

CHAPTER 6

CONDITIONAL PROCESS MODELING

Using Structural Equation Modeling to Examine Contingent Causal Processes

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There is much theoretical and applied value to research that attempts to establish whether there is a causal relation between two variables X and Y . But establishing a causal association is rarely sufficient for broad understanding and application. We better understand some phenomenon and can better use that understanding when we can answer not only whether X affects Y , but also *how* X exerts its effect on Y , and *when* X affects Y and when it does not, or does so strongly as opposed to weakly. The “how” question relates to the underlying psychological, cognitive, or biological process, mechanism, or causal chain of events that links X to Y , whereas the “when” question pertains to what might be considered the boundary conditions of the causal association—under what circumstances, or for which types of people, does X exert an effect on Y and under what circumstances, or for which type of people, does X not exert an effect?

If it is true, and we believe it is, that all causal associations exist through some kind of causal chain of events and that all effects have at least some boundary conditions, then any study that addresses only the how question or the when question, but not both, is incomplete in significant ways, just as is a study that answers only whether or not an effect exists. A more complete analysis of a phenomenon will address both questions simultaneously, so as to uncover how a sequence of causal events—in terms of the direction, magnitude, or existence of the effect or lack thereof—depends on contextual or individual difference factors. The purpose of this chapter is to introduce some of the important principles and procedures of an analytical approach that does just this, an approach called *conditional process modeling*. After first introducing conditional process modeling and its history, we define some of the important terms and concepts we will use throughout this chapter. We then describe how to convert a conceptual diagram of a conditional process into a statistical model, the parameters of which can then be estimated using a structural equation modeling (SEM) program. We then show how parameters of the model are pieced together to yield estimates of the conditional nature of the process being modeled and how inferences about those estimates are made. We end by discussing some extensions of the procedure we outline into the arena of latent variable modeling.

WHAT IS CONDITIONAL PROCESS MODELING?

Process modeling is undertaken when the goal is to understand, explore, and estimate the mechanism by which some putative causal variable affects an outcome through at least one intermediary variable. *Conditional* process modeling is undertaken when this process is thought to be contingent on additional variables. As described in detail throughout this chapter, it is used to estimate the *direct* and *indirect* pathways through which a variable transmits its effects, as well as to model how the size of those effects depend on (or are *conditional on*) the value(s) of one or more *moderators*.

The term *conditional processing modeling* represents a melding of two ideas both conceptually and analytically: *process modeling* and *moderation analysis*. Judd and Kenny (1981) described *process modeling* in their early seminal piece in *Evaluation Review* as an attempt to “specify the causal chain” (p. 602) through which interventions exert their effects on an outcome of interest. Process modeling is now better known as *mediation analysis*; the label *process modeling* never gained traction. Of course, “mediation analysis” did gain traction following the publication of Baron and Kenny’s extraordinarily popular *Journal of Personality and Social Psychology* article (Baron & Kenny, 1986). We adopt “process modeling” here partly to honor the contributions that Charles Judd, David Kenny, and their early colleagues have

made to the literature in this area, but also because mediation is a controversial term which can invite confusion depending on how it is used and defined (see, e.g., Mathieu & Taylor, 2006).

The *conditional* in “conditional process modeling” stems from moderation analysis and the concept of *interaction* (see, e.g., Aiken & West, 1991; Jaccard & Turrisi, 2003). If an antecedent variable X 's effect on some consequent variable Y depends on a third variable W , we can say that X 's effect is *moderated by* or is *conditional on* W , or that X and W *interact* in influencing Y . In such a case, it is not meaningful to talk about X 's effect on Y without first conditioning that discussion on the value or values of the moderator, W . Typically, when a moderating effect is found in such an analysis, the interaction is probed in order to estimate *conditional effects*, sometimes called *simple slopes* or *simple effects*, which quantify the effect of X on Y at various values of W that have some kind of theoretical or practical (if not arbitrary) meaning (Bauer & Curran, 2005; Hayes & Matthes, 2009).

Although the term *conditional process modeling* is new (see Hayes, 2013), the concept is not. Some of the earliest literature on mediation analysis describes scenarios in which a causal process could be described by combining moderation and process analysis. Judd and Kenny (1981) contemplated the possibility that an experimental treatment could affect the magnitude of a mediation effect by influencing the size of the association between mediator and outcome. James and Brett (1984) showed how a variable's effect on some outcome might be mediated for some people but not others if one or more of the paths in a mediation model is moderated, what they termed *moderated mediation*. Like James and Brett, Baron and Kenny (1986) discussed how a manipulation's effect on a mediator might be moderated, but they also described how a moderated effect of a manipulation on an outcome could be mediated, a phenomenon called *mediated moderation*.

There are several examples of piecemeal approaches to assessing moderated mediation and mediated moderation processes in the literature prior to the turn of the century (e.g., Druley & Townsend, 1998; Tepper, Eisenbach, Kirby, & Potter, 1998). It was not until the publication of a handful of articles starting in 2005, however, that research articles truly and analytically integrating moderation and mediation analysis began to appear in scientific journals. Muller, Judd, and Yzerbyt (2005) provided analytical models and corresponding equations for examining when “mediation is moderated and moderation is mediated” (from the article title). Almost simultaneously but in different journals, Edwards and Lambert (2007) and Preacher, Rucker, and Hayes (2007) published derivations and analytical tools for quantifying and testing hypotheses about what Preacher et al. (2007) called the *conditional indirect effect*—the indirect effect of one variable on another through a third expressed as a function of one or two moderator variables.

Edwards and Lambert also discussed how direct effects in a mediation model, like indirect effects, also can be mathematically modeled as a function of a moderator variable. Finally, a number of articles from David MacKinnon and his colleagues (e.g., Fairchild & MacKinnon, 2009; Morgan-Lopez & MacKinnon, 2006) provided advice and examples for how to appropriately model moderated mediation and mediated moderation effects.

The impact of these articles, gauged by the number of citations they have collectively received, has been large, to say the least. Analyses that simultaneously estimate both moderated and mediated effects in order to better illuminate the boundary conditions of a mediated causal process are now found in the literature with ease (recent examples include Antheunis, Valkenberg, & Peter, 2010; Parade, Leerkes, & Blankson, 2010; Van Dijke & de Cremer, 2010). Yet, as significant as these methodological developments are, to date they have been discussed and applied only in the context of fairly simple models, generally involving a single intervening variable or mediator, with a single indirect effect that is moderated by one or, in a few instances, two variables. In the pages that follow, after first introducing some important concepts and terms, we illustrate the steps involved in the estimation and interpretation of a conditional process model in the context of a fairly general model involving multiple pathways of influence and several moderators, some of which combine multiplicatively in their influence on the process being modeled. Although there is danger in starting with an overly complex framework, we believe that there is advantage in doing so, for illustrating the procedure with a complex model allows us to describe the various rules in constructing and estimating a conditional process model that will apply to both complex and simple models.

Before beginning, it is important to briefly comment on the position we take with respect to statistics, correlation, and causality. Conditional process modeling is a tool for understanding causal processes. Although the example we use here will include an experimental manipulation, many associations later in the sequence of the putative causal chain of events have various noncausal interpretations. Of course, no statistical tool can actually be used to ascertain whether an association is causal, for the inferences one can make stem not from the statistical procedures one uses but how one goes about collecting the data. Furthermore, inferences are products not of mathematics but of mind. We do not object to the modeling of data that stem from a design that affords only limited causal interpretation. However, the researcher should be aware of the limitations of data, and should take pains to ensure that the interpretation (causal or not) is sensible in light of theory or past research on the process being studied.

INDIRECT, DIRECT, AND CONDITIONAL EFFECTS

Before proceeding, we define some important terms and concepts that form the foundation of conditional process modeling. We start with *indirect effect*. Consider the simple causal model displayed in Figure 6.1 (a, left),

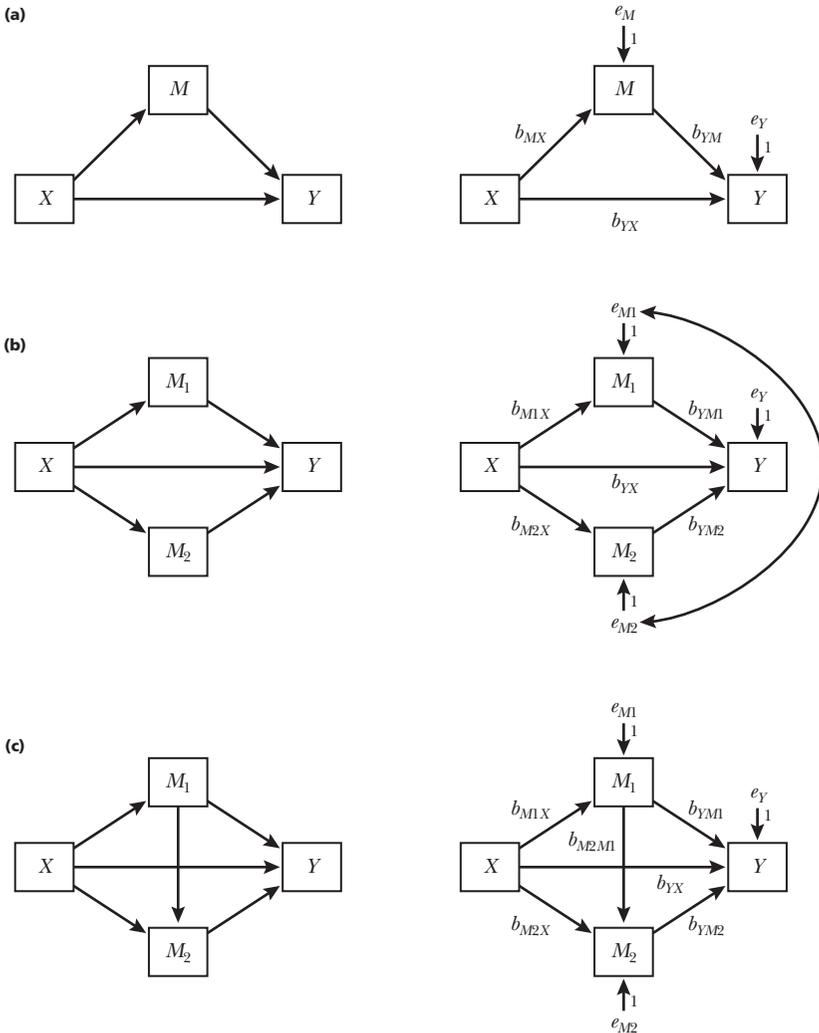


Figure 6.1 Conceptual (left) and statistical models (right) representing (a) simple mediation, (b) parallel multiple mediation, and (c) serial multiple mediation of the effect of X on Y . All variance and intercept parameters, and most covariance parameters, are omitted from this and subsequent figures to reduce clutter.

sometimes called a simple mediation model, represented in the form of a *conceptual model*. A conceptual model will be defined formally later. For now, suffice it to say that it is a visual representation of the process but not a formal path diagram, as one would use in SEM. Suppose M and Y are two observed, continuous measures, whereas X is either an observed continuous or dichotomous variable. Assuming linearity in all relations, the effect of X on both M and Y can be ascertained by estimating the parameters in the linear equations

$$M = i_M + b_{MX}X + e_M \quad (6.1)$$

$$Y = i_Y + b_{YM}M + b_{YX}X + e_Y \quad (6.2)$$

where i_M and i_Y are intercepts and e_M and e_Y are errors in prediction of M and Y , respectively. These coefficients come from the *statistical model* corresponding to the conceptual model of the process, depicted in Figure 6.1 (a, right), meaning the model as it would be specified in a SEM program. The parameters of these two equations, along with relevant variances and covariances, can be estimated using either SEM or two ordinary least squares regressions, the choice of which has little consequence on the results one will get. The indirect effect of X on Y through M is quantified as the product of the coefficients for the paths linking X to Y through M .¹ In this model, the path from X to M is b_{MX} and the path from M to Y is b_{YM} , so the indirect effect is $b_{MX}b_{YM}$. We can say that two cases differing by one unit on X are estimated to differ by $b_{MX}b_{YM}$ units on Y as result of how the difference in X causally influences M , which in turn causes differences in Y .

The indirect effect might not, and typically does not, completely quantify how differences in X map on to differences in Y , for X may affect Y *directly*, that is, independently of the indirect pathway through M . The *direct effect* of X on Y , quantified as b_{YX} , quantifies by how much two cases differing by one unit on X are estimated to differ on Y independent of M , or holding M constant. These two components of X 's influence on Y , the indirect and direct effects, sum to the total effect of X on Y , $b_{MX}b_{YM} + b_{YX}$.

Early in the evolution of mediation or process analysis, great emphasis was placed on tests of significance for each coefficient in Equations 6.1 and 6.2 above as well as the total effect. The goal of early piecemeal approaches was to ascertain whether M meets various criteria outlined by Judd and Kenny (1981) and Baron and Kenny (1986) for establishing M as a mediator of X 's effect either completely (when all but the direct effect is statistically different from zero) or partially (when the direct effect is statistically different from zero but closer to zero than the total effect). Although this approach to mediation analysis remains popular among researchers, it now is perceived by experts in mediation analysis as outdated and has fallen out of

favor. In its place is a greater emphasis on the estimation of, and inferences about, the indirect effect, with much less concern about tests of significance for individual coefficients in the model, the relative sizes or significance of the direct and total effect, or labeling M as a complete or partial mediator (see, e.g., Cerin & MacKinnon, 2009; Hayes, 2009, 2013; Rucker, Preacher, Tormala, & Petty, 2011).

It is often the case, perhaps even typical, that X influences Y through more than one intermediary variable. Figure 6.1(b), depicts a *parallel multiple mediator model* (see, e.g., Preacher & Hayes, 2008) with two intervening variables. Again assuming linearity of all effects, this model requires three equations to estimate the effects of X :

$$M_1 = i_{M1} + b_{M1X}X + e_{M1} \quad (6.3)$$

$$M_2 = i_{M2} + b_{M2X}X + e_{M2} \quad (6.4)$$

$$Y = i_Y + b_{YM1}M_1 + b_{YM2}M_2 + b_{YX}X + e_Y. \quad (6.5)$$

There are two kinds of indirect effect in this model: specific and total. The *specific indirect effect* of X on Y through M_1 is the product of the paths linking X to Y through M_1 , which here is $b_{M1X} b_{YM1}$. The specific indirect effect through M_2 is calculated similarly as $b_{M2X} b_{YM2}$. The sum of the specific indirect effects produces the *total indirect effect* of X : $b_{M1X} b_{YM1} + b_{M2X} b_{YM2}$. The direct effect of X is still b_{YX} and the total effect of X is the sum of the direct and the total indirect effects: $b_{YX} + b_{M1X} b_{YM1} + b_{M2X} b_{YM2}$.

The model in Equations 6.3 through 6.5 assumes no causal association between M_1 and M_2 , unlike the model in Figure 6.1(c). Such a causal association can be estimated by including M_1 in the model of M_2 :

$$M_2 = i_{M2} + b_{M2X}X + b_{M2M1}M_1 + e_{M2}. \quad (6.6)$$

The specific indirect effects for M_1 and M_2 are still defined as above, but this addition to the model produces another specific indirect effect of X , this one through M_1 and M_2 serially, estimated as the product of the three constituent paths: $b_{M1X} b_{M2M1} b_{YM2}$. The total indirect effect of X is the sum of the three specific indirect effects ($b_{M1X} b_{YM1} + b_{M2X} b_{YM2} + b_{M1X} b_{M2M1} b_{YM2}$) and the total effect of X on Y is the sum of the direct effect (b_{YX}) and all indirect effects.

Whereas in mediation analysis, where focus is on estimating and interpreting the indirect effect X on Y through M , in *moderation analysis*, the goal is to ascertain whether X 's effect on Y depends on some third variable W . When evidence of moderation is found, then interpretative focus is placed on estimating and interpreting the *conditional effect* of X on Y given W . The most basic moderation model is depicted graphically in Figure 6.2(a). The

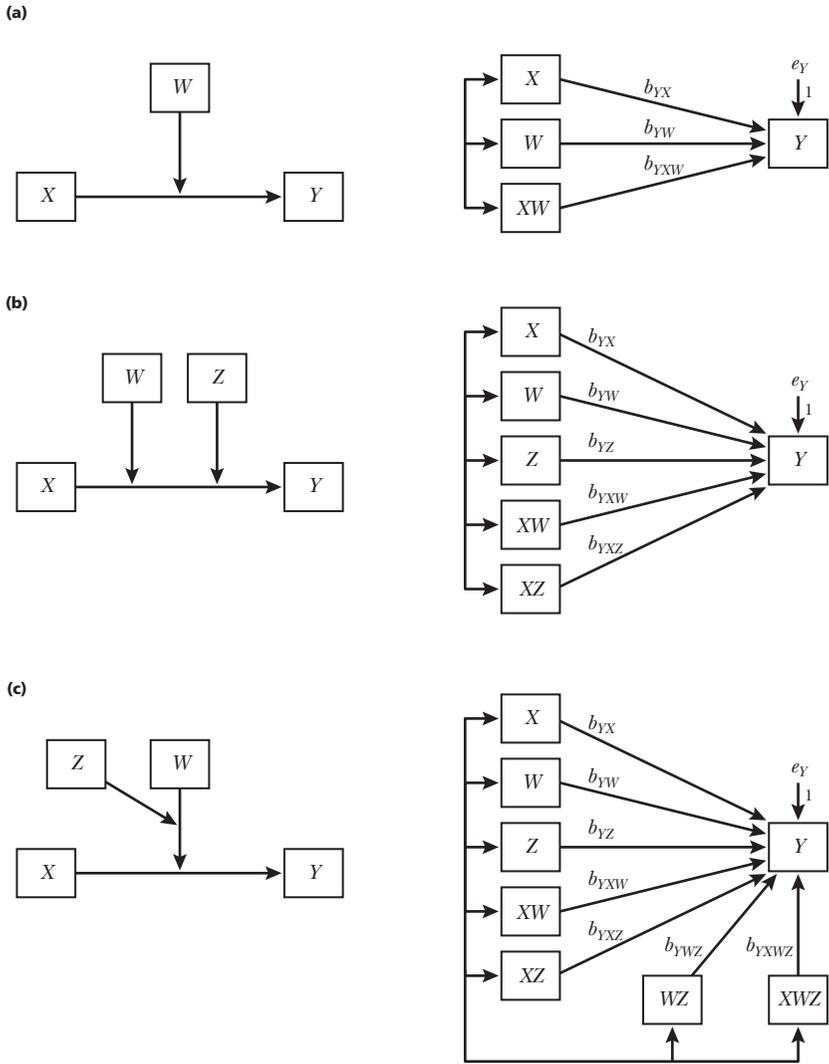


Figure 6.2 Conceptual (left) and statistical models (right) representing (a) simple moderation, (b) additive multiple moderation, and (c) multiplicative multiple moderation.

unidirectional arrow from W to the path linking X to Y represents ‘depends on,’ that is, the effect of X on Y depends on W . Although there are many different forms that moderation can take, the most commonly estimated form and the only form we will address in this chapter is linear moderation, or linear interaction. Assuming X and W are either continuous or dichotomous

mous, the moderating effect of W on the effect of X on Y can be derived by estimating the parameters in the following model:

$$Y = i_Y + b_{YX}X + b_{YW}W + b_{YXW}XW + e_Y \quad (6.7)$$

where XW is the product of W and X . In this model, X 's effect on Y is estimated as a linear function of W , as can be seen by reexpressing Equation 6.7 in an algebraically equivalent form

$$Y = i_Y + (b_{YX} + b_{YXW}W)X + b_{YW}W + e_Y. \quad (6.8)$$

In this model, the conditional effect of X on Y is $b_{YX} + b_{YXW}W$. It is interpreted as the amount by which two cases differing by a unit on X , given a specific value on W , are estimated to differ on Y . For instance, when $W = 2$, two cases who differ by one unit on X are estimated to differ by $b_{YX} + 2b_{YXW}$ units on Y . But when $W = 3$, two differing by one unit on X are estimated to differ by $b_{YX} + 3b_{YXW}$ units on Y . When b_{YXW} is different from zero, it is meaningless to talk about X 's effect using a single point estimate, because its effect is not a constant. It is a function of W , meaning that it depends on W .

The effect of X on Y can be conditional on more than one variable. For the purposes of our discussion of conditional process modeling, there are two scenarios worth mentioning. Figure 6.2(b) depicts X 's effect as additively conditional on both W and Z . Such a scenario would be modeled as

$$Y = i_Y + b_{YX}X + b_{YW}W + b_{YZ}Z + b_{YXW}XW + b_{YXZ}XZ + e_Y. \quad (6.9)$$

This model can be written equivalently as

$$Y = i_Y + (b_{YX} + b_{YXW}W + b_{YXZ}Z)X + b_{YW}W + b_{YZ}Z + e_Y \quad (6.10)$$

which shows how X 's effect is an additive linear function of both W and Z . So the conditional effect of X on Y is $b_{YX} + b_{YXW}W + b_{YXZ}Z$. It depends on both W and Z . For instance, when $W = 2$ and $Z = 3$, the conditional effect of X is $b_{YX} + 2b_{YXW} + 3b_{YXZ}$.

Figure 6.2(c) depicts a situation where X 's effect on Y is multiplicatively conditional on W and Z , modeled as

$$Y = i_Y + b_{YX}X + b_{YW}W + b_{YZ}Z + b_{YXW}XW + b_{YXZ}XZ + b_{YWZ}WZ + b_{YXWZ}XWZ + e_Y \quad (6.11)$$

or, rewritten in equivalent form to reveal the conditional nature of X 's effect on Y :

$$Y = i_Y + (b_{YX} + b_{YXW}W + b_{YXZ}Z + b_{YXWZ}WZ)X + b_{YW}W + b_{YZ}Z + b_{YWZ}WZ + e_Y. \tag{6.12}$$

From Equation 6.12 it can be seen that X 's effect on Y is a function of W , Z , as well as their product: $b_{YX} + b_{YXW}W + b_{YXZ}Z + b_{YXWZ}WZ$. So it is conditional on both W and Z . For instance, when $W = 3$ and $Z = 2$, the conditional effect of X is $b_{YX} + 3b_{YXW} + 2b_{YXZ} + 6b_{YXWZ}$.

If a causal effect can be conditional on a moderator, and indirect and direct effects quantify a variable's causal effect on some variable, then it follows that indirect and direct effects can also be conditional. Figure 6.3 depicts a simple model in which the direct effect of X on Y is moderated by W , as is the indirect effect of X on Y through M . This is what Edwards and Lambert (2007, p. 4) called a "direct effect and second stage moderation model." It is a conditional process model, albeit a rudimentary one, because it includes elements of both moderation and mediation. Of interest in such a model are the conditional direct and indirect effects of X , which can be derived from the coefficients of two linear models

$$M = i_M + b_{MX}X + e_M \tag{6.13}$$

$$Y = i_Y + b_{YX}X + b_{YW}W + b_{YM}M + b_{YXW}XW + b_{YMW}MW + e_Y. \tag{6.14}$$

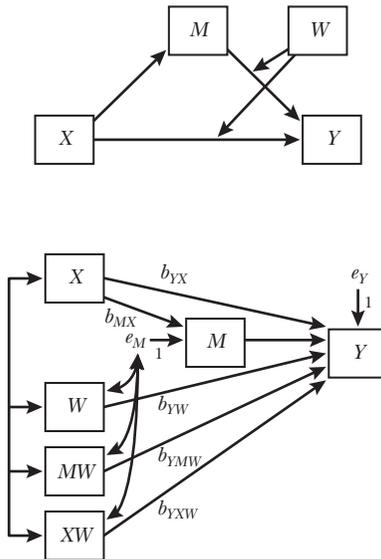


Figure 6.3 Conceptual (top) and statistical model (bottom) representing a form of conditional process model in which the indirect and direct effect of X is moderated by W .

By rewriting Equation 6.14 in an equivalent form,

$$Y = i_Y + (b_{YX} + b_{YXW}W)X + b_{YW}W + b_{YM}M + b_{YMW}MW + e_Y \quad (6.15)$$

it can be seen that the direct effect of X on Y depends on W , for it is a linear function of W : $b_{YX} + b_{YXW}W$. So the direct effect is conditional on W .

Earlier it was shown that the indirect effect of X on Y through M is derived as the product of the coefficients linking X to Y going through M . From Equation 6.13, the effect of X on M is estimated as b_{MX} . But what is the effect of M on Y holding X constant? Unlike earlier, there is no single effect of M on Y , for its value depends on W , as can be seen by rewriting Equation 6.15 in an equivalent form:

$$Y = i_Y + (b_{YX} + b_{YXW}W)X + (b_{YM} + b_{YMW}W)M + b_{YW}W + e_Y. \quad (6.16)$$

Here, the effect of M on Y is a linear function of W : $b_{YM} + b_{YMW}W$. Multiplying the effect of X on M by the conditional effect of M on Y given W yields the *conditional indirect effect* of X on Y : $b_{MX}(b_{YM} + b_{YMW}W)$. So, the indirect effect is not a single quantity defined by the parameter estimates but, instead, is a function of those estimates and that involves W . Substituting a different value of W into this formula for the indirect effect will yield a different estimate of X 's effect on Y through M .

TRANSLATING A CONCEPTUAL MODEL INTO A STATISTICAL MODEL

Our hypotheses and beliefs about the process through which one variable affects another through a causal sequence of events is represented in the form of a diagram we call the *conceptual model*. This conceptual model is not the same as a formal path diagram or structural equation model and should not be interpreted as such. Rather, it visually represents the direct and indirect paths of influence from a causal or antecedent variable that is the focus of the study to the final outcome or consequent variable of interest, as well as which of those causal influences are moderated. The conceptual model uses a visual notation common for representing direct, indirect, and moderated causal effects. A unidirectional arrow represents the direction of causal flow or path from an antecedent variable (where the arrow begins) to a consequent variable (where the arrow ends), and a unidirectional arrow from one variable to another arrow or path represents moderation by that variable. Yet ultimately, this conceptual model must be translated into a formal statistical model—a path diagram or structural equation model—in

order to test hypotheses and quantify the various effects of interest statistically. This section describes this translation process.

Thus far, our discussion of conditional process modeling has been abstract. To make it more concrete, we now introduce a simulated working example to be used throughout the rest of this chapter. This example, conceptually diagrammed in Figure 6.4, closely resembles the example used by Judd and Kenny (1981) in their seminal paper on process modeling. Participants in the study either are ($X = 1$) or are not ($X = 0$) exposed to a mass media campaign designed to lower risks of coronary heart disease, with exposure being determined by random assignment. It is hypothesized that exposure to the campaign will increase knowledge of the dietary causes of heart disease (M_1 , measured such that a higher value corresponds to greater knowledge), and this knowledge will translate into dietary choices that are better for the heart (M_2 , measured such that higher scores correspond to more healthy eating habits). Such choices, in turn, will result in better heart health (Y : measured with various physiological indicators of heart health). It is also proposed that those who use the mass media more often (W , mea-

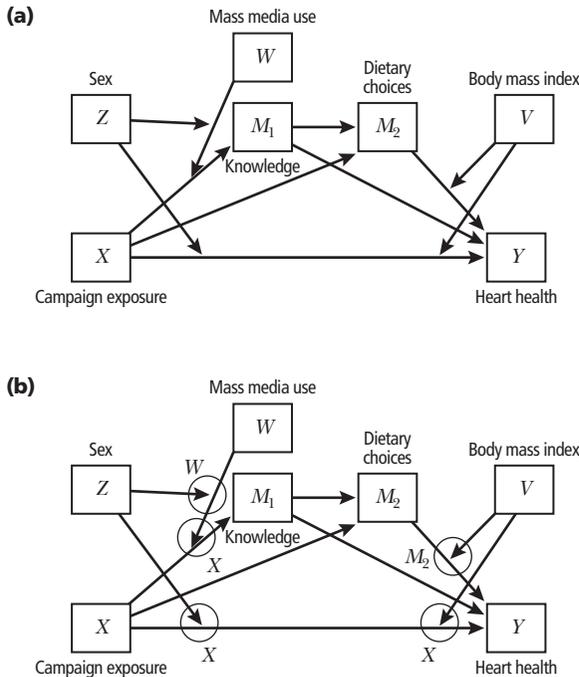


Figure 6.4 (a) The conceptual model for the Heart Health Campaign example; (b) points of moderation represented in the conceptual model following the completion of Step 2.

sured such that higher scores correspond to greater use) are more likely to learn from the mass media campaign, indicating moderation of the effect of the campaign on knowledge, but that this differential effect of exposure as a function of mass media use should depend on sex ($Z = 1$ for males, 0 for females). Further complicating the process, it is expected that the effect of dietary choices on heart health will depend on a person's body mass (V : quantified as BMI, a numerical function of weight and height, such that a higher score represents greater body mass). Finally, it is acknowledged that the campaign could influence heart health through other means. For instance, the campaign could influence dietary choices independent of knowledge, and it could influence heart health through means other than diet. Direct effects of the mass media campaign, however, are expected to independently (i.e., additively rather than multiplicatively) depend on sex as well as body mass. For the sake of this illustration, assume that sex, mass media use, and body mass are measured at the beginning of the campaign, knowledge of the dietary causes of heart disease is measured three months after the start of the campaign, dietary choices three months after that, and, finally, heart health three months still later (i.e., nine months after the start of the campaign). All variables are measured as continua except for sex and exposure to the campaign, which are dichotomous.² The simulated means, variances, and covariances are reported in Table 6.1.

This elaborate process is diagrammed in Figure 6.4(a) as a conceptual model. At the core of this conceptual model is a serial multiple mediator model with three indirect effects of exposure to the campaign (X) on heart health (Y) (via knowledge [M_1], dietary choices [M_2], and knowledge and dietary choices serially), as well as a direct effect. The effect of the campaign on knowledge is diagrammed as moderated by mass media use (W) (represented with the arrow from mass media use to the path linking exposure to knowledge), with the extent of this moderation itself moderated by sex (Z) (represented with the arrow from sex to the arrow linking mass media use to the path linking campaign exposure to knowledge—a three way interaction). In addition, the direct effect of exposure to the campaign is additively moderated by sex and body mass (V) (represented with the arrows from these two variables to the path linking exposure to heart health—two-way interactions). Finally, the effect of dietary choices on heart health is moderated by body mass (i.e., the arrow from body mass to the path linking dietary choices to heart health, also a two-way interaction).

A conceptual model might be incomplete, depending on additional hypotheses one might propose linking variables in the system. For instance, it might be reasonable to expect that there would be sex differences in knowledge. This path of influence from sex to knowledge is missing from the conceptual diagram in Figure 6.4. Although it could be added, it turns out that

TABLE 6.1 Means and Variance-Covariance Matrix for Heart Health Campaign Example

	Mean	Variance	M_1	M_2	Y	W	Z	V	WX	ZX	ZW	ZWX	VM_2	VX	X
M_1	61.113	118.292	118.292	8.303	27.006	1.354	-0.212	0.873	7.045	0.529	-0.204	2.807	233.987	41.827	1.503
M_2	7.906	2.738	8.303	2.738	8.249	0.054	-0.061	0.210	0.532	0.015	-0.234	0.110	76.831	3.248	0.114
Y	25.077	57.496	27.006	8.249	57.496	0.335	-0.067	12.477	3.650	0.454	-0.075	2.026	328.844	30.315	0.852
W	3.954	0.982	1.354	0.054	0.335	0.982	0.015	0.002	0.615	0.026	0.534	0.319	1.363	1.002	0.039
Z	0.495	0.250	-0.212	-0.061	-0.067	0.015	0.250	0.062	-0.030	0.119	0.996	0.483	-1.253	-0.229	-0.009
V	27.272	13.415	0.873	0.210	12.477	0.002	0.062	13.415	-0.013	0.035	0.237	0.124	111.339	7.021	0.015
WX	1.992	4.524	0.432	0.055	0.116	-0.025	-0.001	-0.072	0.244	0.488	0.168	2.194	14.505	27.456	1.008
ZX	0.235	0.180	-0.111	-0.010	0.066	-0.001	0.000	-0.003	0.004	0.062	0.492	0.731	0.636	3.276	0.119
ZW	1.973	4.443	-0.037	-0.018	0.025	-0.012	0.000	-0.011	-0.008	0.010	0.245	2.212	-4.870	-0.405	-0.018
ZWX	0.956	3.184	0.190	0.017	0.078	-0.007	0.010	0.025	-0.003	0.000	-0.006	0.061	3.705	13.304	0.484
VM_1	215.817	3,023.412	0.653	0.492	5.242	-0.121	-0.072	-0.448	0.003	-0.052	-0.011	0.011	37.962	146.748	3.259
VX	13.480	192.934	0.401	0.043	0.923	-0.072	-0.003	-0.003	-0.001	0.016	0.025	-0.002	1.716	3.353	6.824
X	0.494	0.250	1.503	0.114	0.852	0.039	-0.009	0.015	0.001	0.000	-0.001	0.004	0.043	0.000	0.250

Note: In the analysis reported in this manuscript, X, W, Z, V, and M_2 were mean centered and so have means of 0. The Mean and Variance columns are for uncentered data. Diagonal elements are variances after mean centering. Lower triangular elements are covariances after mean centering. Upper triangular elements are covariances before centering. Centering was conducted before computation of products.

doing so is not necessary, for it will end up in the statistical translation of the conceptual model anyway, as will be seen. Although there is room for debate on this point, we recommend that conceptual diagrams depict only those relations—direct, indirect, and moderated—that pertain specifically to the effect of the causal variable that begins the chain of events (exposure to the campaign) leading to the final outcome variable of interest (heart health). Later, additional causal influences in the system can be added to the statistical model if desired prior to estimation of the coefficients in the model.

We do not estimate the conceptual model. Rather, once the conceptual model is constructed, that model must be translated into a statistical model in the form of various linear equations the parameters of which can then be estimated. For novices, this is probably the more difficult part of conditional process modeling. The recipe described below for doing so will seem at first to contain a lengthy list of ingredients, but with an understanding of the basic steps and some practice, the process will become more intuitive and the recipe will not be required in order to complete it.

Step 1: Derive the Number of Linear Models Necessary to Model the Process Statistically

The first step is perhaps the easiest of them all. A conditional process model requires p linear models, one for each consequent variable. Recall that a consequent variable is a variable that receives a causal path, meaning that at least one variable in the model sends an arrow to it. As can be seen in Figure 6.4, there are three consequent variables in this conceptual model, M_1 , M_2 , and Y . So we need $p = 3$ linear models to represent the process, one for each consequent variable.

Step 2: Label the Points of Moderation in the Conceptual Model

The next step is a visual aid that helps in the derivation of the necessary product terms in the p linear models. Using the diagram of the conceptual model, circle any arrowhead that points at another path and label that circle with the name of the variable that sends an arrow *straight through* it. Importantly, make sure that your circle encompasses only a single arrowhead. So when drawing your conceptual diagram, make sure that if a path is moderated by more than one variable, you visually separate the arrowheads pointing at the same path. As can be seen in Figure 6.4(b), there are 5 points of moderation (two for the path from X to Y , one for the path from

M_2 to Y , one for the path from X to M_1 , and one for the path from W to the path from X to M_1).

Step 3: Construct Sequences of Variable Names for Each Consequent

The next step is perhaps the most complicated to explain, but it is simple to complete once understood. This step involves tracing all *valid pathways* from an antecedent variable to a consequent variable while constructing a sequence of variable names as the pathway is traced. A pathway from an antecedent to a given consequent is considered a valid pathway if you can trace from that antecedent to the consequent while never tracing in a direction opposite to the arrow and never passing through another antecedent variable. A sequence of variable names is constructed for a given pathway by “collecting” variable names as you trace along the path from the antecedent to the consequent. A sequence of variable names always begins with the name of the antecedent variable at the starting point, and as you pass through a circle, append the name of the variable with which the circle is labeled to the end of the sequence *unless that variable name already occurs in the sequence you are generating*, in which case you simply ignore it. When you reach a consequent variable, stop. Do not include the name of the consequent variable at which you stop in the sequence.

Consider consequent M_1 . There are three valid pathways from an antecedent variable to M_1 . The first starts at X and passes through a circle labeled X before ending at M_1 . So the sequence for that pathway is “ X ”. We do not count the second X because X already exists in the sequence at the point our tracing intersects the X circle en route to M_1 . The second pathway starts at W and passes through a circle labeled W and then a circle labeled X before ending at M_1 . So the sequence for this second pathway is “ WX ”. We add X to this sequence after W because X did not exist in the sequence up to the point we reached the X circle, but because W already exists in the sequence once we reached the W circle, that W is not included in the sequence. The final pathway starts at Z and passes through a circle labeled W , and then one labeled X , before terminating at M_1 . So the sequence for this last pathway to M_1 is “ ZWX ”. There are no other sequences for consequent M_1 because there are no additional means of tracing from a variable to M_1 without violating one of the two rules above. For instance, although there are two ways to trace from V to M_1 , (V to M_2 to M_1 , and V to Y to M_1), doing so violates not just one but both rules—tracing opposite of the direction an arrow points (M_2 to M_1 or Y to M_1) as well as passing through another variable (M_2 or Y).

Using this same procedure, there are two valid pathways to consequent M_2 , each with only one variable in the sequence: “ M_1 ” and “ X ”. Finally, there are six valid pathways to consequent Y . The sequences for these six pathways are “ X ”, “ ZX ”, “ VX ”, “ M_1 ”, “ M_2 ” and “ VM_2 ”. Clearly, this step can create a lot of information that is hard to keep in memory, so we recommend you implement it on a piece of paper (as in Table 6.2), keeping careful track of what sequences are generated for each consequent so that you can refer to them later.

Step 4: Expansion of Sequences with at Least Three Variable Names

For most conditional process models, Step 4 is not necessary, but it is important to describe this step because you might, on occasion, need to implement it. Let k be defined as the maximum number of variables in any sequence for a given consequent. So for consequent M_1 , $k = 3$, whereas for consequents M_2 and Y , $k = 1$. For any consequent with $k > 2$, take each sequence with at least three variable names and generate all possible combinations of variable names containing at least one fewer variable. In

TABLE 6.2 The Outcome of Various Steps Converting the Conceptual Model into a Statistical Model

	Consequent		
	M_1	M_2	Y
Variable sequences generated at Step 3	X	X	X
	WX	M_1	ZX
	ZWX		M_1
			M_2
			VM_2
			VX
Variable sequences added at Step 4	ZX	None	None
	ZW		
Predictor variable list after completion of Step 5	X	X	X
	W	M_1	V
	Z		Z
	WX		M_1
	ZX		M_2
	ZW		ZX
	ZWX		VX
		VM_2	

most conditional process models to which this step applies, k is likely to be no greater than 3, but we illustrate first with $k = 4$. Suppose a sequence contained the variable names ABCD. All possible combinations of variable names with at least one fewer variable would be ABC, ABD, ACD, BCD, AB, AC, AD, BC, BD, and CD. In the current example, one sequence for consequent M_1 contains the sequence ZWX. So all possible combinations with at least two variables would be ZW, ZX, and WX. As there are no more sequences for any consequent variable with a least 3 variables, we stop.

Once all combinations are generated, add only those combinations generated at this step to the list of sequences for that consequent variable that are not already present in that consequent variable's sequence list. Ignore the order of the variable names in a sequence, meaning, for example, that WX can be considered equivalent to XW. Because we implemented this step only for consequent M_1 , we need to consider only the list of sequences already generated for M_1 . Recall that list contains the sequences "X", "WX" and "ZXW". Step 4 generated "ZX" "ZW", and "XW" by expansion of "ZXW". Ignoring order, "XW" already appears in the sequence list (as "XW") so we ignore it, but "ZX" and "ZW" do not already appear, so we add these two combinations to the list of sequences for consequent M_1 , yielding the list "X", "WX", "ZWX" "ZX", and "ZW". This completes Step 4. It may be helpful to return to Table 6.2 at this point to check understanding.

Step 5. Use the List of Sequences to Generate the Linear Models for Each Consequent

After completion of Step 4 (if applicable), you are ready to construct the linear models for each consequent. The rules for doing so are simple. First, any variable name that appears in any sequence for a given consequent variable should be a predictor variable in the model of that consequent. Then all remaining sequences in the list for a given consequent containing at least two variables should be predictors in the model of that consequent, represented as products of those variables in the sequence. Each linear model should also include an intercept and a random error. Use whatever convention you want for labeling the regression parameters for each predictor variable in each linear model. Here, we use the $b_{\text{TO-FROM}}$ notation used throughout much of this book.

Referring to Table 6.2, the model for consequent variable M_1 should include X, W, Z (because these variables appear at least once in the list of sequences for consequent M_1), as well as WX, ZX, ZW, and ZWX as predictors. Thus,

$$M_1 = i_{M_1} + b_{M_1X}X + b_{M_1W}W + b_{M_1Z}Z + b_{M_1WX}WX + b_{M_1ZX}ZX + b_{M_1ZW}ZW + b_{M_1ZWX}ZWX + e_{M_1}. \quad (6.17)$$

The model for consequent variable M_2 is much simpler, for there are only two sequences in the list and no sequences with two or more variables, so no products of variables. The model is

$$M_2 = i_{M_2} + b_{M_2X}X + b_{M_2M_1}M_1 + e_{M_2}. \quad (6.18)$$

The model for consequent variable Y is somewhat more complex, with five variables that appear somewhere in one of the sequences: X , Z , M_1 , M_2 , and V . In addition, there are three products: ZX , VX , and VM_2 . So the model is

$$Y = i_Y + b_{YX}X + b_{YZ}Z + b_{YM_1}M_1 + b_{YM_2}M_2 + b_{YV}V + b_{YZX}ZX + b_{YVX}VX + b_{YVM_2}VM_2 + e_Y. \quad (6.19)$$

Step 6: Fine-Tuning the Models

These three linear equations are a mathematical representation of the conceptual model. At this point, it is worth taking a look at the equations and examining whether something important might be missing. For example, notice that the model assumes that there is no effect of body mass (V) on dietary choices (M_2). The researcher might feel that it is important to allow body mass to affect diet, or to at least statistically control for body mass when estimating the effect of dietary knowledge (M_1) on dietary choices (M_2). In that case, add V as a predictor to the model of M_2 (Equation 6.18). Of course, if this effect had been represented in the conceptual model, then V would have ended up as a predictor of M_2 . It is also common in some fields to partial out various demographics to adjust all coefficients for the possibility that the associations they represent are spurious. For instance, we could include age or income as a predictor variable to all three models, and all coefficients would be adjusted for the influence of age and income on all consequent variables and path estimates leading to them.

It is important, however, that no variables are *removed* from the model generated from the first five steps, at least not yet. Some of the predictor variables in the linear models generated from this procedure are there because they are analytically necessary in order to properly estimate the moderated effects. For instance, the three way interaction between X , W , and Z in the model of M_1 requires the inclusion of all three two-way interactions in that model as well as X , W , and Z , and the inclusion of the interactions between X and Z and X and V in the model of Y requires the inclusion of X , W , and V in the model. So, for example, even though we might have no basis for believing that there are likely to be sex differences in knowledge or heart health, Z must be in the model of M_1 and Y because Z is a component

of an interaction involving Z in both models. Excluding Z from the model of M_1 and Y would be equivalent to constraining its coefficient in these models to zero. Unless the corresponding parameters being estimated actually are zero, excluding these variables from the model would result in a biased estimate of the interaction. Later, if some of the higher order interactions are found to be nonsignificant, they (and other terms in the model that are analytically necessary for proper estimation of those interactions) could be deleted.

We also believe that in models that include indirect effects, all corresponding direct effects should be included in initial model estimation regardless of whether those effects are expected theoretically or otherwise. In this example, the model being estimated includes all possible direct and indirect effects from campaign exposure to heart health. However, one could imagine scenarios in which one's conceptual model includes only some of the possible direct effects. For instance, suppose there were no a priori basis for believing that knowledge influences heart health directly. We recommend including this direct effect in the model anyway, for if this a priori belief is wrong, a failure to include this direct effect can bias the estimation of the conditional indirect effects from knowledge to heart health through dietary choices as well as the conditional direct effects of exposure to the campaign on heart health. Testing causal processes requires the estimation of both direct and indirect effects, or at least an openness to the existence of both types of effects. Excluding a direct effect represents a constraint in the model—that the direct effect is zero. We believe it is better to let the data tell you whether a direct effect is zero rather than forcing it to be so. The loss in parsimony that results if you are right is a small price to pay when it means less biased estimation of indirect effects in the event you are wrong, and the decision to include these untheorized direct effects can always be undone if an initial estimation reveals no evidence of such effects. Of course, if you heed this advice, there would be no need to make this change when fine-tuning the model, for your conceptual model will include all direct effects and they will appear in the equations after following the steps above.

Once you develop a familiarity with how to represent a process or a set of hypotheses in the form of a conceptual model, this fine-tuning process will no longer be necessary. With experience working with conditional process modeling, your conceptual model will probably translate into a statistical model that, with the exception of some terms included by analytical necessity to model a moderation process, exactly corresponds to the causal process you intend to model, with all necessary terms in the linear equations to quantify and test hypotheses of interest.

MODEL ESTIMATION

With the linear models representing the conceptual model now derived, it is possible to start estimating the parameters of the full conditional process model itself. In this section we describe our approach to estimation. We recognize that there is some room for debate about the steps we describe here, and that different analysts might approach the task differently depending on their own personal philosophy about model fitting and data exploration.

The conceptual model is holistic, in that it is an omnibus proposal about a set of associations, some unmoderated and some moderated, that link various antecedent variables to consequent variables. Of course, it is possible that the data might not be consistent with parts of the model. For example, if there is no evidence after estimation of the full model that the effect of dietary choices on heart health is moderated by body mass, then it would be sensible to reconceptualize the model without this moderated path. Doing so would change the linear models derived earlier, of course.

We believe that prior to moving forward with estimation of a full structural equation model, it is sensible to first estimate the linear model for each of the consequents in ordinary least squares regression or in an SEM program in order to ascertain whether the highest order interaction(s) that the process presumes exists actually are in evidence when subjected to empirical test. If the predicted interaction does emerge, then all is well with that component of the model. But if an interaction that the conceptual model and the theory or hypotheses that gave rise to it turns out not to have support in the data (as gauged by a hypothesis test on the corresponding parameter in the model), then remove it from the conceptual model and rederive the linear model for the consequent variable given the new conceptual model. For example, suppose an OLS regression analysis of the model for consequent M_1 (knowledge) revealed no statistically significant three-way interaction between X , W , and Z . In that case, it would be reasonable to reconceptualize the model to fix the interaction between X and W to be constant across values of Z . In terms of the conceptual model, this would involve removing the path from Z to the path from W moderating the effect of X on M_1 . Following the steps described above, this would yield a new, much simpler linear model for M_1 : $M_1 = i_{M_1} + b_{M_1X}X + b_{M_1W}W + b_{M_1WX}WX$. This model could then be estimated in OLS to support (or refute) that the effect of X on M_1 varies as a function of W . Suppose it does. Then one can proceed to the next consequent variable, and so forth, iteratively estimating and reestimating the model of each consequent until one settles on a model for each that is consistent with the data.

When we approached the first step of model estimation in this fashion, we found that the three-way interaction in the model of M_1 was statistical-

ly significant, implying that the moderation of X 's effect by W was indeed moderated by Z . All other two-way interactions in this model are necessary for the proper estimation of the three-way interaction, so they must be retained in this model, as must the lowest order terms in the model (X , W , and Z) regardless. We also found that all four of the expected two-way interactions were statistically significant in the model of Y . So there was no need to reconceptualize the process by changing moderated paths to unmoderated ones.

A couple of comments are in order about this procedure. First, it will often be the case that this model modification stage, which we have described as occurring after the translation of the conceptual model into a set of linear models, actually will have occurred first. That is, an investigator may take a set of known moderated relations revealed from a set of initial and simpler analyses and piece them together in the form of a full conditional process model in order to estimate conditional direct and indirect effects. In fact, we believe this is probably more consistent with the way that researchers actually proceed. One's *a priori* beliefs about what the data will reveal are often tempered by the reality revealed in initial analyses, and those beliefs modified in light of what is now known before they are pieced together into a conditional process model.

Second, the procedure we just described sounds a lot like data mining, and there is always some danger to letting empirical criteria rather than theory or hypotheses overly influence the modeling process. At the same time, there is value to being sensitive to what preliminary analyses are suggesting and modifying one's original model as needed in order to bring it into closer alignment with what those preliminary analyses are telling you. In this case, because we are proposing making a model simpler by removing interactions that the data do not support, the dangers of overfitting described by, for example, MacCallum, Roznowski, and Necowitz (1992) are considerably reduced or eliminated. That is, the approach we describe here emphasizes parsimony over maximizing model fit, so it is somewhat more conservative relative to one which adds terms to a model because doing so improves fit. But because a null hypothesis can never be proven true, the possibility of model misspecification cannot be ruled out after such model pruning. The data analyst's own philosophy about how to balance parsimony with the dangers of misspecification will have to govern this stage of the modeling, along with knowledge of the substantive literature and theory guiding the investigation.

Once one has settled on a set of linear models that include moderated effects that are consistent with the data, those models can be pieced together and the parameters of the models estimated simultaneously in a structural equation model. If interest were solely on the parameter estimates and tests of significance for individual paths, then this would be unnecessary. Gener-

ally, little new is learned by estimating the paths simultaneously rather than in individual analyses of each consequent variable. However, the estimation and testing of conditional direct and indirect effects generally involves combinations or *functions* of parameters, as will be discussed in the next section, as well as the ability to conduct inferential tests on specific outputs from those functions. This requires an SEM program that can combine parameter estimates from the linear models and provide the information needed to compute conditional indirect effects.

Figure 6.5 depicts the conditional process model in the form of a formal path diagram. Each linear model is represented with unidirectional paths leading to the consequent from each of the predictor variables. We also explicitly include the covariances between variables that never serve the role of consequent (i.e., exogenous variables). In many SEM programs (such as Mplus), these covariances are estimated by default, but some programs (such as AMOS) require the user to explicitly estimate them by drawing them in the diagram or including them in the model code. It is important to become familiar with the chosen program's defaults before proceeding further.

We use Mplus to estimate the parameters of this model because the program has some powerful features for combining parameter estimates together into new estimates, something we will take advantage of when esti-

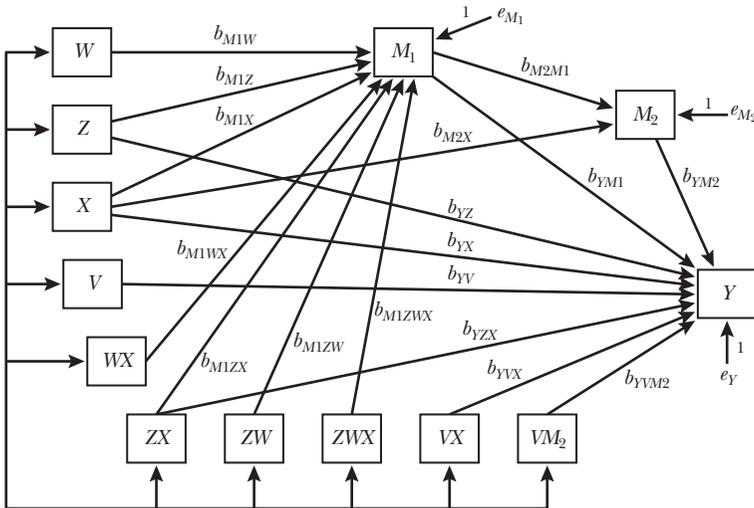


Figure 6.5 The statistical model for the Heart Health example in path diagram form. Not pictured, and also optional in full or in part (see the text), are 16 covariances involving model error covariances: $e_{M1} \leftrightarrow V$, $e_{M1} \leftrightarrow VX$, $e_{M1} \leftrightarrow VM_2$, $e_{M2} \leftrightarrow W$, $e_{M2} \leftrightarrow Z$, $e_{M2} \leftrightarrow V$, $e_{M2} \leftrightarrow WX$, $e_{M2} \leftrightarrow ZX$, $e_{M2} \leftrightarrow ZW$, $e_{M2} \leftrightarrow ZWX$, $e_{M2} \leftrightarrow VX$, $e_{M2} \leftrightarrow VM_2$, $e_Y \leftrightarrow W$, $e_Y \leftrightarrow WX$, $e_Y \leftrightarrow ZW$, $e_Y \leftrightarrow ZWX$.

mating conditional indirect effects and conducting inferential tests on their values given the data. The Mplus code we used can be found in Appendix A. A few comments on this code are in order. First, in the DEFINE section at the top, notice that we have chosen to mean center all variables that are involved in product terms, and we did so prior to computing products. There is some debate in the OLS regression literature as to whether such mean centering is necessary or desirable when estimating models with interactions (e.g., Echambi & Hess, 2007; Hayes, 2013; Kromrey & Foster-Johnson, 1998; Shieh, 2011). Our position is that in OLS regression the decision to center or not is one of personal choice and preference. But in structural equation modeling we recommend mean centering, as this can decrease the likelihood of convergence problems. Centering can either increase or decrease fit, depending on constraints in the model involving product terms. If the variables have means near zero to start with, such centering will generally have no effect. In this case, we found that the model diagrammed in Figure 6.5 (*without* the covariances noted in the figure caption) fit quite abysmally without centering [CFI = 0.254, RMSEA = 0.500, $\chi^2(16) = 3214.158$, $p < 0.001$] but exceedingly well when we centered the variables involved in products [CFI > 0.999, RMSEA < .001, $\chi^2(16) = 10.392$, $p = 0.845$] even though the parameter estimates and standard errors for the product terms were identical.

The linear models are spelled out in the MODEL section of the code, with consequent variables regressed ON predictors. Symbols in parentheses are labels attached to parameter estimates that are used later in the MODEL CONSTRAINT section for building estimates of conditional direct and indirect effects, to be discussed in the next section. The MODEL section in this code also includes covariances between consequent variable residuals and variables in the model that do not send a path to the corresponding consequent (there are 16 of these covariances: $e_{M1} \leftrightarrow V$, $e_{M1} \leftrightarrow VX$, $e_{M1} \leftrightarrow VM_2$, $e_{M2} \leftrightarrow W$, $e_{M2} \leftrightarrow Z$, $e_{M2} \leftrightarrow V$, $e_{M2} \leftrightarrow WX$, $e_{M2} \leftrightarrow ZX$, $e_{M2} \leftrightarrow ZW$, $e_{M2} \leftrightarrow ZWX$, $e_{M2} \leftrightarrow VX$, $e_{M2} \leftrightarrow VM_2$, $e_Y \leftrightarrow W$, $e_Y \leftrightarrow WX$, $e_Y \leftrightarrow ZW$, $e_Y \leftrightarrow ZWX$). Mplus will fix these at zero by default, so if they are desired, they must be explicitly stated in the model code. Freely estimating these yields a perfectly fitting model because the model is just identified. The parameter estimates for all paths can be found in Table 6.3.

The inclusion of these additional covariances is a matter of personal choice and modeling philosophy. A failure to include some or all of them can result in a misspecified model that may fit poorly. But a model can fit poorly for a number of reasons. In a model with observed variables only (i.e., no latent variables), poor fit can be the result of paths or covariances that are fixed to zero, or, for parsimony-adjusted measures of fit, the inclusion of paths that are unnecessary, meaning that they can be fixed to zero. Fixing a covariance between two variables to zero implies that the two variables are unassociated after accounting for the other effects in the

TABLE 6.3. Estimates of Structural Coefficients from Maximum Likelihood Estimation of the Model in Figure 6.5

	Consequent					
	M_1		M_2		Y	
X	b_{M1X}	5.735 (0.732)	b_{M2X}	0.038 (0.108)	b_{YX}	2.086 (0.333)
W	b_{M1W}	1.236 (0.370)				
Z	b_{M1Z}	-0.828 (0.732)			b_{YZ}	0.320 (0.321)
V					b_{YV}	0.886 (0.044)
M_1			b_{M2M1}	0.069 (0.005)	b_{YM1}	-0.006 (0.017)
M_1					b_{YM2}	2.865 (0.109)
WX	b_{M1WX}	1.966 (0.740)				
ZX	b_{M1ZX}	-1.919 (1.464)			b_{YZX}	1.609 (0.641)
ZW	b_{M1ZW}	0.151 (0.740)				
ZWX	b_{M1ZWX}	3.150 (1.481)				
VX					b_{YVX}	0.180 (0.088)
VM_2					b_{YVM2}	0.104 (0.026)

model. It might be that this assumption is false, either because the variables share an unmodeled common cause, or one variable affects the other. One could, based on an examination of the size and significance of these covariances when they are freely estimated, modify parts of the model by including causal paths that are not a part of the original model. Alternatively, one could be agnostic about causal influence by modeling associations unaccounted for by the causal model through the estimation of covariance terms. Inclusion of a causal path merely because it improves model fit moves one into dangerous territory when it comes to the problem of capitalizing on chance that MacCallum et al. (1992) warned about. On the other hand, one should recognize that by excluding paths from a model of a given consequent variable (by fixing the path to zero) and freely estimating the covariance instead, there is the potential of introducing bias in the estimation of all other paths to that consequent variable. Some personal judgment is sometimes necessary when making the choice. The danger of

overfitting can be counteracted by cross-validating the results one obtains on a new sample, or withholding half of the data for initial model fitting and then applying the empirically modified model to the other half for the sake of confirmation of the modified model.

In this example, and in the Mplus code in Appendix A, we do include all covariances listed above and in the caption in Figure 6.5. This model is saturated and so fits perfectly. However, the addition of these covariances did not significantly improve the model, $\chi^2(16) = 10.398$, and the estimates of model parameters were very similar compared to when these covariances were fixed to zero. But this should not be considered a general phenomenon. Model fit as well as parameter estimates can be different, even dramatically so, depending on the choice.

DERIVATION AND INFERENCES ABOUT (CONDITIONAL) DIRECT AND INDIRECT EFFECTS

With the parameters of the statistical model estimated, direct and indirect effects can be calculated, inferential tests about their size conducted, and their substantive meaning interpreted. In this section, we describe how to derive the expressions for the direct and indirect effects, estimate them for specific values of the moderators, and conduct inferential tests on their values.

Deriving Direct and Indirect Effects as Functions

The first step is to derive the mathematical expressions for the indirect and direct effects. In models with no moderators of any of the paths, this step is not necessary because the direct and indirect effects are constants (i.e., not functions of moderators) that are frequently provided in output from SEM programs. But when one or more of the direct or indirect effects is moderated, these effects become conditional and so are functions of parameters and moderators. SEM programs do not know what those functions are, so the user must specify what they are in order to obtain estimates of the conditional direct and/or indirect effects and inferential tests.

To facilitate the derivation process, it is helpful to label the paths from X to Y (directly or through an intermediary variable) in the diagram of the conceptual model with their corresponding parameters from the statistical model so that the direct and indirect effects can be built easily using the conceptual diagram. First, for those paths in the conceptual model that are unmoderated, label the paths with the corresponding parameter from the statistical model. In this example, there are three such unmoderated paths that are involved in the transmission X 's effect to Y in the conceptual mod-

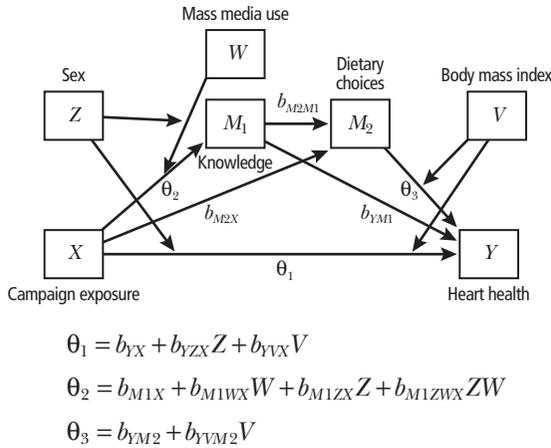


Figure 6.6 The conceptual model with path labels for derivation of (conditional) direct and indirect effects.

el. The first is from X to M_2 , which we labeled b_{M_2X} in the statistical model. The second is from M_1 to Y , which is b_{YM_1} . The third is from M_1 to M_2 , or $b_{M_2M_1}$ in the statistical model. These labels are attached to the corresponding paths in Figure 6.6. Second, for each of the moderated paths (there are three in this example), there is no single parameter in the statistical model that defines it. Rather, these paths are functions of at least two parameters and one or more moderators, as discussed in the “Direct, Indirect, and Conditional Effects” section above. So pick any arbitrary symbol for the moderated paths and attach those labels to those paths in the conceptual diagram. In Figure 6.6, we use θ_1 , θ_2 , and θ_3 for the paths from X to Y , X to M_1 , and M_2 to Y , respectively.

The next step is to define θ_1 , θ_2 , and θ_3 in terms of the parameters from the statistical model. In general terms, our goal is to derive the mathematical expression for an antecedent variable’s path to a given consequent. This is done by extracting and summing the terms from the linear model of the consequent that involve the antecedent variable and then factoring the antecedent out of the sum. This requires a little algebraic manipulation of the linear model for the consequent to which the moderated path points.

Starting with θ_1 , which is the moderated path from X to Y , we know from earlier (Equation 6.19) that the linear model for Y is

$$Y = i_Y + b_{YX}X + b_{YZ}Z + b_{YM_1}M_1 + b_{YM_2}M_2 + b_{YV}V + b_{YZX}ZX + b_{YVX}VX + b_{YVM_2}VM_2 + e_Y \tag{6.20}$$

To derive θ_1 , first extract all terms involving the antecedent (X) and sum them, as such

$$b_{YX}X + b_{YZX}ZX + b_{YVX}VX \quad (6.21)$$

and then, using the distributive law of multiplication, factor the antecedent out of the sum:

$$(b_{YX} + b_{YZX}Z + b_{YVX}V)X. \quad (6.22)$$

The expression for the path of interest, θ_1 is the factor that doesn't contain the antecedent variable. So in this case,

$$\theta_1 = b_{YX} + b_{YZX}Z + b_{YVX}V. \quad (6.23)$$

The same procedure is followed for deriving θ_2 and θ_3 . θ_2 is the moderated path from X to M_1 . Extracting and summing the terms involving X from the linear model for M_1 (Equation 6.17) yields

$$b_{M_1X}X + b_{M_1WX}WX + b_{M_1ZX}ZX + b_{M_1ZWX}ZWX \quad (6.24)$$

which factors to

$$(b_{M_1X} + b_{M_1WX}W + b_{M_1ZX}Z + b_{M_1ZWX}ZW)X \quad (6.25)$$

yielding

$$\theta_2 = b_{M_1X} + b_{M_1WX}W + b_{M_1ZX}Z + b_{M_1ZWX}ZW. \quad (6.26)$$

For θ_3 , M_2 is the antecedent and Y is the consequent. Extracting and summing the terms involving M_2 from the linear model for Y (Equation 6.19) and then factoring out M_2 leaves

$$\theta_3 = b_{YM_2} + b_{YVM_2}V. \quad (6.27)$$

Having derived expressions for all of the paths in the conceptual model, we can now define the direct and indirect effects. The direct effect is simple, for it is merely the path from X to Y without going through any intermediary variables. We already derived this. It is just θ_1 . Arbitrarily labeling the direct effect as δ ,

$$\delta = \theta_1 = b_{YX} + b_{YZX}Z + b_{YVX}V. \quad (6.28)$$

For the indirect effect(s), we trace all possible routes from X to Y going through at least one intermediary variable, multiplying products of paths as we go. As discussed earlier, there are three *specific* indirect effects. The first one goes from X to M_1 to Y . Arbitrarily using γ to denote an indirect effect,

$$\gamma_{M_1} = \theta_2 b_{Y M_1} = (b_{M_1 X} + b_{M_1 W X} W + b_{M_1 Z X} Z + b_{M_1 Z W X} ZW) b_{Y M_1}. \quad (6.29)$$

Observe that γ_{M_1} is a function of both W and Z . Thus, it is a conditional indirect effect. The second specific indirect effect goes from X to M_2 to Y , so

$$\gamma_{M_2} = b_{M_2 X} \theta_3 = b_{M_2 X} (b_{Y M_2} + b_{Y V M_2} V). \quad (6.30)$$

This conditional indirect effect is a function of only V . The last indirect effect goes from X through both M_1 and M_2 before ending at Y . Multiplying all three paths yields

$$\gamma_{M_1 M_2} = \theta_2 b_{M_2 M_1} \theta_3 = (b_{M_1 X} + b_{M_1 W X} W + b_{M_1 Z X} Z + b_{M_1 Z W X} ZW) b_{M_2 M_1} (b_{Y M_2} + b_{Y V M_2} V). \quad (6.31)$$

This conditional indirect effect is a function of all three moderators, W , Z , and V .

Quantification and Visualization of Conditional Direct and Indirect Effects

Equations 6.28 through 6.31 express the direct and indirect effects in terms of parameters of the model. But we can be more specific than these symbolic representations because the estimation of the model has yielded empirically-derived estimates of the corresponding parameters. Substituting the parameter estimates into their appropriate places yields

$$\hat{\delta} = 2.086 + 1.609Z + 0.180V \quad (6.32)$$

$$\hat{\gamma}_{M_1} = (5.735 + 1.966W - 1.919Z + 3.150ZW) (-0.006) \quad (6.33)$$

$$\hat{\gamma}_{M_2} = 0.038(2.865 + 0.104V) \quad (6.34)$$

$$\hat{\gamma}_{M_1 M_2} = (5.735 + 1.966W - 1.919Z + 3.150ZW) (0.069) (2.865 + 0.104V). \quad (6.35)$$

With these functions for the direct and indirect effects expressed now in terms of moderators and parameter estimates, we can finally answer such

questions as “What are the direct and indirect effects of the campaign on heart health?” But because the direct and indirect effects are moderated, the answer is “It depends.” In order to answer this question, it has to be phrased more specifically, such as, “What are the direct and indirect effects of the campaign on heart health among men of moderate body mass but who use the mass media relatively little?” To answer this question, we can employ a procedure similar to the computation of simple slopes or conditional effects in moderated multiple regression. We simply select values of the moderators corresponding to those values on which we want to condition the estimates, and substitute them into Equations 6.32 through 6.35. For instance, if we arbitrarily define a person who uses the mass media relatively little as someone one standard deviation below the sample mean, and someone with moderate body mass as a person with average BMI, then the indirect and direct effects of the campaign for a male ($Z = 0.505$ after mean centering, as centering was done prior to computation of products) of average body mass ($V = 0$ after mean centering) and who uses the mass media relatively little ($W = -0.992$ after mean centering) are, according to Equations 6.32 through 6.35,

$$\hat{\delta} = 2.086 + 1.609(0.505) + 0.180(0) = 2.899$$

$$\hat{\gamma}_{M1} = [5.735 + 1.966(-0.992) - 1.919(0.505) + 3.150(0.505)(-0.992)] \\ (-0.006) = -0.007$$

$$\hat{\gamma}_{M2} = 0.038 [2.865 + 0.104(0)] = 0.109$$

$$\hat{\gamma}_{M1M2} = [5.735 + 1.966(-0.992) - 1.919(0.505) + 3.150(0.505)(-0.992)] \\ (0.069) [2.865 + 0.104(0)] = 0.245$$

Because X is coded with a difference of 1 between those who did (-0.494) and did not (0.506) get exposure to the campaign, these direct and indirect effects can be interpreted as the average difference in heart health resulting from these effects. So among men of average body mass who use the mass media relatively little, those who were exposed to the campaign are $\hat{\delta} = 2.899$ healthy heart units higher than those not exposed to the campaign directly as a result of the exposure. They are $\hat{\gamma}_{M1M2} = 0.245$ units higher as a result of the effect on the campaign on knowledge which in turn affected dietary choices which then affected heart health. Furthermore, they are $\hat{\gamma}_{M2} = 0.109$ units higher as a result of the effect of the campaign on dietary choices independent of knowledge, which in turn affected heart health, but $|\hat{\gamma}_{M1}| = |-0.007| = 0.007$ units lower as a result of the effect of the

campaign on knowledge, which in turn affected heart health independent of dietary choices.

In order to better grasp how the indirect and direct effects are contingent on the moderators, it is useful to complete these computations for various combinations of moderator values, simply by using the functions for the direct and indirect effects, substituting in the moderator values, and doing the computation. Table 6.4 displays the direct and indirect effects for people defined by combinations of sex, body mass, and use of mass media. For these computations, we use the convention of defining “relatively low,” “moderate,” and “relatively high” on continuous variables as one standard deviation below the sample mean, the sample mean, and one standard deviation above the mean, respectively. These are admittedly arbitrary, and other values could be used.

Another useful means of displaying the moderated nature of the direct and indirect effects is through a visual representation, such as a line graph (see, e.g., Bauer, Preacher, & Gil, 2006; Edwards & Lambert, 2007; Preacher et al., 2007). A graph for the indirect effect of the campaign through M_1 and M_2 can be found in Figure 6.7 using separate plots for levels of media use and separate lines for males and females. A comparable plot for the direct effect accompanies the plot of the indirect effect. Represented this way, it is clear that both direct and indirect effects increase with increasing body mass. That is, the campaign seems more effective, both directly and indirectly, among those with higher body mass. It is also apparent that the campaign seems to affect men directly more than indirectly through knowledge and diet choices, but also that this indirect effect is larger among men who use the mass media more. Among women, however, the indirect effect does not seem to depend on extent of mass media use. Finally, the difference in this indirect effect of the campaign between men and women varies by media use, with the larger sex differences among those who use the mass media relatively less.

Our discussion above, as well as Figure 6.7, neglects the specific indirect effects through knowledge only ($\hat{\gamma}_{M1}$) and through dietary choices only ($\hat{\gamma}_{M2}$). Comparable visual depictions of these effects could be generated. However, as can be seen in Table 6.4, these effects are rather small and, it turns out, not statistically different from zero regardless of the choice of moderator value(s). It is to the topic of statistical inference that we turn next.

Statistical Inference: Probing the Direct and Indirect Effects at Levels of the Moderator(s)

In this section thus far, focus has been on the derivation of the functions for the conditional direct and indirect effects, and using those functions given the parameter estimates and specific values of the moderator to quan-

TABLE 6.4. Direct and Indirect Effects of the Heart Health Campaign for Various Combinations of Sex, Mass Media Use, and Body Mass. (Bootstrap Interval Estimates in Parentheses)

Sex (Z)	Media (W)	BMI (V)	Indirect Effects			Direct Effect ($\hat{\delta}$)
			M_1 ($\hat{\gamma}_{M1}$)	M_2 ($\hat{\gamma}_{M2}$)	M_1 and M_2 ($\hat{\gamma}_{M1M2}$)	
Female	Low	Low	-0.038 (-0.271, 0.177)	0.094 (-0.434, 0.606)	1.077* (0.561, 1.688)	1.133* (0.394, 1.906)
Female	Moderate	Low	-0.040 (-0.275, 0.192)	0.094 (-0.434, 0.606)	1.146* (0.779, 1.589)	1.200* (0.555, 1.860)
Female	High	Low	-0.043 (-0.299, 0.205)	0.094 (-0.434, 0.606)	1.215* (0.720, 1.809)	1.267* (0.544, 2.024)
Female	Low	Moderate	-0.038 (-0.271, 0.177)	0.109 (-0.509, 0.690)	1.242* (0.645, 1.933)	1.313* (0.459, 2.185)
Female	Moderate	Moderate	-0.040 (-0.275, 0.192)	0.109 (-0.509, 0.690)	1.322* (0.904, 1.814)	1.390* (0.652, 2.123)
Female	High	Moderate	-0.043 (-0.299, 0.205)	0.109 (-0.509, 0.690)	1.401* (0.810, 2.045)	1.486* (0.624, 2.317)
Female	Low	High	-0.038 (-0.271, 0.177)	0.123 (-0.580, 0.777)	1.407* (0.719, 2.187)	1.493* (0.521, 2.485)
Female	Moderate	High	-0.040 (-0.275, 0.192)	0.123 (-0.580, 0.777)	1.497* (1.015, 2.068)	1.581* (0.748, 2.423)
Female	High	High	-0.043 (-0.299, 0.205)	0.123 (-0.580, 0.777)	1.588* (0.937, 2.327)	1.669* (0.715, 2.635)
Male	Low	Low	-0.007 (-0.122, 0.033)	0.094 (-0.434, 0.606)	0.212 (-0.263, 0.706)*	0.299 (-0.404, 0.986)*
Male	Moderate	Low	-0.029 (-0.203, 0.135)	0.094 (-0.434, 0.606)	0.817* (0.481, 1.223)	0.883* (0.283, 1.524)

(continued)

TABLE 6.4. Direct and Indirect Effects of the Heart Health Campaign for Various Combinations of Sex, Mass Media Use, and Body Mass. (Bootstrap Interval Estimates in Parentheses) (continued)

Sex (Z)	Media (W)	BMI (V)	Indirect Effects			Direct Effect ($\hat{\delta}$)	
			M ₁ ($\hat{\gamma}_{M1}$)	M ₂ ($\hat{\gamma}_{M2}$)	M ₁ and M ₂ ($\hat{\gamma}_{M1M2}$)		Total
Male	High	Low	-0.050 (-0.349, 0.233)	0.094 (-0.434, 0.606)	1.422* (0.936, 1.427)	1.466* (0.762, 2.261)	2.239*
Male	Low	Moderate	-0.007 (-0.122, 0.033)	0.109 (-0.509, 0.690)	0.245* (-0.307, 0.806)	0.346* (-0.468, 1.133)	2.899*
Male	Moderate	Moderate	-0.029 (-0.203, 0.135)	0.109 (-0.509, 0.690)	0.942* (0.559, 1.389)	1.022* (0.326, 1.731)	2.899*
Male	High	Moderate	-0.050 (-0.349, 0.233)	0.109 (-0.509, 0.690)	1.640* (1.072, 2.312)	1.699* (0.884, 2.574)	2.899*
Male	Low	High	-0.007 (-0.122, 0.033)	0.123 (-0.580, 0.777)	0.277* (-0.347, 0.914)	0.393* (-0.528, 1.283)	3.558*
Male	Moderate	High	-0.029 (-0.203, 0.135)	0.123 (-0.580, 0.777)	1.067* (0.625, 1.564)	1.162* (0.376, 1.957)	3.558*
Male	High	High	-0.050 (-0.349, 0.233)	0.123 (-0.580, 0.777)	1.858* (1.205, 2.608)	1.931* (1.007, 2.931)	3.558*

* $p < 0.05$, assuming normality of the sampling distribution of the indirect effect. Bootstrap confidence intervals are bias corrected and based on 10,000 bootstrap samples.
Note: "Low," "moderate," and "high" media use and body mass were defined as one standard deviation below the sample mean, the mean, and one standard deviation above the mean, respectively. After mean centering, these correspond to -0.992, 0, and 0.992 for media use, and -3.665, 0, and 3.665 for body mass. Mean centered values of sex were used for males ($Z = 0.505$) and females ($Z = -0.495$). The total indirect effect is the sum of the three specific indirect effects.

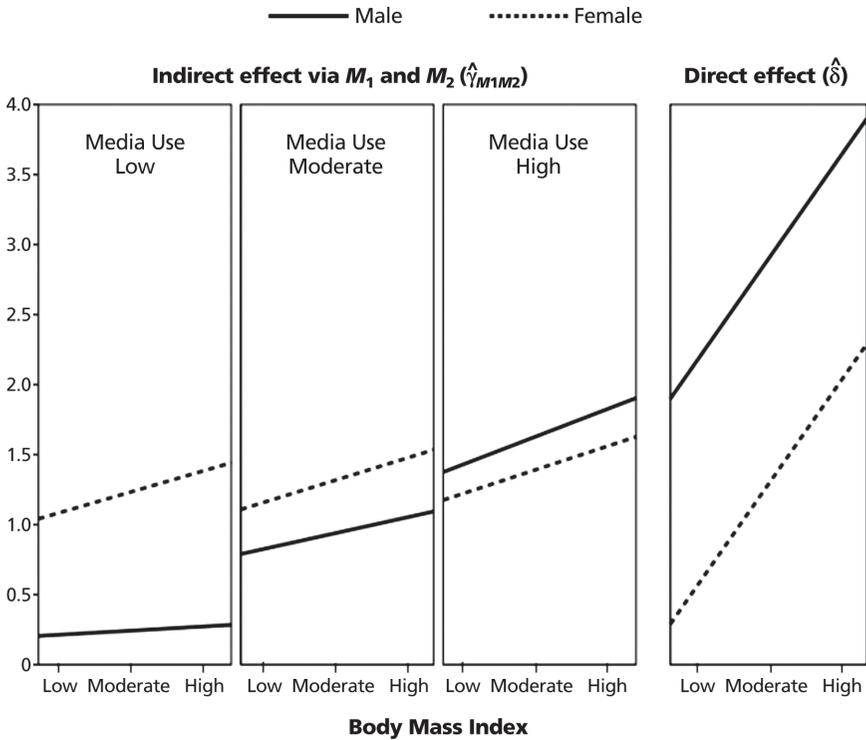


Figure 6.7 A visual representation of the conditional direct effect of the campaign and the conditional indirect effect through both knowledge and diet choices.

tify these effects. But our discussion has been purely *descriptive*. Although an estimate of, say, $\gamma_{M_1M_2}$ for a given combination of moderator values may be descriptively different from zero, that does not mean that one can claim its population value is. There are inherent limitations of thinking dichotomously as “zero” or “not zero”, but interpretation and discussion is often made simpler by focusing on those effects that are demonstrably different from zero while ignoring those that are not. Inferences for conditional direct and indirect effects can be conducted in a manner analogous to inferences about conditional effects or “simple slopes” in multiple regression with interactions. There are two approaches we will focus on, but only one that we can wholeheartedly endorse.

The first approach bases the inference on a ratio of the estimate of the effect for a given combination of moderator values to the standard error of that estimate. For example, to test the hypothesis that the population indirect effect of the campaign through knowledge and dietary choices ($\gamma_{M_1M_2}$) for females ($Z = -0.495$) relatively high in body mass ($V = 3.665$) and who

use the mass media relatively little ($W = -0.995$), first derive the indirect effect using the parameter estimates from the full model (Equation 6.35):

$$\hat{\gamma}_{M1M2} = (5.735 + 1.966W - 1.919Z + 3.150ZW) (0.069) \\ (2.865 + 0.104V) = 1.322$$

Next, divide this ratio by an estimate of the standard error (obtained via the delta method and provided automatically by most SEM software; Sobel, 1982). If you assume the sampling distribution of the conditional indirect effect is normal, a p -value to test the null hypothesis that the population conditional indirect effect is zero for that combination of moderator values can be obtained from the standard normal distribution. In this case

$$Z = \frac{1.322}{0.231} = 5.723, p < .001.$$

Alternatively, a confidence interval can be constructed as the point estimate plus or minus about two standard errors, or (0.870, 1.776). Either way, we can claim that this conditional indirect effect is statistically different from zero. This process can be repeated for any or all direct and indirect effects of interest for various combinations of the moderator values. Doing so for the estimates of all effects reported in Table 6.4 reveals that the conditional indirect effect of the campaign through either knowledge (γ_{M1}) alone or dietary choices (γ_{M2}) alone is not different from zero for any of these combinations of moderator values. However, the conditional indirect effect through both knowledge and dietary choices (γ_{M1M2}) is positive and statistically different from zero except for males who use the mass media relatively little. Finally, the direct effect (δ) of the campaign is positive and statistically different from zero for all but women relatively low in body mass.

These computations can be automated in some SEM programs. We like to use Mplus in part because it makes this process simple. The “MODEL CONSTRAINT” section of the Mplus code in Appendix A constructs estimates of conditional indirect effects for various combinations of sex, mass media use, and body mass by implementing Equations 6.32 through 6.35 and generating the resulting estimates in the output.

We have not provided computational formulas for conditional direct and indirect effects here for four reasons. First, the formulas for conditional *direct* effects can be found in sources on estimating and probing interactions in linear regression, and the same formulas apply here (see e.g., Aiken & West, 1991; Bauer & Curran, 2005; Hayes & Matthes, 2009). Second, the derivation and computation of standard errors for conditional direct and indirect effects can be exceedingly complex, involving linear combinations of parameters, moderator values, and covariances between

parameter estimates. Preacher et al. (2007) provide formulas for the standard errors for five relatively simple conditional indirect effects, and even those are more complicated than anyone would want to try to implement by hand. Third, a good SEM program that can estimate functions of parameters will also provide standard errors for those functions, and often Z and p -values as well. Mplus, for example, generates standard errors using the delta method in the output that results from the MODEL CONSTRAINT section of the code.

But most important, we choose not to dwell on the computation of standard errors for conditional indirect effects because we do not recommend the use of this approach to inference. The problem with this approach is similar to the problem with the Sobel test when assessing indirect effects in mediation models without moderation. This test assumes the sampling distribution of the conditional indirect effect is normal. Conditional indirect effects are products of parameters that are themselves roughly normally distributed. But the sampling distribution of the product of random normal variables is not itself normal. Rather, the sampling distributions of products tend to be skewed and heavy tailed, meaning that p -values and confidence intervals that are based on the assumption of normality and symmetry can be inaccurate. Thus, we recommend bootstrap-derived confidence intervals as alternative approach to inference about conditional indirect effects that respects the nonnormality and asymmetry of the sampling distribution of the conditional indirect effect. Bootstrap confidence intervals for indirect effects have already been widely studied and recommended over the Sobel test in the context of the kinds of mediation models depicted in Figure 6.2 (see e.g., Hayes, 2009; Preacher & Hayes, 2004, 2008; Shrout & Bolger, 2002), as well as in conditional process models more rudimentary than those discussed here (Edwards & Lambert, 2007; Preacher et al., 2007). Furthermore, the evidence suggests that this approach is superior to the normal theory approach described above as well as many other tests for mediation that have been proposed in the mediation literature (Briggs, 2006; MacKinnon, Lockwood, & Williams, 2004; Williams & MacKinnon, 2008). There is no reason to believe these findings would not apply just as much to testing hypotheses about conditional indirect effects.

To construct a bootstrap confidence interval for a conditional indirect effect, the parameters of the model must be repeatedly estimated, each time after taking a random sample with replacement from the data equal in size to the original sample. In each resampling of the data, after estimation of the model parameters, the indirect effects are estimated as discussed above. Repeated many times (at least 1,000 times, but more is better; we recommend 5,000 to 10,000) this procedure empirically generates an approximation of the sampling distribution of the effect of interest. Interval estimates of the indirect (conditional or unconditional) effects are then constructed

by finding the estimates in the distribution over repeated resampling that define the upper and lower 2.5th percentiles (for a 95% confidence interval) in the bootstrap sampling distribution of the effect. Inferences from such *percentile-based* bootstrap confidence intervals are made by ascertaining whether the confidence interval contains zero. If a 95% confidence interval does not contain zero, then the conclusion is that one can say with 95% confidence that the population conditional indirect effect is different from zero. However, if the confidence interval straddles zero, then one cannot claim that the population conditional indirect effect is different from zero, at least not with 95% confidence. Although not literally a hypothesis test, a failure for the confidence interval to contain zero can be interpreted much like one would interpret the outcome of a hypothesis test. One can reject the null hypothesis that the population conditional indirect effect is zero in favor of the alternative that it is different from zero.

Because the end points of the confidence interval are derived based on percentiles of an empirically generated distribution, there is no expectation or requirement that the endpoints are equidistant from the point estimate, as is true when constructing confidence intervals assuming the sampling distribution is normal. This reflects the reality that the sampling distribution of a product of normal variables is not typically symmetric. For this reason, interval estimates based on bootstrapping are often called “asymmetric confidence intervals.” The endpoints can also be adjusted in various ways to produce bias corrected or bias corrected and accelerated confidence intervals (see, e.g., Efron & Tibshirani, 1993), although there is some debate in the literature about just how much of a difference these adjustments make to the validity and power of this approach to inference.

Of course, because it is so computationally intensive and repetitive, bootstrapping would be entirely impractical if it were not for the availability of statistical programs that implement this method. Fortunately, some SEM programs have bootstrapping routines built in. For instance, Mplus can produce percentile and bias corrected confidence intervals for model parameters as well as indirect effects. When combined with its MODEL CONSTRAINT function, both point and bootstrap confidence intervals for conditional indirect effects are easily calculated.

In the Mplus code in Appendix A, bootstrapping is implemented in the ANALYSIS and OUTPUT section of the code. Removing the comment symbol (“!”) from “Bootstrap = 10000” tells Mplus to bootstrap all parameter estimates, including functions of parameters generated in the MODEL CONSTRAINT code, using 10,000 bootstrap samples. Removing the comment from “cinterval (bbootstrap)” tells Mplus to produce bias corrected bootstrap confidence intervals for all those parameters and functions of parameters. For instance, earlier we calculated the point estimate for the conditional indirect effect of the campaign through knowledge and dietary

choices for women high in body mass but low in mass media use as 1.322. Bootstrapping a confidence interval in Mplus yielded an interval estimate of (0.904, 1.814). So we can claim confidently that this indirect effect is positive and different from zero *without* making the assumption of normality of the sampling distribution. Bootstrap confidence intervals generated by Mplus using the code in Appendix A can be found in Table 6.4. As can be seen, these confidence intervals produce inferences that are largely identical to the normal theory tests, which will tend to occur with relatively large effects or in large samples such as this one.

EXTENSIONS TO LATENT VARIABLES

One of the primary benefits of SEM is that it grants researchers the ability to correct for the attenuating effects of measurement error by using latent variables with multiple observed indicators. Theoretically, latent variables are error-free representations of the constructs of interest. Because it is rarely possible to obtain error-free measurements of constructs, and because measurement error can drastically reduce the power for detecting an effect, it is usually to the researcher's advantage to substitute latent variables for measured variables when possible. Including latent variables in mediation models (without moderation) is straightforward, and indirect effects can be quantified and tested in exactly the same way as with measured variables (see e.g., Cheung & Lau, 2008; Coffman & MacCallum, 2005; Lau & Cheung, 2012).

If the ultimate outcome variable Y is latent, no fundamental changes are necessary to the procedures outlined in the preceding. The researcher merely needs to define a latent variable F_Y with multiple observed indicators, then substitute F_Y in the model for Y in all discussions and procedures describe above. However, conditional process models present special challenges when the researcher wishes to use latent variables as antecedent or moderator variables. Recall that conditional process models require the researcher to compute products of antecedent variables. We achieved this earlier in Mplus by using a DEFINE statement to create additional variables that were equal to products of variables already present in the data set. The primary challenge here is that it is not possible to literally compute the product of two latent variables, or the product of a latent variable and an observed variable, for inclusion in a model—by definition, latent variables are unobserved.

Fortunately, methodologists have developed ways to estimate interaction effects in the latent variable context (see chapter by Marsh, Wen, Hau, & Nagengast in this volume, as well as Schumacker & Marcoulides, 1998, for overviews of several approaches). A method implemented in Mplus is

termed the *latent moderated structural equations* (LMS) approach (Klein & Moosbrugger, 2000). Details behind exactly how this method proceeds are well beyond the scope of this chapter. Suffice it to say the LMS method is computationally intensive and requires numerical integration. On the other hand, LMS is easier to implement than competing methods because (1) it does not require the researcher to create products of measured indicators and (2) it is straightforward to invoke in Mplus. LMS results in unbiased, efficient estimates of interaction effects that are robust to departures from normality, and with unbiased standard errors.

We illustrate such a model with a brief example. In a recent study, Parker, Nouri, and Hayes (2011) explored the relation between perceptions of distributive justice (fairness of outcomes one receives at the company) and turnover intentions (likelihood of resigning) in a sample of 110 employees of accounting firms. Specifically, they hypothesized that perceptions of distributive justice (F_X ; 4 indicators) influences turnover intentions (F_Y ; three indicators) indirectly through promotion instrumentality (F_M ; three indicators), which is the perception that good performance is rewarded through promotion. They further hypothesized that this indirect effect would be moderated because job performance (F_W ; six indicators) was expected to influence the relation between promotion instrumentality and turnover intention (see Figure 6.8 for the conceptual model; this model corresponds to “Model 3” in Preacher et al., 2007 and the “second stage moderation model” in Edwards & Lambert, 2007). In Parker et al. (2011), all four constructs were measured by averaging responses to the items within each scale, but they could also be treated as latent variables with items as indicators. In the syntax provided in Appendix B, all four constructs are represented as latent variables with items as indicators (see Figure 6.8 for the statistical model in the form of a structural equation model). In Mplus, a latent product (here, $F_W F_M$) is created by use of the XWITH command, which also instructs Mplus to employ the LMS method. In this example, as in the published version using observed variables, the interaction between promotional instrumentality and job performance was statistically significant. Thus, it is sensible to estimate conditional indirect effects, which for this model would be $b_{F_M F_X} [b_{F_Y F_M} + b_{F_Y (F_W F_M)}] F_W$. When the moderator is latent, then conditional values may be chosen by using the latent moderator’s mean (typically zero by default) and standard deviation (the square root of its estimated or fixed variance; here that is 1), just as with observed moderators. The conditional indirect effects of distributive justice on turnover intentions via promotion instrumentality at one SD below the mean (–1), the mean (0), and one SD above the mean (1) of latent job performance moderator were, respectively, 0.023, –0.355, and –0.734.

To determine whether these effects differ significantly from zero, we constructed 95% confidence intervals using a Monte Carlo method de-

Should
be “ F_W ”

fects for each conditional value of job performance. The 2.5th and 97.5th percentiles of these distributions can serve as limits of 95% CIs. In the present case, these CIs were [0.431, -0.376] for low job performance, [-0.025, -0.766] for mean job performance, and [-0.175, -1.444] for high job performance.³ Because the first CI includes zero while the latter two exclude zero, we conclude that the latter two conditional indirect effects differ significantly from zero at $\alpha = .05$. This pattern of significance mirrors the results reported by Parker et al. (2011). This method of confidence interval construction has the nice property that the indirect effects are not assumed to be normally distributed (or even symmetric).

CONCLUSION

The goal of this chapter was to describe the basic principles of conditional process modeling in an SEM framework. This analytical approach combines properties of moderation and mediation analysis with the goal of estimating and understanding the various pathways, direct and indirect, moderated and unmoderated, through which a putative causal agent influences an outcome. Our example was chosen in order to illustrate the fundamental steps in building a complex model, estimating the parameters in SEM software, constructing the effects of interest, and testing hypotheses. But these steps generalize to the estimation of simple models already discussed in the moderated mediation literature, and they can also be applied to latent variable models or models that mix observed and latent variables, as illustrated in the previous section. Although our focus has been exclusively on models with continuous consequent variables, in theory this approach can be extended to any analysis that can be parameterized in terms of a generalized linear model.

APPENDIX A

The Mplus code below estimates the path model in Figure 6.5, produces conditional direct and indirect effects from the resulting parameter estimates for various combinations of the moderators, and conducts inferential tests of those conditional effects. Removing the exclamation point (!) from the ANALYSIS and OUTPUT sections of the code implements bootstrapping for inference about conditional effects.

```

DATA:
  FILE IS C:\cpm2.txt;
  FORMAT IS free;
VARIABLE:
  NAMES ARE x w z v m1 m2 y;
  USEVARIABLES ARE x w z v m1 m2 y wx zx wz wzx vm2 vx;
DEFINE:
  !mean center variables involved in products;
  v = v-27.272;
  w = w-3.954;
  m2 = m2-7.906;
  z = z-0.495;
  x = x-0.494;
  !create product variables;
  wx = w*x*w;
  zx = z*x;
  wz = w*z;
  wzx = w*z*x;
  vm2 = v*m2;
  vx = v*x;
ANALYSIS:
  !bootstrap = 10000;
MODEL:
  m1 ON x (m1x)
        w
        z
        wx (m1wx)
        zx (m1zx)
        wz
        wzx (m1wzx);
  m2 ON x (m2x)
        m1 (m2m1);
  y ON x (yx)
        v
        z
        m1 (ym1)
        m2 (ym2)
        vm2 (yvm2)
        zx (yzx)
        vx (yvx);

```

```

M1 WITH v vx vm2;
M2 WITH w z v wx zx wz wzx vx vm2;
Y WITH w wx wz wzx;
X;
MODEL CONSTRAINT:
  new (cdir1 cdir2 cdir3 cdir4 cdir5 cdir6
       cind1 cind2 cind3 cind4 cind5 cind6
       cind7 cind8 cind9 cind10 cind11 cind12
       cind13 cind14 cind15 cind16 cind17 cind18
       cind21 cind22 cind23 cind24 cind25 cind26
       cind31 cind32 cind33 ctind1 ctind2 ctind3
       ctind4 ctind5 ctind6 ctind7 ctind8 ctind9
       ctind10 ctind11 ctind12 ctind13 ctind14 ctind15
       ctind16 ctind17 ctind18
       zmale zfemale vlow vave vhigh wlow wave whigh);
  zmale = 0.505;
  zfemale = -0.495;
  vlow = -3.665;
  vave = 0;
  vhigh = +3.665;
  wlow = -0.992;
  wave = 0;
  whigh = 0.992;

  !cdir1-cdir6 are conditional direct effects of x on y at;
  !values of z and v;

  cdir1 = yx+yzx*zmale+yvx*vlow;
  cdir2 = yx+yzx*zfemale+yvx*vlow;
  cdir3 = yx+yzx*zmale+yvx*vave;
  cdir4 = yx+yzx*zfemale+yvx*vave;
  cdir5 = yx+yzx*zmale+yvx*vhigh;
  cdir6 = yx+yzx*zfemale+yvx*vhigh;

  !cind1-18 are conditional indirect effects via M1 and M2 at;
  !values of z, w, and v;

  cind1 = (m1x+m1wx*wlow+m1zx*zmale+m1wzx*wlow*zmale)*m2m1*
           (ym2+yvm2*vlow);
  cind2 = (m1x+m1wx*wlow+m1zx*zmale+m1wzx*wlow*zmale)*m2m1*
           (ym2+yvm2*vave);
  cind3 = (m1x+m1wx*wlow+m1zx*zmale+m1wzx*wlow*zmale)*m2m1*
           (ym2+yvm2*vhigh);
  cind4 = (m1x+m1wx*wave+m1zx*zmale+m1wzx*wave*zmale)*m2m1*
           (ym2+yvm2*vlow);
  cind5 = (m1x+m1wx*wave+m1zx*zmale+m1wzx*wave*zmale)*m2m1*
           (ym2+yvm2*vave);
  cind6 = (m1x+m1wx*wave+m1zx*zmale+m1wzx*wave*zmale)*m2m1*
           (ym2+yvm2*vhigh);
  cind7 = (m1x+m1wx*whigh+m1zx*zmale+m1wzx*whigh*zmale)*m2m1*
           (ym2+yvm2*vlow);
  cind8 = (m1x+m1wx*whigh+m1zx*zmale+m1wzx*whigh*zmale)*m2m1*
           (ym2+yvm2*vave);

```

```

cind9 = (m1x+m1wx*whigh+m1zx*zmale+m1wzx*whigh*zmale)*m2m1*
      (ym2+yvm2*vhigh);
cind10 = (m1x+m1wx*wlow+m1zx*zfemale+m1wzx*wlow*zfemale)*m2m1*
      (ym2+yvm2*vlow);
cind11 = (m1x+m1wx*wlow+m1zx*zfemale+m1wzx*wlow*zfemale)*m2m1*
      (ym2+yvm2*vave);
cind12 = (m1x+m1wx*wlow+m1zx*zfemale+m1wzx*wlow*zfemale)*m2m1*
      (ym2+yvm2*vhigh);
cind13 = (m1x+m1wx*wave+m1zx*zfemale+m1wzx*wave*zfemale)*m2m1*
      (ym2+yvm2*vlow);
cind14 = (m1x+m1wx*wave+m1zx*zfemale+m1wzx*wave*zfemale)*m2m1*
      (ym2+yvm2*vave);
cind15 = (m1x+m1wx*wave+m1zx*zfemale+m1wzx*wave*zfemale)*m2m1*
      (ym2+yvm2*vhigh);
cind16 = (m1x+m1wx*whigh+m1zx*zfemale+m1wzx*whigh*zfemale)*
      m2m1*(ym2+yvm2*vlow);
cind17 = (m1x+m1wx*whigh+m1zx*zfemale+m1wzx*whigh*zfemale)*
      m2m1*(ym2+yvm2*vave);
cind18 = (m1x+m1wx*whigh+m1zx*zfemale+m1wzx*whigh*zfemale)*
      m2m1*(ym2+yvm2*vhigh);

!cind21-28 are conditional indirect effects via M1 only at;
!values of z, w;

cind21 = (m1x+m1wx*wlow+m1zx*zmale+m1wzx*wlow*zmale)*ym1;
cind22 = (m1x+m1wx*wlow+m1zx*zfemale+m1wzx*wlow*zfemale)*ym1;
cind23 = (m1x+m1wx*wave+m1zx*zmale+m1wzx*wave*zmale)*ym1;
cind24 = (m1x+m1wx*wave+m1zx*zfemale+m1wzx*wave*zfemale)*ym1;
cind25 = (m1x+m1wx*whigh+m1zx*zmale+m1wzx*whigh*zmale)*ym1;
cind26 = (m1x+m1wx*whigh+m1zx*zfemale+m1wzx*whigh*zfemale)*ym1;

!cind31-33 are conditional indirect effects via M2 only at;
!values of v;

cind31 = m2x*(ym2+yvm2*vlow);
cind32 = m2x*(ym2+yvm2*vave);
cind33 = m2x*(ym2+yvm2*vhigh);

!ctind1-ctind18 are total indirect effects at values of z, w, ;
!and v;

ctind1 = cind1+cind21+cind31;
ctind2 = cind2+cind21+cind32;
ctind3 = cind3+cind21+cind33;
ctind4 = cind4+cind23+cind31;
ctind5 = cind5+cind23+cind32;
ctind6 = cind6+cind23+cind33;
ctind7 = cind7+cind25+cind31;
ctind8 = cind8+cind25+cind32;
ctind9 = cind9+cind25+cind33;
ctind10 = cind10+cind22+cind31;
ctind11 = cind11+cind22+cind32;

```

```

ctind12 = cind12+cind22+cind33;
ctind13 = cind13+cind24+cind31;
ctind14 = cind14+cind24+cind32;
ctind15 = cind15+cind24+cind33;
ctind16 = cind16+cind26+cind31;
ctind17 = cind17+cind26+cind32;
ctind18 = cind18+cind26+cind33;

```

OUTPUT:

```
!cinterval (bcbootstrap)
```

APPENDIX B

The Mplus code below estimates the statistical model in Figure 6.8, produces conditional direct and indirect effects from the resulting parameter estimates for various values of the moderator, and conducts inferential tests of those conditional effects.

DATA:

```
FILE IS data.dat;
```

VARIABLE:

```

NAMES ARE dj1 dj2 dj3 dj4 pi1 pi2 pi3
jp1 jp2 jp3 jp4 jp5 jp6 to1 to2 to3;
USEVARIABLES ARE dj1 dj2 dj3 dj4 pi1 pi2 pi3
jp1 jp2 jp3 jp4 jp5 jp6 to1 to2 to3;

```

ANALYSIS:

```

TYPE IS RANDOM;
ALGORITHM IS INTEGRATION;

```

MODEL:

```

dj BY dj1* dj2-dj4;
pi BY pi1* pi2-pi3;
to BY to1* to2-to3;
jp BY jp1* jp2-jp6;
dj@1 pi@1 to@1 jp@1;
pijp | pi XWITH jp;
pi ON dj (mx);
to ON pi (ym);
to ON dj jp;
to ON pijp (ywm);
pi WITH jp;

```

MODEL CONSTRAINT:

```

NEW (eff1 eff2 eff3);
eff1 = mx*(ym+ywm*(-1));
eff2 = mx*(ym+ywm*0);
eff3 = mx*(ym+ywm*1);

```

NOTES

1. We use the terms *path*, *coefficient*, and *regression coefficient* synonymously to refer to the weight for an antecedent variable in a model of a consequent variable. In all discussions here, these weights are in unstandardized rather than standardized form, although in some cases the mathematics applies equally to both forms.
2. The data we use here were fabricated for pedagogical purposes. None of the findings we report here should be interpreted as substantiated by actual empirical evidence.
3. R and SPSS code that accomplishes the Monte Carlo estimation are available from either author by request.

REFERENCES

- Aiken, L. S., & West, S. G. (1991). *Multiple regression: Testing and interpreting interactions*. Thousand Oaks, CA: Sage Publications.
- Antheunis, M. L., Valkenberg, P. M., & Peter, J. (2010). Getting acquainted through social network sites: Testing a model of online uncertainty reduction and social attraction. *Computers in Human Behavior, 26*, 100–109.
- Baron, R. M., & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. *Journal of Personality and Social Psychology, 51*, 1173–1182.
- Bauer, D. J., & Curran, P. J. (2005). Probing interactions in fixed and multilevel regression: Inferential and graphical techniques. *Multivariate Behavioral Research, 40*, 373–400.
- Bauer, D. J., Preacher, K. J., & Gil, K. M. (2006). Conceptualizing and testing random indirect effects and moderated mediation in multilevel models. New procedures and recommendations. *Psychological Methods, 11*, 142–163.
- Briggs, N. E. (2006). *Estimation of the standard error and confidence interval of the indirect effect in multiple mediator models*. Unpublished doctoral dissertation, The Ohio State University, Columbus, OH
- Cerin, E., & MacKinnon, D. P. (2009). A commentary on current practice in mediating variable analysis in behavioural nutrition and physical activity. *Public Health Nutrition, 12*, 1182–1188.
- Cheung, G. W., & Lau, R. S. (2008). Testing mediation and suppression effects of latent variables: Bootstrapping with structural equation models. *Organizational Research Methods, 11*, 296–325.
- Coffman, D. L., & MacCallum, R. C. (2005). Using parcels to convert path models into latent variable models. *Multivariate Behavioral Research, 40*, 235–259.
- Druley, J. A., & Townsend, A. L. (1998). Self-esteem as a mediator between spousal support and depressive symptoms: A comparison of healthy individuals and individuals coping with arthritis. *Health Psychology, 17*, 255–261.
- Echambi, R., & Hess, J. D. (2007). Mean-centering does not alleviate collinearity problems in moderated multiple regression models. *Marketing Research, 26*, 438–445.

- Edwards, J. R., & Lambert, L. S. (2007). Methods for integrating moderation and mediation: A general analytical framework using moderated path analysis. *Psychological Methods, 12*, 1–22.
- Efron, B., & Tibshirani, R. (1993). *An introduction to the bootstrap*. New York, NY: Chapman & Hall.
- Fairchild, A. J., & MacKinnon, D. P. (2009). A general model for testing mediation and moderation effects. *Prevention Science, 10*, 87–99.
- Hayes, A. F. (2009). Beyond Baron and Kenny: Statistical mediation analysis in the new millennium. *Communication Monographs, 76*, 408–420.
- Hayes, A. F. (2013). *Introduction to mediation, moderation, and conditional process analysis*. New York, NY: The Guilford Press.
- Hayes, A. F., & Matthes, J. (2009). Computational procedures for probing interactions in OLS and logistic regression: SPSS and SAS implementations. *Behavior Research Methods, 41*, 924–936.
- Jaccard, J., & Turrisi, R. (2003). *Interaction effects in multiple regression* (2nd ed.). Thousand Oaks, CA: Sage Publications.
- James, L. R., & Brett, J. M. (1984). Mediators, moderators, and tests for mediation. *Journal of Applied Psychology, 69*, 307–321.
- Judd, C. M., & Kenny, D. A. (1981). Process analysis: Estimating mediation in treatment evaluations. *Evaluation Review, 5*, 602–619.
- Klein, A., & Moosbrugger, H. (2000). Maximum likelihood estimation of latent interaction effects with the LMS method. *Psychometrika, 65*, 457–474.
- Kromrey, J. D., & Foster-Johnson, L. (1998). Mean centering in moderated multiple regression: Much ado about nothing. *Educational and Psychological Measurement, 58*, 42–67.
- Lau, R. S., & Cheung, G. W. (2012). Estimating and comparing specific mediation effects in complex latent variable models. *Organizational Research Methods, 15*, 3–16.
- MacCallum, R. C., Roznowski, M., & Necowitz, L. B. (1992). Model modification in covariance structure analysis: The problem of capitalizing on chance. *Psychological Bulletin, 111*, 490–504.
- MacKinnon, D. P., Lockwood, C. M., & Williams, J. (2004). Confidence limits for the indirect effect: Distribution of the product and resampling methods. *Multivariate Behavioral Research, 39*, 99–128.
- Mathieu, J. E., & Taylor, S. R. (2006). Clarifying conditions and decision points for mediational type inferences in Organizational Behavior. *Journal of Organizational Behavior, 27*, 1031–1056.
- Morgan-Lopez, A., & MacKinnon, D. P. (2006). Demonstration and evaluation of a method for assessing mediated moderation. *Behavior Research Methods, 38*, 77–89.
- Muller, D., Judd, C. M., & Yzerbyt, V. Y. (2005). When mediation is moderated and moderation is mediated. *Journal of Personality and Social Psychology, 89*, 852–863.
- Parade, S. H., Leerkes, E. M., & Blankson, A. (2010). Attachment to parents, social anxiety, and close relationships of female students over the transition to college. *Journal of Youth and Adolescence, 39*, 127–137.

- Parker, R., Nouri, H., & Hayes, A. F. (2011). Distributive justice, promotion instrumentality, and turnover intentions in public accounting firms. *Behavioral Research in Accounting*, *23*, 169–186
- Preacher, K. J., & Hayes, A. F. (2004). SPSS and SAS procedures for estimating indirect effects in simple mediation models. *Behavior Research Methods, Instruments, and Computers*, *36*, 717–731
- Preacher, K. J., & Hayes, A. F. (2008). Asymptotic and resampling methods for assessing and comparing indirect effects in multiple mediator models. *Behavior Research Methods*, *40*, 879–891.
- Preacher, K. J., Rucker, D. D., & Hayes, A. F. (2007). Assessing moderated mediation hypotheses: Theory, methods, and prescriptions. *Multivariate Behavioral Research*, *42*, 185–227.
- Rucker, D. D., Preacher, K. J., Tormala, Z. L., & Petty, R. E. (2011). Mediation analysis in social psychology: Current practices and new recommendations. *Social and Personality Psychology Compass*, *5/6*, 359–371.
- Schumacker, R. E., & Maroulides, G. A. (1998). *Interaction and nonlinear effects in structural equation modeling*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Shieh, G. (2011). Clarifying the role of mean centering in multicollinearity of interaction effects. *British Journal of Mathematical and Statistical Psychology*, *64*, 462–477.
- Shrout, P. E., & Bolger, N. (2002). Mediation in experimental and nonexperimental studies: New procedures and recommendations. *Psychological Methods*, *7*, 422–445.
- Sobel, M. E. (1982). Asymptotic confidence intervals for indirect effects in structural equation models. In S. Leinhardt (Ed.), *Sociological methodology 1982* (pp. 290–312). Washington, DC: American Sociological Association.
- Tepper, B. J., & Eisenbach, R. J., Kirby, S. L., & Potter, P. W. (1998). Test of a justice-based model of subordinates' resistance to downward influence attempts. *Group and Organization Management*, *23*, 144–160.
- Van Dijke, M., & de Cremer, D. (2010). Procedural fairness and endorsement of prototypical leaders: Leader benevolence or follower control? *Journal of Experimental Social Psychology*, *46*, 85–96.
- Williams, J., & MacKinnon, D. P. (2008). Resampling and distribution of the product methods for testing indirect effects in complex models. *Structural Equation Modeling: A Multidisciplinary Journal*, *15*, 23–51.