Online Appendices to Multiple Local Solutions and Geomin Rotation

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The online supplementary appendices contain R code for conducting Geomin rotation from multiple random starts and additional eight tables and five figures that supplement the paper: *Multiple Local Solutions and Geomin Rotation*.

Appendix A

R code

```
library(GPArotation)
library(plyr)
# The following three functions are separate R files.
source('Align.Matrix.R')
source('MultipleGeomin.R')
source('CompareSolutions.R')
data(Thurstone, package = "GPArotation") # for box26
res <- MultipleGeomin(unrotated = box26, epsilon = .01, K = 100, plot = T,
cex=.5)
res
res2 <- cfQ(box26, kappa = 1/nrow(box26), maxit = 10000) # obliqe CF-varimax
res3 <- oblimin(box26, maxit = 10000) # oblique oblimin
res4 <- quartimin(box26, maxit = 10000) # oblique quartimin</pre>
otherSolutions <- list(res2$loadings, res3$loadings, res4$loadings)
allSolutions <- c(res$loadings, otherSolutions) # list of all rotated
loadings
CompareSolutions(allSolutions, compare = 'First')
```

Appendix B

Tables

Table A1: The global solutions at different levels of ε for Model M_{3a} .

Table A2: The global solutions at different levels of ε for Model M_4 .

Table A3: The congruence coefficients between factors of solutions at different values of ε for all models.

Table A4: The congruence coefficients between factors of local solutions at $\varepsilon = .01$ for Model M_{3b} .

Table A5: All solutions at $\varepsilon = .001$ for Model M_4 .

Table A6: The numbers of solutions in simulated samples at different levels of ε for Model M_{3a} .

Table A7: The numbers of solutions in simulated sample at different levels of ε for Model M_4 .

Table A8: The percentages of simulated samples with congruence coefficients greater than 0.92 for all columns across different values of ε for all models.

	$\hat{\mathbf{\Lambda}}^{\mathrm{I}} \; (\varepsilon = .02)$			$\hat{\mathbf{\Lambda}}^{\mathrm{I}} \; (\varepsilon = .01)$			$\hat{\mathbf{\Lambda}}^{\mathrm{I}} \; (\varepsilon = .001)$		
	Verb	Arith	Spat	Verb	Arith	Spat	Verb	Arith	Spat
WrdMean	.929	034	015	.931	036	014	.925	043	.008
SntComp	.757	.151	022	.758	.150	023	.759	.147	016
OddWrds	.818	.018	.088	.818	.016	.088	.814	.009	.106
MxdArit	038	1.004	049	038	1.008	058	005	1.020	118
Remndrs	.044	.806	.059	.043	.809	.052	.069	.816	.007
MissNum	.141	.753	.061	.141	.756	.054	.164	.762	.014
Gloves	057	.166	.545	060	.165	.545	057	.158	.540
Boots	.047	.033	.725	.044	.032	.727	.041	.019	.734
Hatchts	.019	036	.890	.016	038	.892	.009	054	.905
Factor Correlations									
Verb	1.000			1.000			1.000		
Arith	.567	1.000		.571	1.000		.556	1.000	
Spat	.401	.438	1.000	.405	.448	1.000	.400	.504	1.000

Table A1

Factor loading and correlation matrices of the global solutions with different ε values for M_{3a} .

	$\hat{\mathbf{\Lambda}}^{\mathrm{I}} \ (\varepsilon = .02)$					$\hat{\mathbf{\Lambda}}^{\mathrm{I}} \ (\varepsilon = .01)$			$\hat{\mathbf{\Lambda}}^{\mathrm{I}} \ (\varepsilon = .001)$			
	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4
1	.662	.051	.039	.057	.664	.048	.043	.050	.679	.031	.038	.043
2	.514	.026	004	021	.515	.024	002	025	.523	.016	007	033
3	.423	.097	.067	.015	.423	.094	.070	.011	.436	.080	.069	.011
4	.633	.132	021	014	.635	.130	019	020	.644	.116	019	025
5	.062	.758	.116	071	.057	.755	.119	072	.067	.727	.149	032
6	.001	.808	043	.076	.004	.810	041	.073	.002	.780	002	.112
7	021	.847	.070	068	026	.846	.072	069	020	.817	.109	024
8	.156	.554	.166	012	.151	.550	.171	014	.167	.523	.190	.017
9	.011	.862	085	.061	.014	.864	084	.058	.009	.835	041	.097
10	221	.041	.879	.044	242	.022	.894	.043	176	006	.873	.088
11	.066	.061	.464	.238	.062	.051	.476	.233	.102	.023	.467	.257
12	.184	141	.735	029	.166	158	.747	032	.226	180	.717	010
13	.416	.050	.498	099	.402	.037	.507	103	.447	.015	.489	087
14	017	.121	007	.513	002	.123	.001	.505	.001	.098	.012	.514
15	.100	.014	021	.485	.114	.015	014	.477	.119	006	008	.478
16	.366	051	055	.483	.383	050	048	.473	.389	072	047	.465
17	143	.060	.153	.658	128	.058	.165	.649	113	.027	.172	.666
18	.263	130	.249	.422	.270	136	.260	.412	.298	161	.248	.415
19	.092	.087	.089	.361	.101	.086	.096	.354	.113	.063	.101	.363
20	.281	.326	025	.235	.289	.326	020	.229	.294	.301	005	.240
21	.386	.026	.351	.101	.382	.017	.359	.095	.416	006	.347	.106
22	.288	.313	021	.242	.296	.313	016	.236	.301	.288	001	.247
23	.394	.285	.106	.137	.397	.281	.112	.130	.413	.255	.120	.143
24	040	.282	.449	.217	045	.273	.461	.212	008	.240	.463	.248

Table A2

Factor loading matrices of the global solutions $\hat{\mathbf{\Lambda}}^{\mathrm{I}}$ with different ε values for M_4 .

M_{3a} : Holzinger Unpublished									
arepsilon	Verb	Arith	Spat						
.02 and $.01$	1	1	.9999						
.01 and .001	.9994	.9999	.9975						
.02 and .001	.9994	.9998	.9968						
M_{3b} : Box Dar	M_{3b} : Box Data								
arepsilon	Н	L	W						
.02 and $.01$	1	1	1						
.01 and .001	.9999	1	1						
.02 and .001	.9999	1	1						
M_4 : Holzinge	M_4 : Holzinger-Swineford 24-Test								
arepsilon	F1	F2	F3	F4					
.02 and $.01$.9995	.9998	.9999	.9999					
.01 and .001	.9971	.9987	.9983	.9964					
.02 and .001	.9980	.9979	.9979	.9967					

	h	1	W
$\hat{m{\Lambda}}^{\rm I}$ and $\hat{m{\Lambda}}^{\rm II}$.661	.658	.743
$\hat{m{\Lambda}}^{\rm I}$ and $\hat{m{\Lambda}}^{\rm III}$.635	.671	.652
$\hat{m{\Lambda}}^{\mathrm{I}}$ and $\hat{m{\Lambda}}^{\mathrm{IV}}$.705	.530	.672
$\hat{\boldsymbol{\Lambda}}^{\mathrm{II}}$ and $\hat{\boldsymbol{\Lambda}}^{\mathrm{III}}$.290	.689	.697
$\hat{\boldsymbol{\Lambda}}^{\mathrm{II}}$ and $\hat{\boldsymbol{\Lambda}}^{\mathrm{IV}}$.757	.940	.724
$\hat{\mathbf{\Lambda}}^{\mathrm{III}}$ and $\hat{\mathbf{\Lambda}}^{\mathrm{IV}}$.828	.826	.999

Table A4

Congruence coefficients among local solutions for M_{3b} at $\varepsilon = .01$.

$\hat{m{\Lambda}}^{ ext{I}}$					$\hat{m{\Lambda}}^{ ext{II}}$				$\hat{m{\Lambda}}^{ m III}$			
	F1	$F2^{\frac{1}{2}}$	<u>•</u> F3	F4	F1	F2	F3	F4	F1	F2	— F3	F4
1	.679	.031	.038	.043	.673	.030	.064	005	.520	.005	.022	.276
2	.523	.016	007	033	.515	.013	.009	063	.433	002	017	.132
3	.436	.080	.069	.011	.415	.079	.092	021	.357	.063	.058	.155
4	.644	.116	019	025	.638	.114	002	059	.528	.090	032	.186
5	.067	.727	.149	032	.006	.728	.169	030	.155	.700	.145	005
6	.002	.780	002	.112	.015	.785	006	.123	011	.742	004	.166
7	020	.817	.109	024	069	.819	.120	012	.079	.789	.107	018
8	.167	.523	.190	.017	.107	.524	.221	.000	.198	.500	.182	.079
9	.009	.835	041	.097	.029	.840	051	.115	001	.794	043	.154
10	176	006	.873	.088	410	005	.997	.005	015	.018	.861	018
11	.102	.023	.467	.257	.020	.028	.540	.186	.021	.017	.453	.317
12	.226	180	.717	010	.017	182	.831	099	.318	161	.701	.000
13	.447	.015	.489	087	.283	.011	.575	157	.506	.016	.472	.002
14	.001	.098	.012	.514	.108	.110	.015	.479	302	.068	.004	.644
15	.119	006	008	.478	.224	.005	004	.438	199	035	018	.636
16	.389	072	047	.465	.502	062	040	.411	.009	108	062	.710
17	113	.027	.172	.666	018	.043	.196	.609	457	.000	.161	.780
18	.298	161	.248	.415	.316	153	.297	.339	.021	180	.232	.591
19	.113	.063	.101	.363	.160	.072	.120	.322	107	.041	.091	.484
20	.294	.301	005	.240	.341	.307	.003	.213	.106	.266	015	.407
21	.416	006	.347	.106	.336	006	.412	.037	.335	017	.331	.242
22	.301	.288	001	.247	.349	.293	.007	.218	.108	.253	012	.417
23	.413	.255	.120	.143	.403	.258	.149	.102	.282	.226	.106	.314
24	008	.240	.463	.248	092	.246	.531	.191	046	.230	.452	.278
		^				^	**			^	3.73	
	Б.		IV				V	D .			VI Fra	D .
	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4
1	.449	F2 .015	F3 .038	.380	.359	F2 .009	F3	.623	.832	F2 .006	F3 301	.016
2	.449 .378	F2 .015 .005	F3 .038008	.380 .226	.359 .320	F2 .009 .001	F3 .023 002	.623 .418	.832 .625	F2 .006 012	F3301297	0.016 -0.022
2 3	.449 .378 .304	F2 .015 .005 .077	F3 .038008 .069	.380 .226 .211	.359 .320 .229	F2 .009 .001 .068	F3 .023 002 .064	.623 .418 .385	.832 .625 .542	F2 .006 012 .075	F3301297178	.016 022 015
2 3 4	.449 .378 .304 .452	F2 .015 .005 .077 .111	F3 .038008 .069023	.380 .226 .211 .285	.359 .320 .229 .384	F2 .009 .001 .068 .099	F3 .023 002 .064 011	.623 .418 .385 .522	.832 .625 .542 .767	F2 .006 012 .075 .079	F3301297178366	.016 022 015 .000
2 3 4 5	.449 .378 .304 .452	F2 .015 .005 .077 .111 .790	F3 .038008 .069023 .137	.380 .226 .211 .285 116	.359 .320 .229 .384 .008	F2 .009 .001 .068 .099 .739	F3 .023002 .064011 .184	.623 .418 .385 .522	.832 .625 .542 .767 .101	F2 .006 012 .075 .079 .754	F3301297178366 .007	.016 022 015 .000 011
2 3 4 5 6	.449 .378 .304 .452 .075 080	F2 .015 .005 .077 .111 .790 .839	F3 .038008 .069023 .137021	.380 .226 .211 .285 116 .032	.359 .320 .229 .384 .008 079	F2 .009 .001 .068 .099 .739	F3 .023002 .064011 .184 .011	.623 .418 .385 .522 .000	.832 .625 .542 .767 .101 .003	F2 .006 012 .075 .079 .754	F3301297178366 .007 .033	.016 022 015 .000 011 .172
2 3 4 5 6 7	.449 .378 .304 .452 .075 080	F2 .015 .005 .077 .111 .790 .839 .889	F3 .038008 .069023 .137021 .093	.380 .226 .211 .285 116 .032 156	.359 .320 .229 .384 .008 079 040	F2 .009 .001 .068 .099 .739 .789	F3 .023002 .064011 .184 .011 .149	.623 .418 .385 .522 .000 .059 076	.832 .625 .542 .767 .101 .003 014	F2 .006 012 .075 .079 .754 .771	F3301297178366 .007 .033 .028	.016 022 015 .000 011 .172 .023
2 3 4 5 6 7 8	.449 .378 .304 .452 .075 080 .002	F2 .015 .005 .077 .111 .790 .839 .889 .564	F3 .038008 .069023 .137021 .093 .185	.380 .226 .211 .285 116 .032 156 .007	.359 .320 .229 .384 .008 079 040	F2 .009 .001 .068 .099 .739 .789 .833 .526	F3 .023002 .064011 .184 .011 .149 .202	.623 .418 .385 .522 .000 .059 076	.832 .625 .542 .767 .101 .003 014	F2 .006 012 .075 .079 .754 .771 .838	F3301297178366 .007 .033 .028 .022	.016 022 015 .000 011 .172 .023 016
2 3 4 5 6 7 8	.449 .378 .304 .452 .075 080 .002 .128 075	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898	F3 .038008 .069023 .137021 .093 .185064	.380 .226 .211 .285 116 .032 156 .007	.359 .320 .229 .384 .008 079 040 .037 061	F2 .009 .001 .068 .099 .739 .789 .833 .526	F3 .023002 .064011 .184 .011 .149 .202021	.623 .418 .385 .522 .000 .059 076 .144	.832 .625 .542 .767 .101 .003 014 .241	F2 .006 012 .075 .079 .754 .771 .838 .557	F3301297178366 .007 .033 .028 .022007	.016 022 015 .000 011 .172 .023 016
2 3 4 5 6 7 8 9	.449 .378 .304 .452 .075 080 .002 .128 075 014	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008	F3 .038008 .069023 .137021 .093 .185064 .909	.380 .226 .211 .285 116 .032 156 .007 .020	.359 .320 .229 .384 .008 079 040 .037 061	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845	F3 .023002 .064011 .184 .011 .149 .202021 .814	.623 .418 .385 .522 .000 .059 076 .144 .045 008	.832 .625 .542 .767 .101 .003 014 .241 .000	F2 .006 012 .075 .079 .754 .771 .838 .557 .816	F3301297178366 .007 .033 .028 .022007 .702	.016022015 .000011 .172 .023016 .185334
2 3 4 5 6 7 8 9 10	.449 .378 .304 .452 .075 080 .002 .128 075 014	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008	F3 .038008 .069023 .137021 .093 .185064 .909 .485	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254	.359 .320 .229 .384 .008 079 040 .037 061 317 171	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388	.623 .418 .385 .522 .000 .059 076 .144 .045 008	.832 .625 .542 .767 .101 .003 014 .241 .000 .004	F2 .006 012 .075 .079 .754 .771 .838 .557 .816 .218	F3301297178366 .007 .033 .028 .022007 .702 .405	.016022015 .000011 .172 .023016 .185334018
2 3 4 5 6 7 8 9 10 11 12	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003	.359 .320 .229 .384 .008 079 040 .037 061 317 171	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263	F2 .006 012 .075 .079 .754 .771 .838 .557 .816 .218 .132 012	F3301297178366 .007 .033 .028 .022007 .702 .405	.016022015 .000011 .172 .023016 .185334018357
2 3 4 5 6 7 8 9 10 11 12 13	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003	.359 .320 .229 .384 .008 079 040 .037 061 317 171 .001	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443	F2 .006 012 .075 .079 .754 .771 .838 .557 .816 .218 .132 012	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025	.016022015 .000011 .172 .023016 .185334018357289
2 3 4 5 6 7 8 9 10 11 12 13 14	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295 .446 294	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187 .018	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003 .039	.359 .320 .229 .384 .008 079 040 .037 061 317 171 .001 .209 283	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006 .081	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484100	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248 .341	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443 .643	F2 .006012 .075 .079 .754 .771 .838 .557 .816 .218 .132012 .114 .097	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025 .327	.016022015 .000011 .172 .023016 .185334018357289 .393
2 3 4 5 6 7 8 9 10 11 12 13 14 15	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295 .446 294 194	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187 .018 .082032	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508 .009010	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003 .039 .566	.359 .320 .229 .384 .008079040 .037061317171 .001 .209283189	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006 .081027	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484100117	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248 .341 .459	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443 .643 .065	F2 .006012 .075 .079 .754 .771 .838 .557 .816 .218 .132012 .114 .097017	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025 .327 .235	.016022015 .000011 .172 .023016 .185334018357289 .393 .364
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295 .446 294 194 006	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187 .018 .082032111	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508 .009010049	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003 .039 .566 .601	.359 .320 .229 .384 .008079040 .037061317171 .001 .209283189018	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006 .081027102	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484100117157	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248 .341 .459 .532	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443 .643 .065 .198	F2 .006012 .075 .079 .754 .771 .838 .557 .816 .218 .132012 .114 .097017105	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025 .327 .235 .065	.016022015 .000011 .172 .023016 .185334018357289 .393 .364 .363
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295 .446 294 194 006 430	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187 .018 .082032111 .003	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508 .009010049 .177	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003 .039 .566 .601 .737	.359 .320 .229 .384 .008079040 .037061317171 .001 .209283189018460	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006 .081027102 .007	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484100117157	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248 .341 .459 .532 .753	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443 .643 .065 .198 .514 016	F2 .006012 .075 .079 .754 .771 .838 .557 .816 .218 .132012 .114 .097017105 .071	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025 .327 .235 .065 .586	.016022015 .000011 .172 .023016 .185334018357289 .393 .364 .363 .427
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295 .446 294 194 006 430	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187 .018 .082032111 .003198	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508 .009010049 .177 .262	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003 .039 .566 .601 .737 .659	.359 .320 .229 .384 .008079040 .037061317171 .001 .209283189018460098	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006 .081027102 .007188	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484100117157 .015 .133	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248 .341 .459 .532 .753 .513	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443 .643 .065 .198 .514 016	F2 .006012 .075 .079 .754 .771 .838 .557 .816 .218 .132012 .114 .097017105 .071115	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025 .327 .235 .065 .586 .273	.016022015 .000011 .172 .023016 .185334018357289 .393 .364 .363 .427 .180
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295 .446 294 194 006 430 .015 114	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187 .018 .082032111 .003198 .050	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508 .009010049 .177 .262 .102	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003 .039 .566 .601 .737 .659 .593	.359 .320 .229 .384 .008079040 .037061317171 .001 .209283189018460098153	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006 .081027102 .007188 .048	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484100117157 .015 .133 .016	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248 .341 .459 .532 .753 .513 .657	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443 .643 .065 .198 .514 016 .470	F2 .006012 .075 .079 .754 .771 .838 .557 .816 .218 .132012 .114 .097017105 .071115 .080	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025 .327 .235 .065 .586 .273 .232	.016022015 .000011 .172 .023016 .185334018357289 .393 .364 .363 .427 .180 .235
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295 .446 294 194 006 430 .015 114	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187 .018 .082032111 .003198 .050 .308	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508 .009010049 .177 .262 .102013	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003 .039 .566 .601 .737 .659 .593 .441	.359 .320 .229 .384 .008079040 .037061317171 .001 .209283189018460098153 .032	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006 .081027102 .007188 .048	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484100117157 .015 .133 .016046	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248 .341 .459 .532 .753 .513 .657 .427	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443 .643 .065 .198 .514 016 .470 .202	F2 .006012 .075 .079 .754 .771 .838 .557 .816 .218 .132012 .114 .097017105 .071115 .080 .282	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025 .327 .235 .065 .586 .273 .232017	.016022015 .000011 .172 .023016 .185334018357289 .393 .364 .363 .427 .180 .235 .216
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295 .446 294 194 006 430 .015 114 .057	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187 .018 .082032111 .003198 .050 .308016	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508 .009010049 .177 .262 .102013 .361	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003 .039 .566 .601 .737 .659 .593 .441 .383 .268	.359 .320 .229 .384 .008079040 .037061317171 .001 .209283189018460098153 .032 .115	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006 .081027102 .007188 .048 .289022	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484100117157 .015 .133 .016046 .305	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248 .341 .459 .532 .753 .513 .657 .427	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443 .065 .198 .514 016 .470 .202 .377	F2 .006012 .075 .079 .754 .771 .838 .557 .816 .218 .132012 .114 .097017105 .071115 .080 .282 .058	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025 .327 .235 .065 .586 .273 .232017 .073	.016022015 .000011 .172 .023016 .185334018357289 .393 .364 .363 .427 .180 .235 .216082
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295 .446 294 194 006 430 .015 114 .057 .289	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .018187 .018 .082032111 .003198 .050 .308016 .292	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508 .009010049 .177 .262 .102013 .361009	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003 .039 .566 .601 .737 .659 .593 .441 .383 .268	.359 .320 .229 .384 .008079040 .037061317171 .001 .209283189018460098153 .032 .115 .032	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006 .081027102 .007188 .048 .289022 .274	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484100117157 .015 .133 .016046 .305045	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248 .341 .459 .532 .753 .513 .657 .427 .452 .478	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443 .643 .065 .198 .514 016 .470 .202 .377 .597	F2 .006012 .075 .079 .754 .771 .838 .557 .816 .218 .132012 .114 .097017105 .071115 .080 .282 .058 .268	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025 .327 .235 .065 .586 .273 .232017 .073014	.016022015 .000011 .172 .023016 .185334018357289 .393 .364 .363 .427 .180 .235 .216082 .218
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21	.449 .378 .304 .452 .075 080 .002 .128 075 014 .007 .295 .446 294 194 006 430 .015 114 .057	F2 .015 .005 .077 .111 .790 .839 .889 .564 .898 .008 .018187 .018 .082032111 .003198 .050 .308016	F3 .038008 .069023 .137021 .093 .185064 .909 .485 .751 .508 .009010049 .177 .262 .102013 .361	.380 .226 .211 .285 116 .032 156 .007 .020 145 .254 003 .039 .566 .601 .737 .659 .593 .441 .383 .268	.359 .320 .229 .384 .008079040 .037061317171 .001 .209283189018460098153 .032 .115	F2 .009 .001 .068 .099 .739 .789 .833 .526 .845002 .012188 .006 .081027102 .007188 .048 .289022	F3 .023002 .064011 .184 .011 .149 .202021 .814 .388 .676 .484100117157 .015 .133 .016046 .305	.623 .418 .385 .522 .000 .059 076 .144 .045 008 .351 .248 .341 .459 .532 .753 .513 .657 .427	.832 .625 .542 .767 .101 .003 014 .241 .000 .004 .263 .443 .065 .198 .514 016 .470 .202 .377	F2 .006012 .075 .079 .754 .771 .838 .557 .816 .218 .132012 .114 .097017105 .071115 .080 .282 .058	F3301297178366 .007 .033 .028 .022007 .702 .405 .341 .025 .327 .235 .065 .586 .273 .232017 .073	.016022015 .000011 .172 .023016 .185334018357289 .393 .364 .363 .427 .180 .235 .216082

Table A5

Factor loading matrices of the local solutions for M_4 at $\varepsilon=.001$.

Co	$\underline{\text{nditions}}$	$\varepsilon = .02$	$\varepsilon = .01$	$\varepsilon = .001$
N	h			
60	low	1 (93.0)	1 (82.0)	3 (19.1)
	medium	1(96.7)	1(86.4)	3(21.7)
	high	1(99.4)	1(95.6)	2(30.2)
	wide	1(94.4)	1 (83.3)	3(19.3)
100	low	1 (96.3)	1 (89.3)	3 (19.2)
	medium	1(99.0)	1(95.4)	3(21.7)
	high	1(100.0)	1(99.2)	2(34.9)
	wide	1(96.7)	1 (89.9)	3(21.6)
200	low	1 (98.1)	1 (94.6)	2 (29.0)
	medium	1(99.9)	1(99.2)	2(34.8)
	high	1 (100.0)	1 (100.0)	2(35.5)
	wide	1(99.2)	1(97.2)	2(29.7)
300	low	1 (99.2)	1 (96.5)	2 (31.7)
	medium	1(100.0)	1(99.9)	2(36.7)
	high	1(100.0)	1(100.0)	2(35.2)
	wide	1 (100.0)	1(99.0)	2(35.2)
500	low	1 (99.7)	1 (98.4)	2 (33.3)
	medium	1 (100.0)	1 (100.0)	1(50.7)
	high	1 (100.0)	1 (100.0)	2(36.3)
	wide	1 (100.0)	1(99.9)	1(51.0)
Note	$N = \sin$	nple size. h	= commun	ality.

Table A6

Median numbers of sample solutions for Model M_{3a} . Proportions of samples with the median number of solutions in parenthesis.

<u>Co</u>	nditions	$\varepsilon = .02$	$\varepsilon = .01$	$\varepsilon = .001$
N	h			
60	low	1 (51.5)	3 (19.1)	≥ 10 (—)
	medium	1(70.7)	2(26.9)	$\geq 10 \; ()$
	high	1(92.1)	1(65.9)	$\geq 10 \; ()$
	wide	1(66.6)	2(27.1)	$\geq 10 \; ()$
100	low	1 (93.4)	2(29.5)	≥ 10 (—)
	medium	1 (89.9)	1(62.8)	$\geq 10 \; ()$
	high	1(98.4)	1(82.0)	$\geq 10 \; ()$
	wide	1 (84.4)	1(52.4)	$\geq 10 \; ()$
200	low	1 (88.4)	1(65.5)	≥ 10 (—)
	medium	1(98.2)	1(80.7)	$\geq 10 \; ()$
	high	1(99.9)	1(93.4)	$\geq 10 \; ()$
	wide	1(95.3)	1(72.2)	$\geq 10 \; ()$
300	low	1 (95.9)	1 (81.4)	≥ 10 (—)
	medium	1(99.6)	1(89.9)	$\geq 10 \; ()$
	high	1 (100.0)	1(93.8)	$\geq 10 \; ()$
	wide	1(98.5)	1(79.4)	$\geq 10 \; ()$
500	low	1 (99.7)	1 (93.7)	9 (8.5)
	medium	1(99.7)	1(95.2)	$\geq 10 \; ()$
	high	1 (100.0)	1(97.6)	$\geq 10 \; ()$
	wide	1(99.6)	1 (88.0)	$\geq 10 \ ()$
Note	$N = \sin$	nple size. h	= commu	nality.

Table A7

Median numbers of sample solutions for Model M_4 . For 100 random starts, the program count up to 10 local solutions.

-				Pair of	ε Values		
Co	$\underline{\text{nditions}}$	M_{3a}		Λ	I_{3b}	M_4	
N	h	.02/.01	.01/.001	.02/.01	.01/.001	.02/.01	.01/.001
60	low	97.6	77.2	90.9	78.6	82.0	53.8
	medium	98.0	82.6	87.6	76.6	93.3	65.8
	high	99.7	96.4	88.7	88.0	98.0	74.8
	wide	97.8	80.9	88.7	68.7	92.1	64.3
100	low	97.9	84.3	87.3	78.7	88.2	55.6
	medium	99.5	90.3	85.2	79.8	96.5	67.0
	high	100.0	98.9	93.8	97.7	99.6	78.1
	wide	99.3	87.9	87.0	62.2	95.6	69.2
200	low	99.4	90.1	89.7	80.6	94.1	70.3
	medium	100.0	98.5	83.3	90.7	99.2	70.1
	high	100.0	99.8	99.5	100.0	99.8	78.1
	wide	99.6	94.4	89.5	46.0	98.2	69.8
300	low	99.4	92.5	87.0	81.3	98.2	79.6
	medium	100.0	99.0	91.5	97.6	99.9	71.8
	high	100.0	99.9	100.0	100.0	100.0	79.3
	wide	99.9	96.9	91.8	26.8	99.8	69.8
500	low	99.9	96.7	87.9	88.9	99.8	88.0
	medium	100.0	100.0	97.9	99.9	100.0	71.1
	high	100.0	100.0	100.0	100.0	100.0	83.5
	wide	100.0	99.1	94.5	13.6	100.0	73.8

Note. N = sample size. h = communality.

Table A8

Percentages of samples with congruence coefficients > .92 across sample global solutions at different values of ε .

Appendix C

Figures

Figure C1: The percentages of simulated samples which produced congruence coefficients greater than 0.98 with the population global solutions at $\varepsilon = .01$ for Model M_{3a} .

Figure C2: The percentages of simulated samples which produced congruence coefficients greater than 0.92 with the population global solutions at $\varepsilon = .01$ for Model M_{3a} .

Figure C3: The percentages of simulated samples which produced congruence coefficients greater 0.92 with the population global solutions at $\varepsilon = .01$ for Model M_{3b} .

Figure C4: The percentages of simulated samples which produced congruence coefficients greater 0.98 with the population global solutions at $\varepsilon = .01$ for Model M_4 .

Figure C5: The percentages of simulated samples which produced congruence coefficients greater 0.92 with the population global solutions at $\varepsilon = .01$ for Model M_4 .

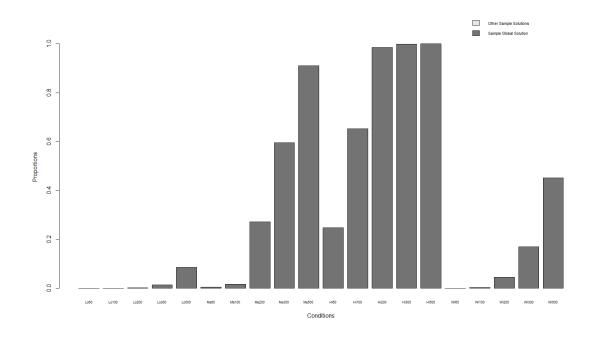


Figure C1. Percentages of samples whose solutions had all congruence coefficients > .98 with the population global solutions Λ^I for M_{3a} at $\varepsilon = .01$.

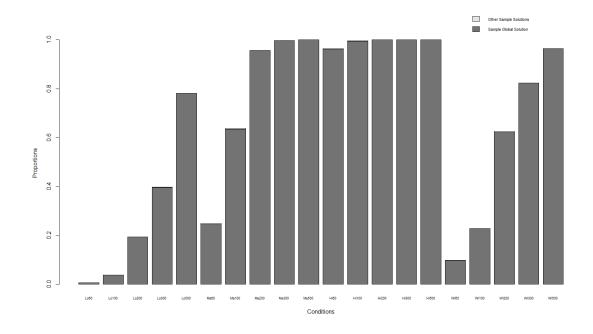


Figure C2. Percentages of samples whose solutions had all congruence coefficients > .92 with the population global solutions Λ^I for M_{3a} at $\varepsilon = .01$.

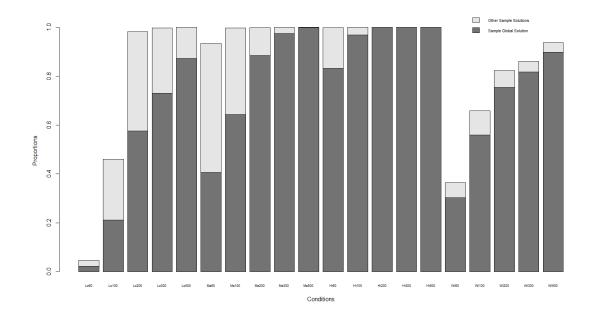


Figure C3. Percentages of samples whose solutions had all congruence coefficients > .92 with the population global solutions Λ^I for M_{3b} at $\varepsilon = .01$.

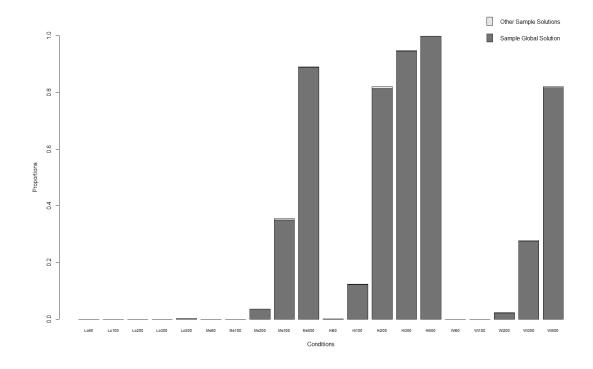


Figure C4. Percentages of samples whose solutions had all congruence coefficients > .98 with the population global solutions Λ^I for M_4 at $\varepsilon = .01$.

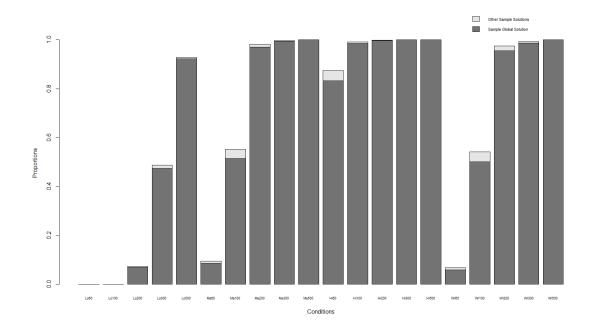


Figure C5. Percentages of samples whose solutions had all congruence coefficients > .92 with the population global solutions Λ^I for M_4 at $\varepsilon = .01$.