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CHI-SQUARE

The term *chi-square* (χ^2) refers to a distribution, a variable that is χ^2-distributed, or a statistical test employing the χ^2 distribution. A χ^2 distribution with k degrees of freedom (*df*) has mean k, variance $2k$, and mode $k - 2$ (if $k > 2$), and is denoted χ^2_{df}. Much of its usefulness in statistical inference derives from the fact that the sample variance of a normally distributed variable is χ^2-distributed with $df = N - 1$. All χ^2 distributions are asymmetrical, right-skewed, and non-negative. Owing to the broad utility of the χ^2 distribution, tabled χ^2 probability values can be found in virtually every introductory statistics text.

X^2 TEST FOR POPULATION VARIANCES

A test of the null hypothesis that $\sigma^2 = \sigma_0^2$ (e.g., H_0: $\sigma^2 = 1.8$) is conducted by obtaining the sample variance s^2, computing the test statistic

$$G = \frac{(N-1)s^2}{\sigma_0^2} \qquad (1)$$

and consulting values of the χ^2_{N-1} distribution. For a two-tailed test, G is compared to the critical values associated with the lower and upper $(50 \times \alpha)$% of the χ^2_{N-1} distribution. Rejection implies, with confidence $1 - \alpha$, that the sample is not drawn from a normally distributed population with variance σ_0^2.

X^2 TESTS OF GOODNESS OF FIT AND INDEPENDENCE

The χ^2 goodness of fit test compares two finite frequency distributions—one a set of observed frequency counts in C categories, the other a set of counts expected on the basis of theory or chance. The statistic

$$G = \sum_{i=1}^{C} \frac{(O_i - E_i)^2}{E_i} \qquad (2)$$

is computed, where O_i and E_i are, respectively, the observed and expected frequencies for category i given a fixed total sample size N. G is approximately χ^2-distributed with $df = C - 1$. If the null hypothesis of equality is rejected, the test implies a statistically significant departure from expectations.

This test can be extended to test the null hypothesis that several frequency distributions are independent. For example, given a 3×4 contingency table of frequencies, where $R = 3$ rows (conditions) and $C = 4$ columns (categories), G may be computed as

$$G = \sum_{j=1}^{R} \sum_{i=1}^{C} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} \qquad (3)$$

and compared against a $\chi^2_{(R-1)(C-1)}$ distribution. Expected frequencies are computed as the product of the marginal totals for column i and row j divided by N. Rejection of the null hypothesis implies that not all rows (or columns) were sampled from independent populations. This test may be extended to any number of dimensions.

These χ^2 tests have been found to work well with average expected frequencies as low as 2. However, these tests are inappropriate if the assumption of independent observations is violated.

COMPARISON OF DISTRIBUTIONS

A common application of χ^2 is to test the hypothesis that a sample's parent population follows a particular continuous probability density function. The test is conducted by first dividing the hypothetical distribution into C "bins" of equal width. The frequencies expected for each bin (E_i) are approximated by computing the probability of randomly selecting a case from that bin and multiplying by N. Observed frequencies (O_i) are obtained by using the same bin limits in the observed distribution. The one-tailed test is conducted by using equation 2 and comparing the result to the critical value drawn from a χ^2_{C-1} distribution. Note that the number of bins, and points of division between bins, must be chosen arbitrarily, yet these decisions can have a large impact on conclusions.

The χ^2 distribution has many other applications in the social sciences, including Bartlett's test of homogeneity of variance, Friedman's test for median differences, tests for heteroscedasticity, nonparametric measures of association, and likelihood ratios. In addition, χ^2 statistics form the basis for many model fit and selection indices used in latent variable analyses, item response theory, logistic regression, and other advanced techniques. All of these methods involve the evaluation of the discrepancy between a model's implications and observed data.

SEE ALSO *Distribution, Normal*

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CHOICE IN ECONOMICS

The theory of choice, individual and social, was mainly developed by economists, with crucial contributions from psychologists, political scientists, sociologists, mathematicians, and philosophers.

Individual choice concerns the selection by an individual of alternatives from a set. In standard microeconomic theory, the individual is supposed to have a preference over a set (or a utility function, that is, a numerical representation of the preference). A standard behavioral assumption asserts that the individual selects the best alternatives according to his or her preference. This implies that the preference and the set of alternatives have appropriate mathematical properties. This is the case, for instance, if the set of alternatives is *finite* (the number of alternatives is a positive integer) and the preference is a *weak ordering* (a ranking of the alternatives from the most preferred to the least preferred with possible ties). When the set of alternatives is the standard budget set of microeconomics, the selection is still possible when appropriate topological assumptions are made on the weak ordering and the space of goods. The selected alternatives are the *demand set*. If there is a single alternative, it is the *individual demand*. It will depend, given a preference, on the budget set that is defined by the individual's wealth and the prices. For a given wealth, as a consequence, demand depends on prices. The behavioral maximization assumption is illuminatingly discussed by Amartya Sen (2002).

Although in microeconomics the standard direction is from preference (or utility) to choice (or demand), *revealed preference theory* reverses this direction. It is alleged that choice is observable, but preference is not. In revealed preference theory, choice is supposed to reveal preference. More precisely, if choice satisfies suitable consistency properties, one can retrieve preference. As an example of such a consistency condition, imagine that you are making a choice in a department store that includes a food department. Your choice in the entire store that happens to be food must be identical to the selection of food you would make if you visited only the food department. Given this kind of consistency condition, it is possible to retrieve a preference that is a weak ordering.

Uncertainty in individual choice differs whether it is *objective uncertainty*, à la John von Neumann (1903–1957) and Oskar Morgenstern (1902–1977), or *subjective uncertainty*, à la Leonard Savage (1917–1971). This entry will discuss only objective uncertainty. In this case, the recourse to utility functions is imperative. The set of alternatives is the set of probabilities over prospects—say, lotteries if the prospects are prizes. The individual has a preference given, for instance, by a weak ordering over the set of lotteries. With a utility function representing a weak ordering (which is possible given appropriate conditions), the only property of the real numbers one can use is the ordering property ("greater than or equal to"). The utility functions are said to be ordinal. They are unique up to a strictly increasing transformation. Over lotteries (with further assumptions), one obtains a utility function (called the *von Neumann-Morgenstern utility function*) that satisfies the expected utility hypothesis: The utility of a lottery is equal to the sum of the utilities of the prizes weighted by the probabilities. For instance, in a lottery with two prizes, a bicycle and a car, if the probability to win the bicycle is .99 and the probability to win the car is .01, the utility of the lottery is equal to .99 times the utility of the bicycle plus .01 times the utility of the car. When the expected utility hypothesis is satisfied, the utility function is unique up to an affine positive transformation, and differences of utility become meaningful because these differences can be compared according to the "greater than or equal to" relation. Such utility functions are called *cardinal*. They are used as the basic element of decision theory under risk, where some further assumptions are made on the utility function (concavity, derivability and properties of derivatives).

Social choice is about the selection of alternatives made by a group of individuals. There are obviously two aspects of social choice corresponding to its double origin: voting and social ethics. Although there were precursors in antiquity and medieval times, the birth of social choice theory is generally attributed to the Marquis de Condorcet (1743–1794) and Jean-Charles Borda (1733–1799), two French scholars, at the end of the eighteenth century. The tremendous modern development of this theory stems from the works of Kenneth Arrow and Duncan Black (1908–1991). Individuals are supposed to have preferences over a set of alternatives. Since these preferences are generally conflicting, one must construct rules to obtain a synthetic (or social) preference or a social choice. Arrow's (im)possibility theorem asserts that there does not exist any rule satisfying specified properties. On the other hand, Black's analysis demonstrates that majority rule generates a social preference provided that some homogeneity of individual preferences (single-peakedness) is assumed.