Online Appendices to Multiple Local Solutions and Geomin Rotation

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The online supplementary appendices contain R code for conducting Geomin rotation from multiple random starts and additional eight tables and five figures that supplement the paper: *Multiple Local Solutions and Geomin Rotation*.

Appendix A

R code

```
library(GPArotation)

library(plyr)

# The following three functions are separate R files.

source('Align.Matrix.R')

source('MultipleGeomin.R')

source('CompareSolutions.R')

data(Thurstone, package = "GPArotation") # for box26

res <- MultipleGeomin(unrotated = box26, epsilon = .01, K = 100, plot = T,
cex= .5)

res

res2 <- cfQ(box26, kappa = 1/nrow(box26), maxit = 10000) # obliqe CF-varimax

res3 <- oblimin(box26, maxit = 10000) # oblique oblimin

res4 <- quartimin(box26, maxit = 10000) # oblique quartimin

otherSolutions <- list(res2$loadings, res3$loadings, res4$loadings)

allSolutions <- c(res$loadings, otherSolutions) # list of all rotated
loadings

CompareSolutions(allSolutions, compare = 'First')
```

Appendix B

Tables

Table A1: The global solutions at different levels of ε for Model M_{3a}.

Table A2: The global solutions at different levels of ε for Model M_4.

Table A3: The congruence coefficients between factors of solutions at different values of ε for all models.

Table A4: The congruence coefficients between factors of local solutions at $\varepsilon = .01$ for Model M_{3b}.

Table A5: All solutions at $\varepsilon = .001$ for Model M_4.

Table A6: The numbers of solutions in simulated samples at different levels of ε for Model M_{3a}.

Table A7: The numbers of solutions in simulated sample at different levels of ε for Model M_4.

Table A8: The percentages of simulated samples with congruence coefficients greater than 0.92 for all columns across different values of ε for all models.

	$\hat{\mathbf{\Lambda}}^{\mathrm{I}}$ ($\varepsilon = .02$)			$\hat{\mathbf{\Lambda}}^{\mathrm{I}}$ ($\varepsilon = .01$)			$\hat{\mathbf{\Lambda}}^{\mathrm{I}}$ ($\varepsilon = .001$)		
	Verb	Arith	Spat	Verb	Arith	Spat	Verb	Arith	Spat
WrdMean	.929	−.034	−.015	.931	−.036	−.014	.925	−.043	.008
SntComp	.757	.151	−.022	.758	.150	−.023	.759	.147	−.016
OddWrds	.818	.018	.088	.818	.016	.088	.814	.009	.106
MxdArit	−.038	1.004	−.049	−.038	1.008	−.058	−.005	1.020	−.118
Remndrs	.044	.806	.059	.043	.809	.052	.069	.816	.007
MissNum	.141	.753	.061	.141	.756	.054	.164	.762	.014
Gloves	−.057	.166	.545	−.060	.165	.545	−.057	.158	.540
Boots	.047	.033	.725	.044	.032	.727	.041	.019	.734
Hatchts	.019	−.036	.890	.016	−.038	.892	.009	−.054	.905
Factor Correlations									
Verb	1.000			1.000			1.000		
Arith	.567	1.000		.571	1.000		.556	1.000	
Spat	.401	.438	1.000	.405	.448	1.000	.400	.504	1.000

Table A1

Factor loading and correlation matrices of the global solutions with different ε values for M_{3a}.

	$\hat{\boldsymbol{\Lambda}}^{\mathrm{I}}$ ($\varepsilon = .02$)				$\hat{\boldsymbol{\Lambda}}^{\mathrm{I}}$ ($\varepsilon = .01$)				$\hat{\boldsymbol{\Lambda}}^{\mathrm{I}}$ ($\varepsilon = .001$)			
	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4
1	.662	.051	.039	.057	.664	.048	.043	.050	.679	.031	.038	.043
2	.514	.026	−.004	−.021	.515	.024	−.002	−.025	.523	.016	−.007	−.033
3	.423	.097	.067	.015	.423	.094	.070	.011	.436	.080	.069	.011
4	.633	.132	−.021	−.014	.635	.130	−.019	−.020	.644	.116	−.019	−.025
5	.062	.758	.116	−.071	.057	.755	.119	−.072	.067	.727	.149	−.032
6	.001	.808	−.043	.076	.004	.810	−.041	.073	.002	.780	−.002	.112
7	−.021	.847	.070	−.068	−.026	.846	.072	−.069	−.020	.817	.109	−.024
8	.156	.554	.166	−.012	.151	.550	.171	−.014	.167	.523	.190	.017
9	.011	.862	−.085	.061	.014	.864	−.084	.058	.009	.835	−.041	.097
10	−.221	.041	.879	.044	−.242	.022	.894	.043	−.176	−.006	.873	.088
11	.066	.061	.464	.238	.062	.051	.476	.233	.102	.023	.467	.257
12	.184	−.141	.735	−.029	.166	−.158	.747	−.032	.226	−.180	.717	−.010
13	.416	.050	.498	−.099	.402	.037	.507	−.103	.447	.015	.489	−.087
14	−.017	.121	−.007	.513	−.002	.123	.001	.505	.001	.098	.012	.514
15	.100	.014	−.021	.485	.114	.015	−.014	.477	.119	−.006	−.008	.478
16	.366	−.051	−.055	.483	.383	−.050	−.048	.473	.389	−.072	−.047	.465
17	−.143	.060	.153	.658	−.128	.058	.165	.649	−.113	.027	.172	.666
18	.263	−.130	.249	.422	.270	−.136	.260	.412	.298	−.161	.248	.415
19	.092	.087	.089	.361	.101	.086	.096	.354	.113	.063	.101	.363
20	.281	.326	−.025	.235	.289	.326	−.020	.229	.294	.301	−.005	.240
21	.386	.026	.351	.101	.382	.017	.359	.095	.416	−.006	.347	.106
22	.288	.313	−.021	.242	.296	.313	−.016	.236	.301	.288	−.001	.247
23	.394	.285	.106	.137	.397	.281	.112	.130	.413	.255	.120	.143
24	−.040	.282	.449	.217	−.045	.273	.461	.212	−.008	.240	.463	.248

Table A2

Factor loading matrices of the global solutions $\hat{\boldsymbol{\Lambda}}^{\mathrm{I}}$ *with different* ε *values for* M_4*.*

M_{3a}: Holzinger Unpublished

ε	Verb	Arith	Spat
.02 and .01	1	1	.9999
.01 and .001	.9994	.9999	.9975
.02 and .001	.9994	.9998	.9968

M_{3b}: Box Data

ε	H	L	W
.02 and .01	1	1	1
.01 and .001	.9999	1	1
.02 and .001	.9999	1	1

M_4: Holzinger-Swineford 24-Test

ε	F1	F2	F3	F4
.02 and .01	.9995	.9998	.9999	.9999
.01 and .001	.9971	.9987	.9983	.9964
.02 and .001	.9980	.9979	.9979	.9967

Table A3

Congruence coefficients between columns of solutions at different values of ε

	h	l	w
$\hat{\mathbf{\Lambda}}^{\mathrm{I}}$ and $\hat{\mathbf{\Lambda}}^{\mathrm{II}}$	.661	.658	.743
$\hat{\mathbf{\Lambda}}^{\mathrm{I}}$ and $\hat{\mathbf{\Lambda}}^{\mathrm{III}}$	.635	.671	.652
$\hat{\mathbf{\Lambda}}^{\mathrm{I}}$ and $\hat{\mathbf{\Lambda}}^{\mathrm{IV}}$	.705	.530	.672
$\hat{\mathbf{\Lambda}}^{\mathrm{II}}$ and $\hat{\mathbf{\Lambda}}^{\mathrm{III}}$	.290	.689	.697
$\hat{\mathbf{\Lambda}}^{\mathrm{II}}$ and $\hat{\mathbf{\Lambda}}^{\mathrm{IV}}$	.757	.940	.724
$\hat{\mathbf{\Lambda}}^{\mathrm{III}}$ and $\hat{\mathbf{\Lambda}}^{\mathrm{IV}}$	.828	.826	.999

Table A4

Congruence coefficients among local solutions for M_{3b} at $\varepsilon = .01$.

	$\hat{\mathbf{\Lambda}}^{\mathrm{I}}$				$\hat{\mathbf{\Lambda}}^{\mathrm{II}}$				$\hat{\mathbf{\Lambda}}^{\mathrm{III}}$			
	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4
1	.679	.031	.038	.043	.673	.030	.064	−.005	.520	.005	.022	.276
2	.523	.016	−.007	−.033	.515	.013	.009	−.063	.433	−.002	−.017	.132
3	.436	.080	.069	.011	.415	.079	.092	−.021	.357	.063	.058	.155
4	.644	.116	−.019	−.025	.638	.114	−.002	−.059	.528	.090	−.032	.186
5	.067	.727	.149	−.032	.006	.728	.169	−.030	.155	.700	.145	−.005
6	.002	.780	−.002	.112	.015	.785	−.006	.123	−.011	.742	−.004	.166
7	−.020	.817	.109	−.024	−.069	.819	.120	−.012	.079	.789	.107	−.018
8	.167	.523	.190	.017	.107	.524	.221	.000	.198	.500	.182	.079
9	.009	.835	−.041	.097	.029	.840	−.051	.115	−.001	.794	−.043	.154
10	−.176	−.006	.873	.088	−.410	−.005	.997	.005	−.015	.018	.861	−.018
11	.102	.023	.467	.257	.020	.028	.540	.186	.021	.017	.453	.317
12	.226	−.180	.717	−.010	.017	−.182	.831	−.099	.318	−.161	.701	.000
13	.447	.015	.489	−.087	.283	.011	.575	−.157	.506	.016	.472	.002
14	.001	.098	.012	.514	.108	.110	.015	.479	−.302	.068	.004	.644
15	.119	−.006	−.008	.478	.224	.005	−.004	.438	−.199	−.035	−.018	.636
16	.389	−.072	−.047	.465	.502	−.062	−.040	.411	.009	−.108	−.062	.710
17	−.113	.027	.172	.666	−.018	.043	.196	.609	−.457	.000	.161	.780
18	.298	−.161	.248	.415	.316	−.153	.297	.339	.021	−.180	.232	.591
19	.113	.063	.101	.363	.160	.072	.120	.322	−.107	.041	.091	.484
20	.294	.301	−.005	.240	.341	.307	.003	.213	.106	.266	−.015	.407
21	.416	−.006	.347	.106	.336	−.006	.412	.037	.335	−.017	.331	.242
22	.301	.288	−.001	.247	.349	.293	.007	.218	.108	.253	−.012	.417
23	.413	.255	.120	.143	.403	.258	.149	.102	.282	.226	.106	.314
24	−.008	.240	.463	.248	−.092	.246	.531	.191	−.046	.230	.452	.278

	$\hat{\mathbf{\Lambda}}^{\mathrm{IV}}$				$\hat{\mathbf{\Lambda}}^{\mathrm{V}}$				$\hat{\mathbf{\Lambda}}^{\mathrm{VI}}$			
	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4
1	.449	.015	.038	.380	.359	.009	.023	.623	.832	.006	−.301	.016
2	.378	.005	−.008	.226	.320	.001	−.002	.418	.625	−.012	−.297	−.022
3	.304	.077	.069	.211	.229	.068	.064	.385	.542	.075	−.178	−.015
4	.452	.111	−.023	.285	.384	.099	−.011	.522	.767	.079	−.366	.000
5	.075	.790	.137	−.116	.008	.739	.184	.000	.101	.754	.007	−.011
6	−.080	.839	−.021	.032	−.079	.789	.011	.059	.003	.771	.033	.172
7	.002	.889	.093	−.156	−.040	.833	.149	−.076	−.014	.838	.028	.023
8	.128	.564	.185	.007	.037	.526	.202	.144	.241	.557	.022	−.016
9	−.075	.898	−.064	.020	−.061	.845	−.021	.045	.000	.816	−.007	.185
10	−.014	.008	.909	−.145	−.317	−.002	.814	−.008	.004	.218	.702	−.334
11	.007	.018	.485	.254	−.171	.012	.388	.351	.263	.132	.405	−.018
12	.295	−.187	.751	−.003	.001	−.188	.676	.248	.443	−.012	.341	−.357
13	.446	.018	.508	.039	.209	.006	.484	.341	.643	.114	.025	−.289
14	−.294	.082	.009	.566	−.283	.081	−.100	.459	.065	.097	.327	.393
15	−.194	−.032	−.010	.601	−.189	−.027	−.117	.532	.198	−.017	.235	.364
16	−.006	−.111	−.049	.737	−.018	−.102	−.157	.753	.514	−.105	.065	.363
17	−.430	.003	.177	.659	−.460	.007	.015	.513	−.016	.071	.586	.427
18	.015	−.198	.262	.593	−.098	−.188	.133	.657	.470	−.115	.273	.180
19	−.114	.050	.102	.441	−.153	.048	.016	.427	.202	.080	.232	.235
20	.057	.308	−.013	.383	.032	.289	−.046	.452	.377	.282	−.017	.216
21	.289	−.016	.361	.268	.115	−.022	.305	.478	.597	.058	.073	−.082
22	.059	.292	−.009	.395	.032	.274	−.045	.465	.388	.268	−.014	.218
23	.218	.261	.117	.317	.130	.242	.091	.478	.539	.261	−.059	.080
24	−.070	.256	.476	.164	−.235	.236	.398	.237	.126	.351	.444	.002

Table A5

Factor loading matrices of the local solutions for M_4 *at* $\varepsilon = .001$.

Conditions		$\varepsilon = .02$	$\varepsilon = .01$	$\varepsilon = .001$
N	h			
60	low	1 (93.0)	1 (82.0)	3 (19.1)
	medium	1 (96.7)	1 (86.4)	3 (21.7)
	high	1 (99.4)	1 (95.6)	2 (30.2)
	wide	1 (94.4)	1 (83.3)	3 (19.3)
100	low	1 (96.3)	1 (89.3)	3 (19.2)
	medium	1 (99.0)	1 (95.4)	3 (21.7)
	high	1 (100.0)	1 (99.2)	2 (34.9)
	wide	1 (96.7)	1 (89.9)	3 (21.6)
200	low	1 (98.1)	1 (94.6)	2 (29.0)
	medium	1 (99.9)	1 (99.2)	2 (34.8)
	high	1 (100.0)	1 (100.0)	2 (35.5)
	wide	1 (99.2)	1 (97.2)	2 (29.7)
300	low	1 (99.2)	1 (96.5)	2 (31.7)
	medium	1 (100.0)	1 (99.9)	2 (36.7)
	high	1 (100.0)	1 (100.0)	2 (35.2)
	wide	1 (100.0)	1 (99.0)	2 (35.2)
500	low	1 (99.7)	1 (98.4)	2 (33.3)
	medium	1 (100.0)	1 (100.0)	1 (50.7)
	high	1 (100.0)	1 (100.0)	2 (36.3)
	wide	1 (100.0)	1 (99.9)	1 (51.0)

Note. N = sample size. h = communality.

Table A6

Median numbers of sample solutions for Model M_{3a}. Proportions of samples with the median number of solutions in parenthesis.

Conditions		$\varepsilon = .02$	$\varepsilon = .01$	$\varepsilon = .001$
N	h			
60	low	1 (51.5)	3 (19.1)	$\geq$ 10 (—)
	medium	1 (70.7)	2 (26.9)	$\geq$ 10 (—)
	high	1 (92.1)	1 (65.9)	$\geq$ 10 (—)
	wide	1 (66.6)	2 (27.1)	$\geq$ 10 (—)
100	low	1 (93.4)	2 (29.5)	$\geq$ 10 (—)
	medium	1 (89.9)	1 (62.8)	$\geq$ 10 (—)
	high	1 (98.4)	1 (82.0)	$\geq$ 10 (—)
	wide	1 (84.4)	1 (52.4)	$\geq$ 10 (—)
200	low	1 (88.4)	1 (65.5)	$\geq$ 10 (—)
	medium	1 (98.2)	1 (80.7)	$\geq$ 10 (—)
	high	1 (99.9)	1 (93.4)	$\geq$ 10 (—)
	wide	1 (95.3)	1 (72.2)	$\geq$ 10 (—)
300	low	1 (95.9)	1 (81.4)	$\geq$ 10 (—)
	medium	1 (99.6)	1 (89.9)	$\geq$ 10 (—)
	high	1 (100.0)	1 (93.8)	$\geq$ 10 (—)
	wide	1 (98.5)	1 (79.4)	$\geq$ 10 (—)
500	low	1 (99.7)	1 (93.7)	9 (8.5)
	medium	1 (99.7)	1 (95.2)	$\geq$ 10 (—)
	high	1 (100.0)	1 (97.6)	$\geq$ 10 (—)
	wide	1 (99.6)	1 (88.0)	$\geq$ 10 (—)

Note. N = sample size. h = communality.

Table A7

Median numbers of sample solutions for Model M_4. For 100 random starts, the program count up to 10 local solutions.

		Pair of ε Values					
Conditions		M_{3a}		M_{3b}		M_4	
N	h	.02/.01	.01/.001	.02/.01	.01/.001	.02/.01	.01/.001
60	low	97.6	77.2	90.9	78.6	82.0	53.8
	medium	98.0	82.6	87.6	76.6	93.3	65.8
	high	99.7	96.4	88.7	88.0	98.0	74.8
	wide	97.8	80.9	88.7	68.7	92.1	64.3
100	low	97.9	84.3	87.3	78.7	88.2	55.6
	medium	99.5	90.3	85.2	79.8	96.5	67.0
	high	100.0	98.9	93.8	97.7	99.6	78.1
	wide	99.3	87.9	87.0	62.2	95.6	69.2
200	low	99.4	90.1	89.7	80.6	94.1	70.3
	medium	100.0	98.5	83.3	90.7	99.2	70.1
	high	100.0	99.8	99.5	100.0	99.8	78.1
	wide	99.6	94.4	89.5	46.0	98.2	69.8
300	low	99.4	92.5	87.0	81.3	98.2	79.6
	medium	100.0	99.0	91.5	97.6	99.9	71.8
	high	100.0	99.9	100.0	100.0	100.0	79.3
	wide	99.9	96.9	91.8	26.8	99.8	69.8
500	low	99.9	96.7	87.9	88.9	99.8	88.0
	medium	100.0	100.0	97.9	99.9	100.0	71.1
	high	100.0	100.0	100.0	100.0	100.0	83.5
	wide	100.0	99.1	94.5	13.6	100.0	73.8

Note. N = sample size. h = communality.

Table A8

Percentages of samples with congruence coefficients > .92 across sample global solutions at different values of ε.

Appendix C

Figures

Figure C1: The percentages of simulated samples which produced congruence coefficients greater than 0.98 with the population global solutions at $\varepsilon = .01$ for Model M_{3a}.

Figure C2: The percentages of simulated samples which produced congruence coefficients greater than 0.92 with the population global solutions at $\varepsilon = .01$ for Model M_{3a}.

Figure C3: The percentages of simulated samples which produced congruence coefficients greater 0.92 with the population global solutions at $\varepsilon = .01$ for Model M_{3b}.

Figure C4: The percentages of simulated samples which produced congruence coefficients greater 0.98 with the population global solutions at $\varepsilon = .01$ for Model M_4.

Figure C5: The percentages of simulated samples which produced congruence coefficients greater 0.92 with the population global solutions at $\varepsilon = .01$ for Model M_4.

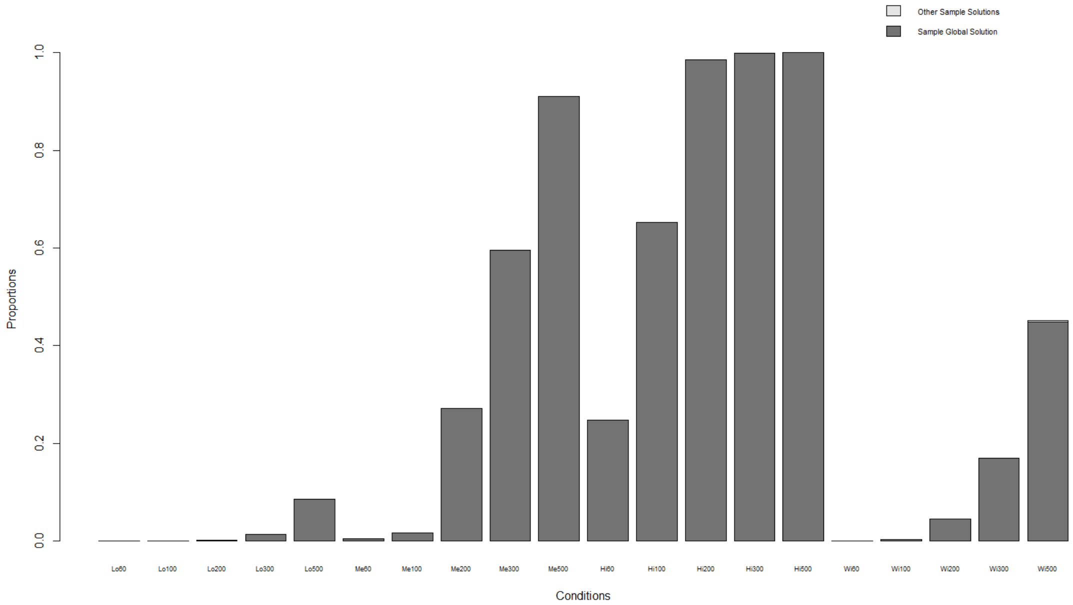

Figure C1. Percentages of samples whose solutions had all congruence coefficients $> .98$ with the population global solutions Λ^I for M_{3a} at $\varepsilon = .01$.

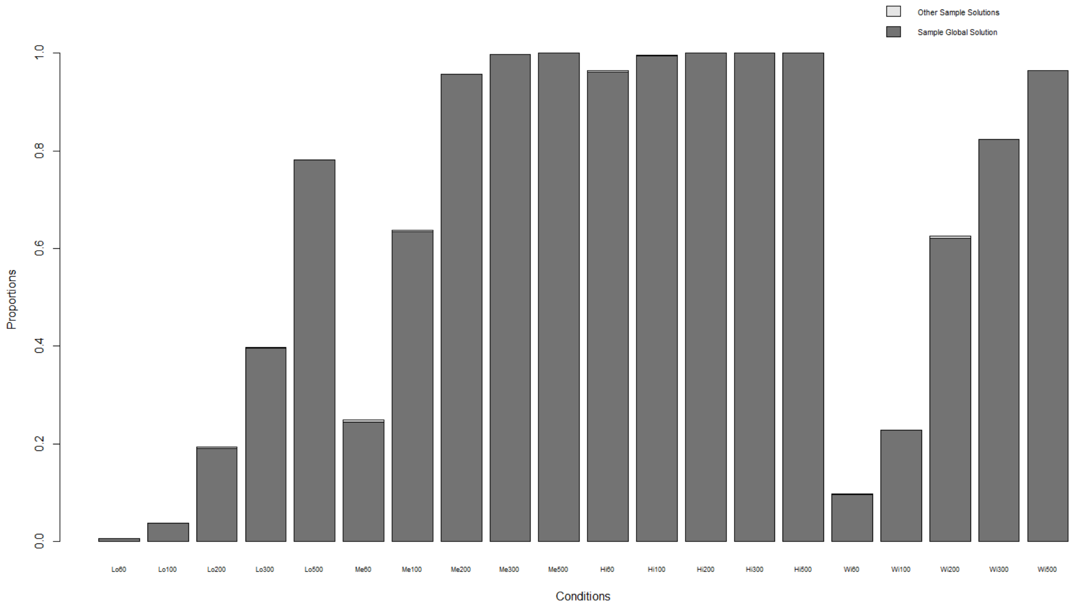

Figure C2. Percentages of samples whose solutions had all congruence coefficients > .92 with the population global solutions Λ^I for M_{3a} at $\varepsilon = .01$.

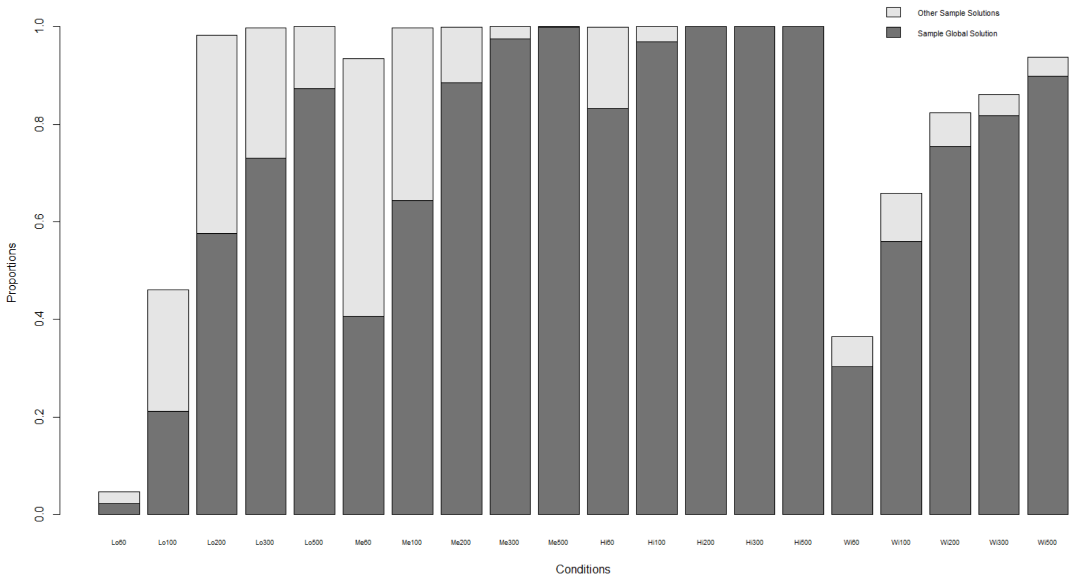

Figure C3. Percentages of samples whose solutions had all congruence coefficients > .92 with the population global solutions Λ^I for M_{3b} at $\varepsilon = .01$.

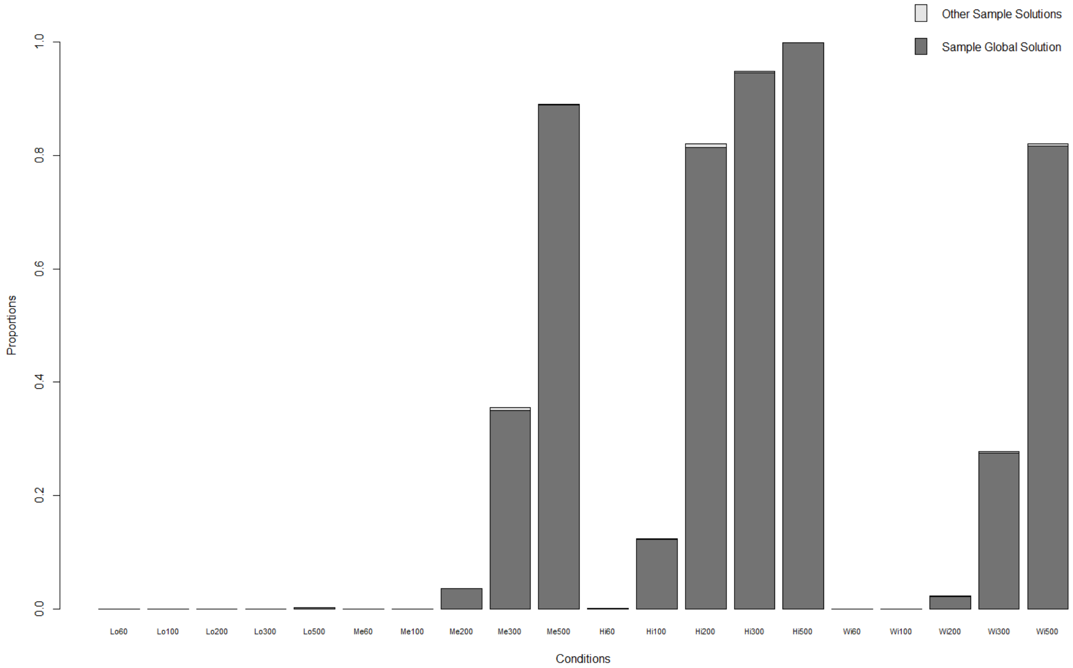

Figure C4. Percentages of samples whose solutions had all congruence coefficients > .98 with the population global solutions Λ^I for M_4 at $\varepsilon = .01$.

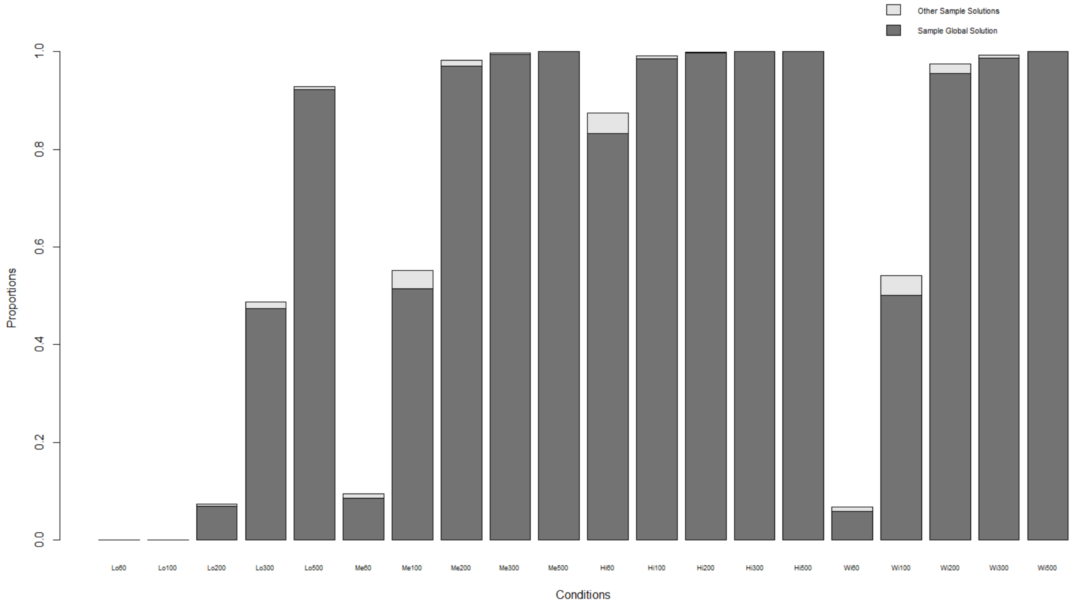

Figure C5. Percentages of samples whose solutions had all congruence coefficients > .92 with the population global solutions Λ^I for M_4 at $\varepsilon = .01$.