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Testing Complex Correlational Hypotheses With Structural Equation Models

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It is often of interest to estimate partial or semipartial correlation coefficients as indexes of the linear association between 2 variables after partialing one or both for the influence of covariates. Squaring these coefficients expresses the proportion of variance in 1 variable explained by the other variable after controlling for covariates. Methods exist for testing hypotheses about the equality of these coefficients across 2 or more groups, but they are difficult to conduct by hand, prone to error, and limited to simple cases. A unified framework is provided for estimating bivariate, partial, and semipartial correlation coefficients using structural equation modeling (SEM). Within the SEM framework, it is straightforward to test hypotheses of the equality of various correlation coefficients with any number of covariates across multiple groups. LISREL syntax is provided, along with 4 examples.

Researchers are often faced with questions about how to test hypotheses about correlation coefficients. Such questions range in complexity from how to compute a simple bivariate correlation coefficient and determine its significance to more complex questions, such as how to test hypotheses about group differences among sets of partial correlations while controlling for several variables. Simpler questions can usually be addressed by referring to popular texts like Cohen and Cohen (1983) or Cohen, Cohen, West, and Aiken (2003), which provide standard errors useful for conducting significance tests and constructing confidence intervals for various correlations and for testing hypotheses about the equality of correlations within and across independent groups. More complicated questions require more involved methods (Olkin & Finn, 1995; Steiger, 1980a, 1980b, 2005).

This article illustrates how to use structural equation modeling (SEM) to test some hypotheses related to bivariate correlations, partial correlations, and

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semipartial correlations. Example code and data are provided for LISREL[1] (Jöreskog & Sörbom, 1996), a popular SEM software package, so that researchers can try these methods on their own and modify the code as necessary to suit individual requirements. In addition, four short applications are provided using real data. First, a taxonomy for the kinds of correlations discussed here is provided.

A TAXONOMY OF CORRELATION COEFFICIENTS

Bivariate Correlations

The *bivariate correlation,* describing the degree of linear relation between x and y, is:

$$r_{xy} = \frac{\operatorname{cov}(x, y)}{sd_x sd_y} \tag{1}$$

where cov(x, y) refers to the covariance of x and y, and sd_x and sd_y are the standard deviations of x and y, respectively. The squared bivariate correlation (r_{xy}^2), or *coefficient of determination*, expresses the proportion of variability in y explained by its linear relation with x, or vice versa. This concept is often illustrated using Venn diagrams. In the diagram in Figure 1, each circle represents the total variability in x, y, and some third variable w. The proportion of variability in y that is explained, or accounted for, by x is $(b + e)/(b + c + e + f)$.

It is often of interest to compute the correlation between two variables while controlling for at least one other variable. Two kinds of correlation coefficients can be used to address this situation: partial and semipartial correlations.

Partial Correlations

Partial correlations express the linear association between x and y after variability in both x and y associated with at least one other variable (w) is removed. The partial correlation is denoted $r_{xy.w}$ to indicate that w has been controlled for, or *partialed out*. One formula for the partial correlation coefficient is:

$$r_{xy.w} = \frac{r_{xy} - r_{xw} r_{yw}}{\sqrt{1 - r_{xw}^2}\sqrt{1 - r_{yw}^2}} \tag{2}$$

The partial correlation coefficient can also be understood as the bivariate correlation between the residuals of x and y after regressing both x and y on w. As with

[1]LISREL was used for the illustrations here, but the same logic may be extended to any of several other SEM packages capable of handling multiple-group models, including AMOS (Arbuckle, 1999), EQS (Bentler, 1997), and Mx (Neale, Boker, Xie, & Maes, 2002). LISREL was chosen because of its popularity and because a student version is available free of charge at the Scientific Software International Web site.

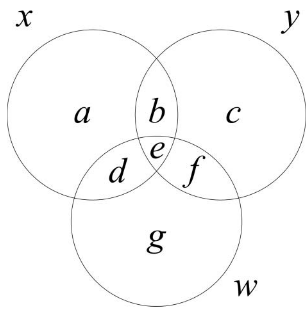

FIGURE 1 Venn diagram representing proportions of variance shared by x, y, and w.

the bivariate correlation, partial correlations may be squared to express the proportion of variance in y explained by x after variability associated with w has been removed from both x and y. In other words, $r^2_{xy.w}$ is the proportion of y variance *not* associated with w that *is* associated with x (Cohen et al., 2003). The Venn diagram in Figure 1 can be used to illustrate this idea; $r^2_{xy.w} = b/(b+c)$.

Semipartial Correlations

When the correlation between x and the part of y from which w has been partialed is desired, a *semipartial* (or *part*) *correlation* may be computed. The formula for the semipartial correlation between x and the partialed y is:

$$r_{x(y.w)} = \frac{r_{xy} - r_{xw} r_{yw}}{\sqrt{1 - r^2_{yw}}} \tag{3}$$

The quantity $r_{x(y.w)}$ can also be thought of as the bivariate correlation between x and the residuals of y after regressing y on w. The squared semipartial correlation expresses the increment in y variance explained by x above and beyond that explained by w. Again with recourse to Figure 1, $r^2_{x(y.w)}$ is equal to $b/(b + c + e + f)$, the increment in explained variance due to x.

The formulae for partial and semipartial correlations may be extended to accommodate more than one partialed variable w. Cohen et al. (2003) provided formulae for all three types of correlation coefficients discussed here with any number of partialed covariates, as well as for standard errors to facilitate hypothesis tests for each of them. The standard errors all assume the same form:

$$SE = \sqrt{\frac{1 - R^2}{N - k - 1}} \tag{4}$$

where k represents one less than the number of variables involved and R^2 represents the square of the correlation of interest, or the variance in y explained by x after partialing potentially several variables from *x*, *y*, or both *x* and *y*. For the special case of a bivariate correlation, the standard error of r_{xy} is thus:

$$SE_{r_{xy}} = \sqrt{\frac{1-R^2}{N-k-1}} = \sqrt{\frac{1-r_{xy}^2}{N-2}} \tag{5}$$

The three types of correlation coefficients discussed here differ only in the role of at least one covariate *w*. For bivariate correlations, *w* is partialed from neither *x* nor *y*. For partial correlations, *w* is partialed from both *x* and *y*; the partial correlation is the correlation of *x* and *y* residuals after regressing both *x* and *y* on *w*. For semipartial correlations, *w* is partialed from either *x* or *y,* but not both.

USING SEM TO COMPUTE CORRELATIONS AND TEST HYPOTHESES

Although it is possible to compute bivariate, partial, and semipartial correlations and their standard errors by hand, it is a time-consuming and error-prone process. Furthermore, tests of cross-group equality among two or more such coefficients are not straightforward. SEM is a general framework that can be used for computing bivariate, partial, and semipartial correlations and for conducting both simple and complex hypothesis tests regarding each type of correlation. Using the SEM framework to test correlational hypotheses has several benefits not available with the traditional approach, primarily the ability to correct for attenuation due to unreliability. An additional advantage of using SEM is that each of the models described here can be embedded in larger structural equation models constructed to explain more complex patterns of relations among sets of variables.

The full LISREL model includes eight parameter matrices involving both exogenous (independent) and endogenous (dependent) variables. For simplicity in this context, it is easier to use a truncated three-matrix model, LISREL Submodel 1 (Jöreskog & Sörbom, 1996) or the factor model, which treats all variables as indicators of exogenous latent variables. The matrix expression for the data model linking the *p* dependent variables in the vector $\mathbf{x}$ $(p \times 1)$ to the *m* latent variables in the vector $\boldsymbol{\xi}$ $(m \times 1)$ is:

$$\mathbf{x} = \boldsymbol{\Lambda}_x \boldsymbol{\xi} + \boldsymbol{\delta} \tag{6}$$

where $\boldsymbol{\Lambda}_x$ $(p \times m)$ is a matrix of loadings linking measured variables to latent variables and $\boldsymbol{\delta}$ $(p \times 1)$ is a vector of residuals. The covariance structure implied by Equation 6 is:

$$\boldsymbol{\Sigma}_{xx} = \boldsymbol{\Lambda}_x \boldsymbol{\Phi} \boldsymbol{\Lambda}'_x + \boldsymbol{\Theta}_\delta \tag{7}$$

where $\mathbf{\Phi}$ ($m \times m$) is the covariance matrix of $\boldsymbol{\xi}$ and $\mathbf{\Theta}_\delta$ ($p \times p$) is the covariance matrix of $\boldsymbol{\delta}$. In more general structural equation models, the diagonal elements of $\mathbf{\Theta}_\delta$ are specified as free parameters to reflect measurement error in the variables. In all the models considered here, $\mathbf{\Theta}_\delta = 0$. In other words, all measured variables will be regarded as perfect, error-free indicators of associated latent variables. In LISREL, the three parameter matrices $\mathbf{\Lambda}_x$, $\mathbf{\Phi}$, and $\mathbf{\Theta}_\delta$ are represented respectively by the two-letter abbreviations LX, PH, and TD. Elements of these matrices are denoted, for example, by LX(3,1) to indicate the element in the third row and first column of $\mathbf{\Lambda}_x$.

LISREL provides point estimates, estimated standard errors, and critical ratios for every estimated parameter. For the single-sample models considered here, all models are saturated. That is, there are no degrees of freedom (*df*) and thus model fit is of no interest. However, when cross-group comparisons are of interest, hypotheses of differences between groups are tested by comparing the fit of two models: one model in which a saturated model is specified within each group (thus $df = 0$) and one model in which at least one key parameter is constrained to equality across groups (thus $df \geq 1$). In these situations, the hypothesis of cross-group equality is tested by comparing the computed χ^2 fit statistic to the critical value of χ^2 for a given significance level (α) and *df* equal to the number of added constraints.

Bivariate Correlations

With the exception of estimating the variance of a single variable, estimating the covariance between two variables is arguably the simplest sort of analysis one can perform in SEM. Computing the correlation between two variables is a straightforward extension of this model in which the variances are rescaled to equal 1 (recall that a bivariate correlation is the covariance of two standardized variables). This is achieved by constraining diagonal elements of PH to equal 1 and estimating elements of LX as scaling parameters. The path diagram corresponding to this model is represented in Figure 2. In this simple model, r_{xy} is estimated as an element of $\mathbf{\Phi}$, specifically PH(2,1). Residual terms (δ_1 and δ_2) with zero variances are included because *x* and *y* are considered error-free indicators of their respective latent variables. This strategy has been described elsewhere, notably by Steiger (1989) and Bentler (1989, p. 152), in the context of testing hypotheses about bivariate correlations within and between groups using EzPath and EQS. Using this strategy, ξ_1 and ξ_2 are termed *alias latent variables* (Steiger, 1989) because they serve as unit-scaled proxies for *x* and *y* so that their covariance can be interpreted as a correlation. LISREL syntax for this model, as well as for most other models discussed in this article, is provided in the Appendix.

Suppose a researcher wishes to test the null hypothesis that ρ_{xy} is equal across two independent groups. This hypothesis may be tested easily by hand using a formula provided by Cohen et al. (2003), but it can also be tested within the SEM framework by conducting a two-group analysis. The model in Figure 2 is specified

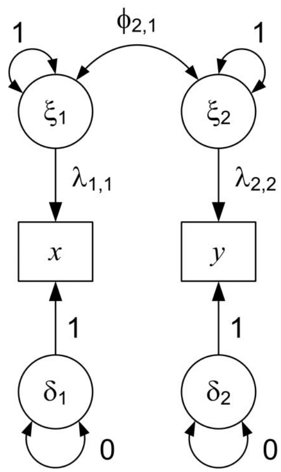

FIGURE 2 Path diagram for a model in which $\phi_{2,1}$ represents the bivariate correlation between x and y.

simultaneously in both groups and the parameter $\phi_{2,1}$ is constrained to equality across groups. The resulting χ^2 fit statistic may be used to gauge the significance of the increase in model misfit after the constraint is added. By rejecting the null hypothesis of equality, it is concluded that the difference in r_{xy} across samples is not due to chance.[2]

Partial Correlations

Computing partial correlations in SEM is similarly straightforward. A conceptual diagram is depicted in Figure 3 (see also Maruyama, 1998, p. 51), in which x and y are both regressed on w, and the residual correlation is the partial correlation of interest. The LISREL path diagram for the same model is depicted in Figure 4. In Figure 4, the residual terms δ_1, δ_2, and δ_3 have zero variances, implying that w, x, and y are error-free indicators of their respective latent variables. The parameters $\lambda_{1,1}$, $\lambda_{2,2}$, and $\lambda_{3,3}$ act as scaling factors so that the latent variances will be 1.0. The alias latent variables ξ_2 and ξ_3 behave like standardized residual terms. Their covariance ($\phi_{3,2}$) is therefore a correlation of residuals after w has been partialed from both x and y.

The model in Figure 4 may be augmented to include any number of control variables like w, and may also be augmented to include any number of other variables whose partial correlations we wish to estimate. As long as the appropriate ele-

[2]The asymptotic χ^2 test provided in SEM agrees quite closely with the test suggested by Cohen et al. (2003), which uses Fisher's r-to-z' transformation to stabilize variance and normalize the correlation coefficient. The p values associated with each test converge as N increases.

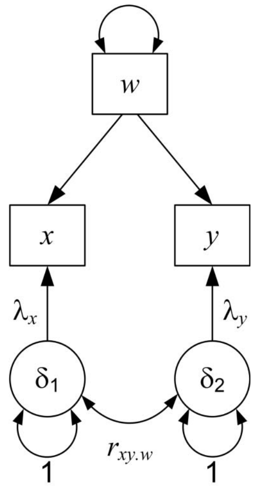

FIGURE 3 Conceptual path diagram in which $r_{xy.w}$ represents the partial correlation between x and y, controlling for w. The parameters λ_x and λ_y are scaling factors such that the residuals (δ_1 and δ_2) will have unit variances and thus $r_{xy.w}$ can be interpreted as a correlation.

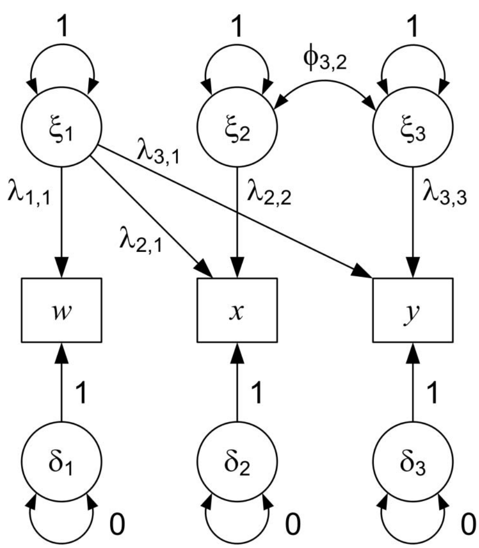

FIGURE 4 Path diagram for a model in which $\phi_{3,2}$ represents the partial correlation between x and y, controlling for w.

ments of $\mathbf{\Lambda}_x$ and $\mathbf{\Phi}$ are specified as free parameters, the results will mirror exactly what would be obtained by hand calculations. As with r_{xy}, group differences in $r_{xy.w}$ may be tested easily within the SEM framework by constraining $r_{xy.w}$ ($\phi_{3,2}$ in Figure 4) to equality across groups and noting any significant increase in χ^2.

Semipartial Correlations

Estimating semipartial correlations in SEM requires a slight modification of the model in Figure 4. Figure 5 includes the effect of *w* on *y* ($\lambda_{3,1}$), but no effect of *w* on *x* because *w* is partialed only from *y*. The extra degree of freedom picked up by adding this constraint is lost again by allowing *w* and *x* to covary by freeing PH(2,1).[3] The semipartial correlation between *x* and *y* partialing *w* from *y*, or $r_{x(y.w)}$, is estimated in the model as PH(3,2). To obtain $r_{y(x.w)}$ rather than $r_{x(y.w)}$, we could constrain LX(3,1) and PH(2,1) to zero and in their place free LX(2,1) and PH(3,1).

As with partial correlations, any number of covariates for *x*, *y*, or both *x* and *y* could be added to the model. We could, for example, estimate the semipartial correlation of *x* and *y* when *w* is partialed from *x* and when *z* is partialed from *y*, if there exists a theoretical reason to do that. Alternatively, we could include several variables whose partial correlation with *y*, controlling for *w*, is desired. Finally, we could conduct cross-group comparisons on any of these models by specifying the same model for independent groups, constraining the parameter of interest ($\phi_{3,2}$ in Figure 5) to equality across groups, and noting any significant increase in χ^2.

Squared Correlations

As noted earlier, the bivariate correlation may be squared to obtain the proportion of variance in *y* explained by *x* (or vice versa). Similarly, the partial correlation $r_{xy.w}$ may be squared to obtain the proportion of variance in *y* explained by *x* once variance due to *w* has been removed from both. The semipartial correlation $r_{x(y.w)}$ may be squared to obtain the increment in *y* variance explained by *x* after variance due to *w* has been accounted for. Each of these squared correlations can be obtained with SEM. For example, to obtain the squared bivariate correlation between *x* and *y* in LISREL, the model in Figure 3 may be augmented to include a new parameter, ϖ, using LISREL's additional parameter (AP) facility. No new free parameters are actually estimated; rather, ϖ is constrained to equal the square of PH(2,1), or r_{xy}^2. If r_{xy}^2 is all that is required, it is simple enough to calculate it by hand. SEM is not well-suited to situations in which it is hypothesized that r_{xy}^2 differs across independent groups (in other words, that group membership moderates the proportion of variance *x* explains in *y*). Confidence intervals and tests are available for such hypotheses, however (see Cohen et al., 2003, p. 88; Olkin & Finn, 1995).

[3]Not freeing PH(2,1) implies that the correlation between *w* and *x* is zero, a constraint that is rarely warranted.

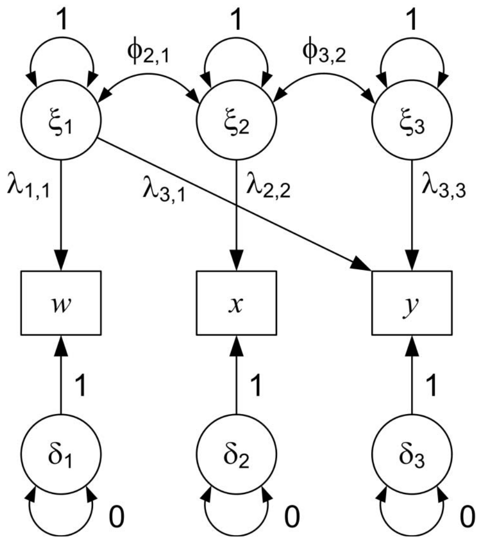

FIGURE 5 Path diagram for a model in which $\phi_{3,2}$ represents the semipartial correlation between x and y, controlling y for w.

TESTING PATTERN HYPOTHESES INVOLVING DEPENDENT CORRELATIONS

Discussion so far has focused on the estimation and testing of correlations (bivariate, partial, and semipartial) within single samples and comparison of these coefficients across independent samples using SEM. However, it is often of interest to compare two or more such coefficients within the same sample. The purpose of this section is to demonstrate how SEM may be employed to test pattern hypotheses involving bivariate, partial, and semipartial correlations when N is large and multivariate normality may be assumed. Pattern hypotheses address questions about the equality of sets of dependent correlations to each other and to point values (Steiger, 1979a, 1980b). The correlations to be compared may or may not share a common variable. Tests for hypotheses involving dependent correlations have been the subject of much research (Aitken, Nelson, & Reinfurt, 1968; Boyer, Palachek, & Schucany, 1983; Choi, 1977; Choi & Wette, 1972; Dunn & Clark, 1969, 1971; Hotelling, 1940; Larzelere & Mulaik, 1977; May & Hittner, 1997; McDonald, 1975; Meng, Rosenthal, & Rubin, 1992; Neill & Dunn, 1975; Olkin, 1966; Olkin & Finn, 1990, 1995; Steiger, 1979a, 1979b; Steiger & Browne, 1984; Williams, 1959; Wolfe, 1976).

As an example, it may be of interest to hypothesize that $\rho_{xy} = \rho_{wz}$ in the population; that is, the linear relation between x and y is the same as that between w and z. The researcher may also hypothesize that two or more sets of correlations are equal. For example, someone might wish to examine the joint hypothesis that the linear relations between x and the variables z and w (ρ_{xz} and ρ_{xw}, respectively) are (a) equal to each other and (b) stable over time: H_0: $\rho_{xy(1)} = \rho_{wx(1)} = \rho_{xy(2)} = \rho_{xw(2)} = \rho_{xy(3)} = \rho_{xw(3)}$. This sort of null hypothesis is easily testable within the SEM framework by placing equality constraints on the desired correlations. An example of this hypothesis, as well as examples of other hypotheses discussed in this article, is presented later.

The method just described also may be used to test pattern hypotheses regarding partial and semipartial correlations. For example, if it is desired to test the equality of sets of correlations while controlling for the influence of some covariate w, a model like those illustrated in Figures 4 and 5 may be specified to include several control variables, with equality constraints placed on the parameters representing partial or semipartial correlations.

Steiger (1980b) noted two drawbacks associated with the likelihood ratio test commonly used in SEM to test pattern hypotheses. The first drawback, processor speed, is no longer the hindrance it once was. Steiger's second objection is that χ^2 is a large-sample statistic that rejects the null hypothesis too often, particularly in small samples; that is, the actual α slightly exceeds the nominal α, and thus the test is overpowered. Steiger (1980a) noted that this problem becomes more pronounced as larger matrices are tested. For testing pattern hypotheses in small samples, researchers should consider using methods outlined by Steiger (1980a, 1980b, 2005) and implemented in his program MULTICORR (Steiger, 1979a). MULTICORR uses the normalizing and variance-stabilizing Fisher's r-to-z' transformation, which often leads to superior small-sample performance. MULTICORR can test many of the hypotheses discussed in this article, but is limited to single-sample tests and can accept correlation matrices no larger than 18 × 18. Steiger (2005) recently made available a *Mathematica* program called WBCORR that can test pattern hypotheses within and between groups using least-squares and asymptotic tests. Comparable hypothesis tests conducted using WBCORR and LISREL typically yield highly similar results. However, MULTICORR and WBCORR were designed to test hypotheses about the equality of bivariate correlations within and (for WBCORR) across groups. They were not programmed with partial, semipartial, or squared correlations in mind. For example, it may be of interest to compare $r_{xy.w}$ at Time 1 with $r_{xy.z}$ at Time 2, where z and w are two different indicators of the same trait. The flexibility of the SEM approach permits such tests. The SEM approach also permits researchers to embed pattern hypotheses in larger models, and has the additional advantage of permitting direct hypothesis tests involving correlations among latent variables with multiple indicators.

EXAMPLES

Four illustrations of the kinds of correlation hypotheses testable within the LISREL framework are provided. The first concerns hypothesized group differences in the magnitude of a partial correlation. The second concerns estimation and comparison of semipartial correlations in three groups. The third and fourth examples concern testing pattern hypotheses within a single sample. LISREL syntax and data for all examples are included in the Appendix.

Example 1

It is commonly believed that temporary employees are less competent than their permanent counterparts, and they are aware of this stereotype. Von Hippel et al. (2005) tested the hypothesis that temporary employees for whom this stereotype was made salient would exhibit a strong defensive relation between impression management and self-reported professional competence. The incompetence stereotype was made salient for one group of temporary employees (the stereotype threat group; $n = 65$) but not for a control group ($n = 49$). Both groups completed an abbreviated version of the impression management subscale from the Balanced Inventory of Desirable Responding (BIDR–IM; Paulhus, 1991) and answered questions about their own competence.

Of central interest was the difference in the strength of the linear relation between impression management and self-reported competence across stereotype threat groups. Duration of employment, scored in weeks, was included as a covariate. In summary, it was of interest to see whether stereotype threat moderated the relation between impression management and self-reported competence after controlling for tenure. To test the hypothesis of no group difference, the model in Figure 4 was applied in both groups simultaneously. The partial correlation in the threat group was $r_{xy.w} = .41$, $p < .001$, but only $r_{xy.w} = .04$, *ns*, in the no-threat group. Consistent with the hypothesis, constraining these partial correlations to equality across groups resulted in a significant decrease in fit, $\chi^2(1, N_1 = 65, N_2 = 49) = 4.24$, $p < .05$. In other words, the data did not support the hypothesis that the partial correlation between impression management and self-reported competence was equal across groups.

Example 2

Consider the situation in which it is of interest to estimate the incremental predictive power of political conservatism in explaining opposition toward abortion, after age has already been taken into account (i.e., the semipartial correlation). Suppose that a researcher hypothesizes that the size of this increment in explanatory ability will differ by political party (Republican, Democrat, and Independent), and is also interested in examining the squared semipartial correlations in each group. These analyses may be undertaken by using the model in Figure 5 and conducting a comparison across three groups representing the three major political parties.

Data were obtained from the National Race and Politics Survey (Sniderman, Tetlock, & Piazza, 1991), a data set made available by the Survey Research Center of the University of California at Berkeley. Data were extracted for 408 Democrats, 459 Republicans, and 411 Independents. The squared semipartial correlation between conservatism and opposition toward abortion, controlling the latter for age, is small but significantly positive for all three parties (Democrats: $r^2_{y(x.w)} = .094$, $p = .027$; Republicans: $r^2_{y(x.w)} = .106$, $p = .027$; Independents: $r^2_{y(x.w)} = .120$, $p = .030$). Constraining unsquared semipartial correlations to equality across all three groups did not result in a significant decrease in fit, $\chi^2(2, N_1 = 408, N_2 = 459, N_3 = 411) = .41, p = .82$. The lack of a significant finding implies that there is not enough evidence to conclude that conservatism uniquely predicts opposition toward abortion to the same degree in the three major parties (controlling for age). Syntax and data for testing this hypothesis are included in the Appendix.

Example 3

Consider the multitrait–multimethod matrix reported by Lawler (1967), in which 113 middle- and top-level managers were assessed in terms of job performance quality (Q), ability (A), and effort (E). Ratings for each of these characteristics were obtained from superiors (S) and peers (P). Self-report data were also collected, but are omitted for simplicity.

$$\mathbf{R} = \begin{array}{r} \\ \text{Quality} \\ \text{Ability} \\ \text{Effort} \\ \text{Quality} \\ \text{Ability} \\ \text{Effort} \end{array} \begin{array}{c} \quad\text{Superior} \qquad\qquad \text{Peers} \\ \left[\begin{array}{ccc|ccc} 1 & & & & & \\ \mathbf{.53} & 1 & & & & \\ \mathbf{.56} & \mathbf{.44} & 1 & & & \\ \hline .65 & \mathbf{.38} & \mathbf{.40} & 1 & & \\ \mathbf{.42} & .52 & \mathbf{.30} & \mathbf{.56} & 1 & \\ \mathbf{.40} & \mathbf{.31} & .53 & \mathbf{.56} & \mathbf{.40} & 1 \end{array}\right] \end{array} \tag{8}$$

One of Campbell and Fiske's (1959) desiderata for convergent validity is that the heterotrait correlations, or the boldfaced correlations among Q, A, and E, should exhibit the same pattern regardless of the assessment method. A strict version of this criterion, requiring pattern correlation equality in addition to pattern similarity, can be represented as the pattern hypothesis:[4]

$$H_0 : \begin{bmatrix} \rho_{AS,QS} = \rho_{AS,QP} = \rho_{QS,AP} = \rho_{AP,QP} \\ \rho_{ES,QS} = \rho_{ES,QP} = \rho_{QS,EP} = \rho_{EP,QP} \\ \rho_{ES,AS} = \rho_{ES,AP} = \rho_{AS,EP} = \rho_{EP,AP} \end{bmatrix} \tag{9}$$

[4]The hypothesized equivalence would be considered overly strict by most standards; it is presented here merely for illustration.

Framed as a structural equation model, the pattern hypothesis in Equation 9 may be tested by placing equality constraints on sets of correlations. Testing the hypothesis in Equation 9 with LISREL yields $\chi^2(9, N = 113) = 36.38, p < .0001$, indicating that the pattern of correlations is not consistent with the strict criterion for convergent validity specified in Equation 9.

Example 4

The Peabody Individual Achievement Test (PIAT; Dunn & Markwardt, 1970) measures performance on a number of scholastic abilities. The PIAT was administered every 2 years, beginning in 1986, as part of the National Longitudinal Survey of Youth (NLSY; Bureau of Labor Statistics, 2000). More precise information on the PIAT cognitive ability assessments is available from Dunn and Dunn (1981) and Dunn and Markwardt (1970). Consider the covariance matrix in Equation 10:

$$\mathbf{S} = \begin{array}{c} \\ \text{Sex} \\ \text{M} \\ \text{R} \\ \text{C} \\ \text{M} \\ \text{R} \\ \text{C} \\ \text{M} \\ \text{R} \\ \text{C} \end{array} \begin{array}{c|ccc|ccc|ccc} & & \text{Age 8} & & & \text{Age 10} & & & \text{Age 12} & \\ .249 & & & & & & & & & \\ \hline .003 & .229 & & & & & & & & \\ .025 & .157 & .283 & & & & & & & \\ .032 & .141 & .213 & .258 & & & & & & \\ \hline -.006 & .146 & .143 & .134 & .223 & & & & & \\ .024 & .171 & .269 & .217 & .178 & .377 & & & & \\ .017 & .137 & .201 & .181 & .143 & .248 & .283 & & & \\ \hline -.009 & .146 & .144 & .128 & .166 & .172 & .149 & .246 & & \\ .028 & .178 & .278 & .231 & .180 & .342 & .257 & .195 & .459 & \\ .012 & .143 & .194 & .174 & .146 & .231 & .203 & .166 & .275 & .316 \end{array} \tag{10}$$

This matrix contains covariances among sex, PIAT Mathematics (M), PIAT Reading Recognition (R), and PIAT Reading Comprehension (C) scores for the same 1,071 children at ages 8, 10, and 12. Suppose an investigator is interested in testing the hypothesis that the partial correlations among these three tests, controlling for sex, remain stable over time. This pattern hypothesis might be represented as:

$$H_0 : \begin{bmatrix} \rho_{R08,M08.SEX} = \rho_{R10,M10.SEX} = \rho_{R12,M12.SEX} \\ \rho_{C08,M08.SEX} = \rho_{C10,M10.SEX} = \rho_{C12,M12.SEX} \\ \rho_{C08,R08.SEX} = \rho_{C10,R10.SEX} = \rho_{C12,R12.SEX} \end{bmatrix} \tag{11}$$

The syntax for the single-group partial correlation model (supplied in the Appendix) may be augmented to include partial correlations among nine variables rather than among only two. Testing this pattern hypothesis with LISREL yields

$\chi^2(6, N = 1{,}071) = 26.29$, $p = .0002$, implying that the partial correlations among M, R, and C do not remain stable over time.

EXTENSIONS

The procedures described here may be extended in several ways. For example, the correlations in question may be compared across more than two groups. A procedure already exists for testing the equality of multiple independent correlations (Chen & Popovich, 2002, p. 22; Cohen & Cohen, 1983, p. 55; Hays, 1963, p. 532), but there is no documented test for multiple-group hypotheses involving partial or semipartial correlations. For example, perhaps the researcher hypothesizes that the partial correlation in one cell of a 2 × 2 factorial design should be higher than that for the other three cells. In that case, the model may be specified separately for each group defined by the 2 × 2 design, and the partial correlation of interest may be constrained to equality across three groups. The null hypothesis that the partial correlation in the fourth group is equal to the correlation in the other three groups can be tested by constraining it equal to the other three and noting any significant increase in χ^2. The procedures described for testing pattern hypotheses may be extended to testing pattern equality across any number of independent groups.

Earlier it was suggested that correlations are often hypothesized to equal particular fixed population values derived from theory or from past research. Usually the point value to which the correlation (bivariate, partial, or semipartial) is compared is zero, but this need not be the case. If, for example, prior theory suggests that $\rho_{x(y.w)} = .3$ and the researcher hypothesizes that $\rho_{x(y.w)}$ under certain circumstances should be higher, the model may be statistically compared to one in which $\rho_{x(y.w)}$ is constrained to equal .3.

Perhaps one of the greatest advantages of testing correlational hypotheses with SEM is the ability to use latent variables with multiple observed indicators. The assumption that variables are measured without error is rarely tenable in practice, so it is often advantageous to represent constructs as latent variables with multiple measured indicators to model unique variance separately from common variance. The power of tests of structural parameters and latent covariances usually increases as a result. The model in Figure 6 illustrates what such a model might look like, in which *x* and *y* are both latent variables, each with three indicators. In this model, $\phi_{3,2}$ represents the partial correlation of two latent variables, η_x and η_y, controlling for *w*. The common factor model (LISREL Submodel 1) cannot be used to test such hypotheses because such models involve structural paths connecting latent variables. Instead, the full LISREL model, or at least LISREL Submodel 3B (the *all-y* model), should be used instead (see Jöreskog & Sörbom, 1996).

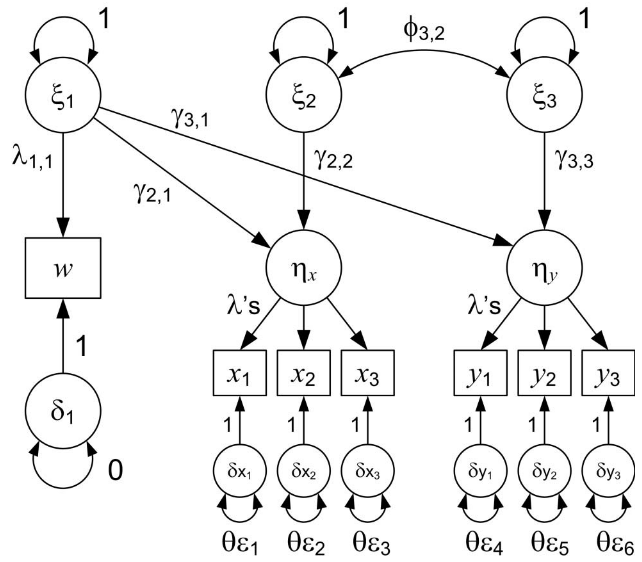

FIGURE 6 Path diagram for a model in which $\phi_{3,2}$ represents the partial correlation between *x* and *y* when both *x* and *y* are latent, controlling for *w*.

DISCUSSION

A unified framework has been provided for estimating unsquared and squared bivariate, partial, and semipartial correlations using SEM. Within the SEM framework, it is straightforward to test hypotheses of the equality of various correlation coefficients with any number of covariates across multiple groups. Examples were provided to illustrate the kinds of analyses discussed, and LISREL syntax was provided to enable researchers to conduct similar analyses.

In this article, correlations have been framed as parameters of structural equation models, with hypotheses concerning cross-group and within-group equality of correlations framed as difference tests using the χ^2 statistic. Two issues bear mentioning. First, these kind of tests are often underpowered in practice. This problem is not a shortcoming of SEM per se; low power presents a problem regardless of whether one uses SEM, WBCORR, or other software-based approaches or conducts difference tests manually. It is mentioned here merely so that researchers will be aware of the problem. Methods exist for conducting a priori power analysis for χ^2 difference tests so that the appropriate minimum sample size can be determined (e.g., MacCallum, Browne, & Cai, 2006).

Second, the procedures here were framed in terms of testing point null hypotheses. However, the American Psychological Association Task Force on Statistical

Inference (Wilkinson & the Task Force on Statistical Inference, 1999) strongly recommended the use of confidence intervals in lieu of significance testing. Normal-theory confidence intervals for correlations can be formed by using the standard errors provided by LISREL, along with appropriate critical values of z, assuming a large sample. However, skewness increases with the magnitude of the correlation. Confidence intervals that assume normality will be less appropriate as the magnitude of the point estimate departs from zero. Another problem is that intervals constructed using standard errors sometimes exceed the logical bounds for a parameter, especially if N is small. One way to circumvent these problems is to normalize r using Fisher's r-to-z' transformation (Fisher, 1946), use the resulting standard errors to compute margins of error, and transform the confidence limits back to the r metric. Unfortunately, the Fisher transform cannot be used in this way to construct intervals for differences between correlations.[5]

In addition to constructing confidence intervals using Fisher's transformation, there are at least three SEM-based alternatives to testing point null hypotheses about correlations. One solution is to use a resampling procedure. For example, AMOS and EQS now come equipped with bootstrapping facilities. No distributional assumptions are necessary when bootstrapping is used to obtain confidence intervals for parameters. Another solution is to gauge the significance of the correlation not by dividing it by its standard error, but by constraining it equal to zero and observing the change in χ^2. Finally, the Mx application provides a new approach to computing intervals that avoids the problem of symmetry. Further details are provided in Neale, Boker, Xie, and Maes (2002) and Neale and Miller (1997).

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[5]I thank an anonymous reviewer for pointing this out.

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APPENDIX

All syntax in this Appendix can be found online at the author's Web site, http://www.quantpsy.org/. In what follows, it is assumed that all input data are in covariance form, but the syntax may be altered to accept raw data instead. Correlation matrices should not be used as input, particularly for models involving multiple groups. Most SEM software applications erroneously treat correlations as if they are distributed as covariances (Cudeck, 1989; Raykov & Marcoulides, 2000).

LISREL syntax for computing the bivariate correlation between *x* and *y*.

```
TI bivariate correlation
DA NI=2 NO=40
CM
0.958365
0.231046 1.163310
MO NX=2 NK=2 LX=DI,FR PH=ST TD=ZE
LK
X Y
ST .5 LX 1 1 LX 2 2 PH 2 1
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
```

LISREL syntax for computing the bivariate correlation between *x* and *y* simultaneously in two groups, with equality constraint in place.

```
GROUP 1 bivariate correlation
DA NG=2 NI=2 NO=40
CM
0.958365
0.231046 1.163310
MO NX=2 NK=2 LX=DI,FR PH=ST TD=ZE
LK
X Y
ST .5 LX 1 1 LX 2 2 PH 2 1
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
GROUP 2 bivariate correlation
DA NI=2 NO=40
CM
0.923433
0.021623 1.263412
MO NX=2 NK=2 LX=DI,FR PH=ST TD=ZE
LK
X Y
ST .5 LX 1 1 LX 2 2 PH 2 1
EQ PH(1,2,1) PH(2,1)
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
```

LISREL syntax for computing the partial correlation between *x* and *y*, controlling both for *w*.

```
TI partial correlation
DA NI=3 NO=40
CM
1.405466
0.633555 0.958365
0.359973 0.231046 1.163310
MO NX=3 NK=3 LX=FU,FI PH=SY,FI TD=ZE
LK
W X Y
FR PH 1 1 PH 3 2 LX 2 1 LX 2 2 LX 3 1 LX 3 3
VA 1 PH 2 2 PH 3 3 LX 1 1
ST .5 PH 1 1 PH 3 2 LX 2 1 LX 2 2 LX 3 1 LX 3 3
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
```

LISREL syntax for computing the partial correlation between *x* and *y* simultaneously in two groups, controlling both *x* and *y* for *w*, with equality constraint in place. This syntax includes the stereotype data discussed in Example 1.

```
GROUP 1 partial correlation
DA NG=2 NI=3 NO=65
CM
1.371222
0.308131 0.875000
0.063412 0.024306 0.085601
MO NX=3 NK=3 LX=FU,FI PH=SY,FI TD=ZE
LK
W X Y
FR PH 1 1 PH 3 2 LX 2 1 LX 2 2 LX 3 1 LX 3 3
VA 1 PH 2 2 PH 3 3 LX 1 1
ST .5 PH 1 1 PH 3 2 LX 2 1 LX 2 2 LX 3 1 LX 3 3
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
GROUP 2 partial correlation
DA NI=3 NO=49
CM
1.593040
0.104162 1.034864
0.043919 0.123677 0.086420
MO NX=3 NK=3 LX=FU,FI PH=SY,FI TD=ZE
LK
W X Y
FR PH 1 1 PH 3 2 LX 2 1 LX 2 2 LX 3 1 LX 3 3
VA 1 PH 2 2 PH 3 3 LX 1 1
ST .5 PH 1 1 PH 3 2 LX 2 1 LX 2 2 LX 3 1 LX 3 3
EQ PH(1,3,2) PH(3,2)
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
```

LISREL syntax for computing the semipartial correlation between *x* and *y*, controlling *y* for *w*.

```
TI semipartial correlation
DA NI=3 NO=40
CM
1.405466
0.633555 0.958365
0.359973 0.231046 1.163310
MO NX=3 NK=3 LX=FU,FI PH=SY,FI TD=ZE
LK
W X Y
FR PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
VA 1 PH 1 1 PH 2 2 PH 3 3
ST .5 PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
```

LISREL syntax for computing the semipartial correlation between *x* and *y* simultaneously in two groups, controlling *y* for *w*, with equality constraint in place.

```
GROUP 1 semipartial correlation
DA NG=2 NI=3 NO=40
CM
1.405466
0.633555 0.958365
0.359973 0.231046 1.163310
MO NX=3 NK=3 LX=FU,FI PH=SY,FI TD=ZE
LK
W X Y
FR PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
VA 1 PH 1 1 PH 2 2 PH 3 3
ST .5 PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
GROUP 2 semipartial correlation
DA NI=3 NO=40
CM
1.357541
0.614773 0.923433
0.265435 0.021623 1.263412
MO NX=3 NK=3 LX=FU,FI PH=SY,FI TD=ZE
LK
W X Y
FR PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
VA 1 PH 1 1 PH 2 2 PH 3 3
ST .5 PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
EQ PH(1,3,2) PH(3,2)
PD
```

```
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
```

LISREL syntax for computing the squared bivariate correlation between *x* and *y*.

```
TI corr_squared
DA NI=2 NO=40
CM
0.958365
0.231046 1.163310
MO NX=2 NK=2 LX=FU,FI PH=SY,FI TD=ZE AP=1
LK
X Y
FR LX 1 1 LX 2 2 PH 2 1
ST .5 LX 1 1 LX 2 2 PH 2 1
VA 1 PH 1 1 PH 2 2
CO PA(1)=PH(2,1)**2
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
```

LISREL syntax for computing the unsquared and squared semipartial correlation between *x* and *y* simultaneously in three groups, controlling *x* for *w*, with equality constraints in place. This syntax includes the data for Democrats, Republicans, and Independents discussed in Example 2.

```
GROUP 1 semi corr Dem
DA NG=3 NI=3 NO=408
CM
2.564644
0.734038 5.394397
0.418448 1.223178 2.489353
MO NX=3 NK=3 LX=FU,FI PH=SY,FI TD=ZE AP=1
LK
W X Y
FR PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
VA 1 PH 1 1 PH 2 2 PH 3 3
ST .5 PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
CO PA(1)=PH(3,2)**2
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
GROUP 2 semi corr Rep
DA NI=3 NO=459
CM
2.476566
0.162600 6.525559
0.175854 1.112253 1.752890
MO NX=3 NK=3 LX=FU,FI PH=SY,FI TD=ZE AP=1
LK
W X Y
```

```
FR PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
VA 1 PH 1 1 PH 2 2 PH 3 3
ST .5 PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
CO PA(1)=PH(3,2)**2
EQ PH(1,3,2) PH(3,2)
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
GROUP 3 semi corr Ind
DA NI=3 NO=411
CM
2.138116
0.160980 5.305798
0.270231 1.174910 2.095828
MO NX=3 NK=3 LX=FU,FI PH=SY,FI TD=ZE AP=1
LK
W X Y
FR PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
VA 1 PH 1 1 PH 2 2 PH 3 3
ST .5 PH 3 1 PH 3 2 LX 1 1 LX 2 2 LX 3 3 LX 2 1
CO PA(1)=PH(3,2)**2
EQ PH(2,3,2) PH(3,2)
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
```

LISREL syntax for testing a pattern hypothesis for bivariate correlations. This syntax includes the multitrait–multimethod data discussed in Example 3.

```
TI bivariate correlation pattern hypothesis
DA NI=6 NO=113
CM
1.00
0.53 1.00
0.56 0.44 1.00
0.65 0.38 0.40 1.00
0.42 0.52 0.30 0.56 1.00
0.40 0.31 0.53 0.56 0.40 1.00
MO NX=6 NK=6 LX=DI,FR PH=ST TD=ZE
LK
QS AS ES QP AP EP
ST .5 LX 1 1 LX 2 2 LX 3 3 LX 4 4 LX 5 5 LX 6 6
ST .5 PH 2 1 PH 3 1 PH 3 2 PH 4 1 PH 4 2 PH 4 3
ST .5 PH 5 1 PH 5 2 PH 5 3 PH 5 4 PH 6 1 PH 6 2 PH 6 3 PH 6 4 PH 6
5
EQ PH 2 1 PH 5 1 PH 4 2 PH 5 4
EQ PH 3 1 PH 6 1 PH 4 3 PH 6 4
EQ PH 3 2 PH 6 2 PH 5 3 PH 6 5
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
```

LISREL syntax for testing a pattern hypothesis for partial correlations. This syntax includes the NLSY PIAT data discussed in Example 4.

```
TI partial correlation pattern hypothesis
DA NI=10 NO=1071
CM
 0.249
 0.003  0.229
 0.025  0.157  0.283
 0.032  0.141  0.213  0.258
-0.006  0.146  0.143  0.134  0.223
 0.024  0.171  0.269  0.217  0.178  0.377
 0.017  0.137  0.201  0.181  0.143  0.248  0.283
-0.009  0.146  0.144  0.128  0.166  0.172  0.149  0.246
 0.028  0.178  0.278  0.231  0.180  0.342  0.257  0.195  0.459
 0.012  0.143  0.194  0.174  0.146  0.231  0.203  0.166  0.275  0.316
MO NX=10 NK=10 LX=FU,FI PH=SY,FI TD=ZE
LK
SEX M08 R08 C08 M10 R10 C10 M12 R12 C12
FR PH 1 1 PH 3 2 PH 4 2 PH 4 3 PH 5 2 PH 5 3 PH 5 4 PH 6 2 PH 6 3
FR PH 6 4 PH 6 5 PH 7 2 PH 7 3 PH 7 4 PH 7 5 PH 7 6 PH 8 2 PH 8 3
FR PH 8 4 PH 8 5 PH 8 6 PH 8 7 PH 9 2 PH 9 3 PH 9 4 PH 9 5 PH 9 6
FR PH 9 7 PH 9 8 PH 10 2 PH 10 3 PH 10 4 PH 10 5 PH 10 6 PH 10 7
FR PH 10 8 PH 10 9 LX 2 1 LX 3 1 LX 4 1 LX 5 1 LX 6 1 LX 7 1 LX 8 1
FR LX 9 1 LX 10 1 LX 2 2 LX 3 3 LX 4 4 LX 5 5 LX 6 6 LX 7 7 LX 8 8
FR LX 9 9 LX 10 10
VA 1 PH 2 2 PH 3 3 PH 4 4 PH 5 5 PH 6 6 PH 7 7 PH 8 8 PH 9 9 PH 10 10
VA 1 LX 1 1
ST .5 PH 1 1 PH 3 2 PH 4 2 PH 4 3 PH 5 2 PH 5 3 PH 5 4 PH 6 2 PH 6 3
ST .5 PH 6 4 PH 6 5 PH 7 2 PH 7 3 PH 7 4 PH 7 5 PH 7 6 PH 8 2 PH 8 3
ST .5 PH 8 4 PH 8 5 PH 8 6 PH 8 7 PH 9 2 PH 9 3 PH 9 4 PH 9 5 PH 9 6
ST .5 PH 9 7 PH 9 8 PH 10 2 PH 10 3 PH 10 4 PH 10 5 PH 10 6 PH 10 7
ST .5 PH 10 8 PH 10 9 LX 2 2 LX 3 3 LX 4 4 LX 5 5 LX 6 6 LX 7 7 LX 8 8
ST .5 LX 9 9 LX 10 10
EQ PH 3 2 PH 6 5 PH 9 8
EQ PH 4 2 PH 7 5 PH 10 8
EQ PH 4 3 PH 7 6 PH 10 9
PD
OU ME=ML ND=4 XM EP=0.00001 IT=1000 NS AD=OFF
```