	
	Y5	0.763	0.026	28.918	0.000	
	Y6	0.872	0.024	35.897	0.000	
	Y7	0.947	0.019	50.688	0.000	
	Y8	0.990	0.010	103.050	0.000	
	Y9	0.999	0.003	336.998	0.000	
A0	WITH					!COVARIANCES OF RANDOM
AX		-8.248	4.168	-1.979	0.048	!COEFFICIENTS
AY		34.624	7.489	4.623	0.000	!
						!
AX	WITH					!
AY		8.864	6.610	1.341	0.180	!
Means						
A0		0.000	0.000	999.000	999.000	!FACTOR MEANS FIXED = 0
AX		0.000	0.000	999.000	999.000	!
AY		0.000	0.000	999.000	999.000	!
Intercepts						
Y1		20.732	0.846	24.510	0.000	!REPEATED MEASURE INTERCEPTS
Y2		25.067	0.764	32.809	0.000	!FIXED EQUAL TO THE TARGET
Y3		28.808	0.726	39.688	0.000	!FUNCTION
Y4		31.954	0.705	45.333	0.000	!
Y5		34.505	0.682	50.579	0.000	!
Y6		36.461	0.651	55.983	0.000	!
Y7		37.821	0.620	60.982	0.000	!
Y8		38.587	0.615	62.745	0.000	!
Y9		38.758	0.675	57.397	0.000	!
Variances						
A0		93.610	11.976	7.816	0.000	!VARIANCE OF INTERCEPT FACTOR
AX		18.019	6.969	2.585	0.010	!VARIANCE OF MAXIMIZER FACTOR
AY		54.708	8.395	6.517	0.000	!VARIANCE OF MAXIMUM FACTOR

Residual Variances

Y1	9.921	0.484	20.494	0.000	!RESIDUAL VARIANCE HELD EQUAL
Y2	9.921	0.484	20.494	0.000	!OVER TIME
Y3	9.921	0.484	20.494	0.000	!
Y4	9.921	0.484	20.494	0.000	!
Y5	9.921	0.484	20.494	0.000	!
Y6	9.921	0.484	20.494	0.000	!
Y7	9.921	0.484	20.494	0.000	!
Y8	9.921	0.484	20.494	0.000	!
Y9	9.921	0.484	20.494	0.000	!

New/Additional Parameters

MA0	20.732	0.846	24.510	0.000	!MEAN INTERCEPT
MAX	7.787	0.412	18.924	0.000	!MEAN MAXIMIZER
MAY	38.772	0.655	59.185	0.000	!MEAN MAXIMUM

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix  
(ratio of smallest to largest eigenvalue) 0.647E-05