LISREL and Mplus code to accompany:

Preacher, K. J., & Hancock, G. R. (June, 2010). *Interpretable reparameterizations of growth curve models*. Invited talk at the Advances in Longitudinal Methods in the Social and Behavioral Sciences conference, The Center for Integrated Latent Variable Research, College Park, MD.

Preacher, K. J., & Hancock, G. R. (2012). On interpretable reparameterizations of linear and nonlinear latent growth curve models. In Harring, J. R., & Hancock, G. R. (Eds.), *Advances in longitudinal methods in the social and behavioral sciences* (pp. 25-58). Charlotte, NC: Information Age Publishing.

Three ways to estimate an aperture parameter

Method 1: Lambda shift method

```
LATENT GROWTH CURVE OF HANCOCK & CHOI DATA, EX. 2
DA NI=4 NO=198 MA=CM
CM
11.000
5.860 13.000
6.205 8.094 14.000
6.103 8.798 10.177 16.000
ME
3.3 3.7 4.0 4.2
MO NY=4 NE=2 LY=FU,FI BE=FU,FI TY=FI AL=FR PS=SY,FI TE=DI,FR AP=1
LE
INT SLP
FR PS 1 1 PS 2 2 !PS 2 1
VA 1 LY 1 1 LY 2 1 LY 3 1 LY 4 1
CO LY 1 2 = 4 - PA 1
CO LY 2 2 = 6 - PA 1
CO LY 3 2 = 7 - PA 1
CO LY 4 2 = 8 - PA 1
PD
OU IT=5000 AD=OFF ND=4
```

Method 2: Phantom variable approach

```
LATENT GROWTH CURVE OF HANCOCK & CHOI DATA, EX. 2
DA NI=4 NO=198 MA=CM
CM
11.000
5.860 13.000
6.205 8.094 14.000
6.103 8.798 10.177 16.000
ME
3.3 3.7 4.0 4.2
MO NY=4 NE=3 LY=FU,FI BE=FU,FI TY=FI AL=FI PS=SY,FI TE=DI,FR
LE
INT SLP PHANTOM
FR AL 1 AL 2
PA PS
 1
 0 1
```

```
 0 0 0
MA PS
 .5
  0 .5
  0 0 0
PA LY
0 0 0
0 0 0
0 0 0
0 0 0
MA LY
1 4 -1
1 6 -1
1 7 -1
1 8 -1
PA BE
 0 0 0
 0 0 0
 0 1 0
MA BE
 0 0 0
 0 0 0
 0 -4.3 0
PD
OU IT=5000 AD=OFF ND=4
```

Method 3: Structured latent curve approach

```
LATENT GROWTH CURVE OF HANCOCK & CHOI DATA, EX. 2
DA NI=4 NO=198 MA=CM
CM
11.000
5.860 13.000
6.205 8.094 14.000
6.103 8.798 10.177 16.000
ME
3.3 3.7 4.0 4.2
MO NY=4 NE=3 LY=FU,FI TY=FI AL=FR PS=SY,FI TE=DI,FR AP=1
LE
INT SLP APERTURE
PA PS
 1
 0 1
 0 0 0
MA PS
 .5
  0 .5
  0  0  0
PA LY
 0 0 0
 0 0 0
 0 0 0
 0 0 0
MA LY
 1 0 0
 1 0 0
```

```
 1 0 0
 1 0 0
CO LY 1 2 = 4 - PA 1
CO LY 2 2 = 6 - PA 1
CO LY 3 2 = 7 - PA 1
CO LY 4 2 = 8 - PA 1
CO LY 1 3 = -1*AL(2)
CO LY 2 3 = -1*AL(2)
CO LY 3 3 = -1*AL(2)
CO LY 4 3 = -1*AL(2)
CO AL 3 = PA 1
PD
OU IT=5000 AD=OFF ND=4
```

A reparameterized Gompertz structured latent growth curve model with random coefficients

Example application to ECLS-K mathematics data (kindergarten through 8th grade). Random coefficients represent the upper asymptote, surge point, and surge slope (see Choi, Hancock, & Harring, 2009) for definitions of these terms.

Choi, J., Harring, J. R., & Hancock, G. R. (2009). Latent growth modeling for logistic response functions. *Multivariate Behavioral Research*, *44*, 620-645.

```
TITLE: ECLS-K math, reparameterized Gompertz curve;
DATA: FILE = eclsk.dat;
VARIABLE: NAMES = id !gender !0=boy, 1=girl; uncomment gender for conditional model
m_fk m_sk m_f1 m_s1 m_s3 m_s5 m_s8;
USEVARIABLES ARE gender m_fk-m_s8;
MISSING = .;
ANALYSIS:
ALGORITHM = INTEGRATION;
INTEGRATION = MONTECARLO;
MODEL:

!Factor Loadings
g0 BY m_fk*(Lg01)
m_sk-m_s8(Lg02-Lg07);
t0 BY m_fk*(Lt01)
m_sk-m_s8(Lt02-Lt07);
g3 BY m_fk*(Lg31)
m_sk-m_s8(Lg32-Lg37);

!Means
[m_fk-m_s8@0]; [g0*147](mu_g0); [t0@0]; [g3*22](mu_g3);

!Variances and covariances
m_fk-m_s8; g0*495 t0*.24 g3; g0 WITH t0 g3; t0 WITH g3;

!Regressions
!g0 t0 g3 ON gender; !uncomment line for conditional model

MODEL CONSTRAINT:
NEW(mu_t0*.7); !Introduce mean of surge point
```

```
!Asymptote loadings
Lg01 = exp(-1*exp(((mu_g3*exp(1)*(mu_t0-0.0))/mu_g0)))
+((mu_g3*exp(1)*(mu_t0-0.0)*exp(mu_g3*exp(1)*(mu_t0-0.0)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-0.0)/mu_g0)))/mu_g0);
Lg02 = exp(-1*exp(((mu_g3*exp(1)*(mu_t0-0.5))/mu_g0)))
+((mu_g3*exp(1)*(mu_t0-0.5)*exp(mu_g3*exp(1)*(mu_t0-0.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-0.5)/mu_g0)))/mu_g0);
Lg03 = exp(-1*exp(((mu_g3*exp(1)*(mu_t0-1.0))/mu_g0)))
+((mu_g3*exp(1)*(mu_t0-1.0)*exp(mu_g3*exp(1)*(mu_t0-1.0)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-1.0)/mu_g0)))/mu_g0);
Lg04 = exp(-1*exp(((mu_g3*exp(1)*(mu_t0-1.5))/mu_g0)))
+((mu_g3*exp(1)*(mu_t0-1.5)*exp(mu_g3*exp(1)*(mu_t0-1.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-1.5)/mu_g0)))/mu_g0);
Lg05 = exp(-1*exp(((mu_g3*exp(1)*(mu_t0-3.5))/mu_g0)))
+((mu_g3*exp(1)*(mu_t0-3.5)*exp(mu_g3*exp(1)*(mu_t0-3.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-3.5)/mu_g0)))/mu_g0);
Lg06 = exp(-1*exp(((mu_g3*exp(1)*(mu_t0-5.5))/mu_g0)))
+((mu_g3*exp(1)*(mu_t0-5.5)*exp(mu_g3*exp(1)*(mu_t0-5.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-5.5)/mu_g0)))/mu_g0);
Lg07 = exp(-1*exp(((mu_g3*exp(1)*(mu_t0-8.5))/mu_g0)))
+((mu_g3*exp(1)*(mu_t0-8.5)*exp(mu_g3*exp(1)*(mu_t0-8.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-8.5)/mu_g0)))/mu_g0);

!Surge point loadings
Lt01 = -1*mu_g3*exp(1)*exp(mu_g3*exp(1)*(mu_t0-0.0)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-0.0)/mu_g0));
Lt02 = -1*mu_g3*exp(1)*exp(mu_g3*exp(1)*(mu_t0-0.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-0.5)/mu_g0));
Lt03 = -1*mu_g3*exp(1)*exp(mu_g3*exp(1)*(mu_t0-1.0)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-1.0)/mu_g0));
Lt04 = -1*mu_g3*exp(1)*exp(mu_g3*exp(1)*(mu_t0-1.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-1.5)/mu_g0));
Lt05 = -1*mu_g3*exp(1)*exp(mu_g3*exp(1)*(mu_t0-3.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-3.5)/mu_g0));
Lt06 = -1*mu_g3*exp(1)*exp(mu_g3*exp(1)*(mu_t0-5.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-5.5)/mu_g0));
Lt07 = -1*mu_g3*exp(1)*exp(mu_g3*exp(1)*(mu_t0-8.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-8.5)/mu_g0));

!Surge slope loadings
Lg31 = -1*exp(1)*(mu_t0-0.0)*exp(mu_g3*exp(1)*(mu_t0-0.0)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-0.0)/mu_g0));
Lg32 = -1*exp(1)*(mu_t0-0.5)*exp(mu_g3*exp(1)*(mu_t0-0.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-0.5)/mu_g0));
Lg33 = -1*exp(1)*(mu_t0-1.0)*exp(mu_g3*exp(1)*(mu_t0-1.0)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-1.0)/mu_g0));
Lg34 = -1*exp(1)*(mu_t0-1.5)*exp(mu_g3*exp(1)*(mu_t0-1.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-1.5)/mu_g0));
Lg35 = -1*exp(1)*(mu_t0-3.5)*exp(mu_g3*exp(1)*(mu_t0-3.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-3.5)/mu_g0));
Lg36 = -1*exp(1)*(mu_t0-5.5)*exp(mu_g3*exp(1)*(mu_t0-5.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-5.5)/mu_g0));
Lg37 = -1*exp(1)*(mu_t0-8.5)*exp(mu_g3*exp(1)*(mu_t0-8.5)/mu_g0)
*exp(-1*exp(mu_g3*exp(1)*(mu_t0-8.5)/mu_g0));
```

The Jenss-Bayley model reparameterized to estimate the effect of cumulative breastfeeding on infant weight at any desired age

We did not have permission to post the Cebu infant data, but we provide Mplus code below to show how to estimate the model.

```
TITLE: cebu growth data (jenss-bayley) with mobile intercept;
DATA: FILE IS cebu_wide_more.dat;
VARIABLE: NAMES ARE id momht rural male age0-age12 br0-br12 h0-h12 w0-w12 b0 b2 b4 b6
b8 b10 b12 b14 b16 b18 b20 b22 b24 xb6 xb8 xb10 xb12 xb14 xb16 xb18 xb20 xb22 xb24;
USEVARIABLES ARE cbf !comment out 'cbf' for unconditional model
x0-x12; !b0-b24 cumulative BF means from age 0m; xb6-xb24 from age 6m
MISSING ARE ALL (-999);
USEOBSERVATIONS ARE id NE 1600044; !omit outlier
DEFINE: !below, choose to model height or weight
!x0=h0;x1=h1;x2=h2;x3=h3;x4=h4;x5=h5;x6=h6;x7=h7;x8=h8;x9=h9;x10=h10;x11=h11;x12=h12;
x0=w0;x1=w1;x2=w2;x3=w3;x4=w4;x5=w5;x6=w6;x7=w7;x8=w8;x9=w9;x10=w10;x11=w11;x12=w12;
!comment out next line for unconditional model
cbf=b12; ! <- SET AGE FOR CUMULATIVE BREASTFEEDING (mos.);
ANALYSIS: ESTIMATOR IS ML; ITERATIONS ARE 10000; !BOOTSTRAP IS 300;
MODEL: [x0-x12@0]; x0-x12*.1(v1); x0-x11 PWITH x1-x12*.027(v2);
fa*1.2; fb*.005; fc*.288; fd*.02; fa WITH fb fc fd; fb WITH fc fd; fc WITH fd;
[fa*9](mfa); [fb*.13](mfb); [fc*1.2](mfc); [fd@0]; ! fa, fb, and fc are linear
fa BY x0-x12@1;
fb BY x0*(b0); fb BY x1-x12(b1-b12);
fc BY x0*(c0); fc BY x1-x12(c1-c12);
fd BY x0*(d0); fd BY x1-x12(d1-d12);
!comment out next line for unconditional model
fa ON cbf*-.53; fb ON cbf*.05; fc ON cbf*-.55; fd ON cbf*-.23;

!jenss-bayley with a mobile intercept
MODEL CONSTRAINT: NEW(mfd*-.334 t0); t0=12; ! <- SET INTERCEPT LOCATION (mos.)

b0=0-t0; c0=exp(mfc+mfd*t0)-exp(mfc+mfd*0);
b1=2-t0; c1=exp(mfc+mfd*t0)-exp(mfc+mfd*2);
b2=4-t0; c2=exp(mfc+mfd*t0)-exp(mfc+mfd*4);
b3=6-t0; c3=exp(mfc+mfd*t0)-exp(mfc+mfd*6);
b4=8-t0; c4=exp(mfc+mfd*t0)-exp(mfc+mfd*8);
b5=10-t0; c5=exp(mfc+mfd*t0)-exp(mfc+mfd*10);
b6=12-t0; c6=exp(mfc+mfd*t0)-exp(mfc+mfd*12);
b7=14-t0; c7=exp(mfc+mfd*t0)-exp(mfc+mfd*14);
b8=16-t0; c8=exp(mfc+mfd*t0)-exp(mfc+mfd*16);
b9=18-t0; c9=exp(mfc+mfd*t0)-exp(mfc+mfd*18);
b10=20-t0; c10=exp(mfc+mfd*t0)-exp(mfc+mfd*20);
b11=22-t0; c11=exp(mfc+mfd*t0)-exp(mfc+mfd*22);
b12=24-t0; c12=exp(mfc+mfd*t0)-exp(mfc+mfd*24);

d0=t0*exp(mfc+mfd*t0)-0*exp(mfc+mfd*0);
d1=t0*exp(mfc+mfd*t0)-2*exp(mfc+mfd*2);
d2=t0*exp(mfc+mfd*t0)-4*exp(mfc+mfd*4);
d3=t0*exp(mfc+mfd*t0)-6*exp(mfc+mfd*6);
d4=t0*exp(mfc+mfd*t0)-8*exp(mfc+mfd*8);
d5=t0*exp(mfc+mfd*t0)-10*exp(mfc+mfd*10);
d6=t0*exp(mfc+mfd*t0)-12*exp(mfc+mfd*12);
d7=t0*exp(mfc+mfd*t0)-14*exp(mfc+mfd*14);
```

```
d8=t0*exp(mfc+mfd*t0)-16*exp(mfc+mfd*16);
d9=t0*exp(mfc+mfd*t0)-18*exp(mfc+mfd*18);
d10=t0*exp(mfc+mfd*t0)-20*exp(mfc+mfd*20);
d11=t0*exp(mfc+mfd*t0)-22*exp(mfc+mfd*22);
d12=t0*exp(mfc+mfd*t0)-24*exp(mfc+mfd*24);

OUTPUT: TECH1 TECH3 STDYX; !CINTERVAL(BCBOOTSTRAP);
```