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No Need to be Discrete: A Method for Continuous Time Mediation Analysis

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Mediation is one concept that has shaped numerous theories. The list of problems associated with mediation models, however, has been growing. Mediation models based on cross-sectional data can produce unexpected estimates, so much so that making longitudinal or causal inferences is inadvisable. Even longitudinal mediation models have faults, as parameter estimates produced by these models are specific to the lag between observations, leading to much debate over appropriate lag selection. Using continuous time models (CTMs) rather than commonly employed discrete time models, one can estimate lag-independent parameters. We demonstrate methodology that allows for continuous time mediation analyses, with attention to concepts such as indirect and direct effects, partial mediation, the effect of lag, and the lags at which relations become maximal. A simulation compares common longitudinal mediation methods with CTMs. Reanalysis of a published covariance matrix demonstrates that CTMs can be fit to data used in longitudinal mediation studies.

Keywords: continuous time models, cross-lagged panel model, exact discrete model, longitudinal mediation, mediation

Mediation analysis has been applied in a vast number of studies due to its correspondence to theoretical predictions about relations among variables. The foundations of mediation analysis, however, have been questioned in the methodological literature. Maxwell and Cole (2007) and Maxwell, Cole, and Mitchell (2011) highlighted that mediation analyses of cross-sectional data frequently produce unexpected results—so much so, that in most practical instances interpretation of mediation analyses performed on cross-sectional data are (at best) questionable. Even longitudinal mediation models are not impervious to criticism; numerous studies have highlighted that the lag between subsequent measurements directly affects observed results (Cole & Maxwell, 2003; Gollob & Reichardt, 1987, 1991; Reichardt, 2011). We discuss how the use of continuous time models (CTMs) could alleviate the dependence of longitudinal mediation models on sampling rate. The incorporation of CTMs into mediation analyses has been suggested, but not implemented, by other authors (e.g., Fritz, 2007; MacKinnon, 2008, p. 230; Maxwell & Cole, 2007; Maxwell et al., 2011); MacKinnon (2008) noted that such models would be "especially promising," but have not yet been developed.

We begin by discussing current methods for assessing mediation in longitudinal data and the key differences between discrete and CTMs. We then present a CTM that could be used for assessing mediation effects similar to those commonly hypothesized by many researchers, requires the same kind of data that are collected for longitudinal mediation research, and can be fit in a structural equation modeling (SEM) program. We focus on methods for interpreting the CTM parameters and calculating the indirect effects for a range of lags. A simulation study and a substantive example highlight similarities and differences between the CTM and longitudinal mediation models.

CURRENT METHODS FOR ASSESSING MEDIATION IN LONGITUDINAL DATA

The simplest mediation model consists of three variables—X, M, and Y. The putative *mediator* M is regressed on the predictor X (with regression slope *a*), and the outcome Y is

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regressed on both X and M (slopes *c'* and *b,* respectively), implying a *direct effect* of X on Y as well as an *indirect effect* ($a \times b$) through M. In this model if X, M, and Y are measured simultaneously—that is, as cross-sectional data—biased estimates of the relationships between variables could result (Maxwell & Cole, 2007; Maxwell et al., 2011).

A popular extension of this simple mediation model is the *cross-lagged panel model* (CLPM). Because this model uses longitudinal data rather than cross-sectional data, it overcomes some of the weaknesses identified by Maxwell and Cole (2007), and consequently has been recommended by methodologists (Cole & Maxwell, 2003; Gollob & Reichardt, 1987, 1991; Maxwell & Cole, 2007; Maxwell et al., 2011; Selig & Preacher, 2009; Wang, Zhang, & Estabrook, 2009). Two common implementations of the CLPM are depicted in Figure 1 (note a difference in the X to Y direct effect), and many substantive examples exist (e.g., Belsky, Fearon, & Bell, 2007; Eisenberg et al., 2005; Ghazarian & Roche, 2010; Hallinger & Heck, 2010; Hawkley, Preacher, & Cacioppo, 2010; Ladd, 2006; Orth, Robins, & Roberts, 2008; Simons-Morton, Chen, Abroms, & Haynie, 2004).

In using the CLPM the researcher must choose a few discrete occasions at which to assess X, M, and Y. The magnitudes of effects, however, vary with lag and consequently any choice of lag constrains the generalizability of results (Selig, Preacher, & Little, 2012). Cole and Maxwell (2003) and Reichardt (2011) noted that there is no "correct" interval that should be used in a given context. Rather, researchers should recognize that the effects of key interest will vary with lag, and are well advised to report effects over several intervals. Even when testing several intervals, the CLPM is typically still restricted to a small number of fixed lags. Furthermore, the lag(s) of most interest for estimating one

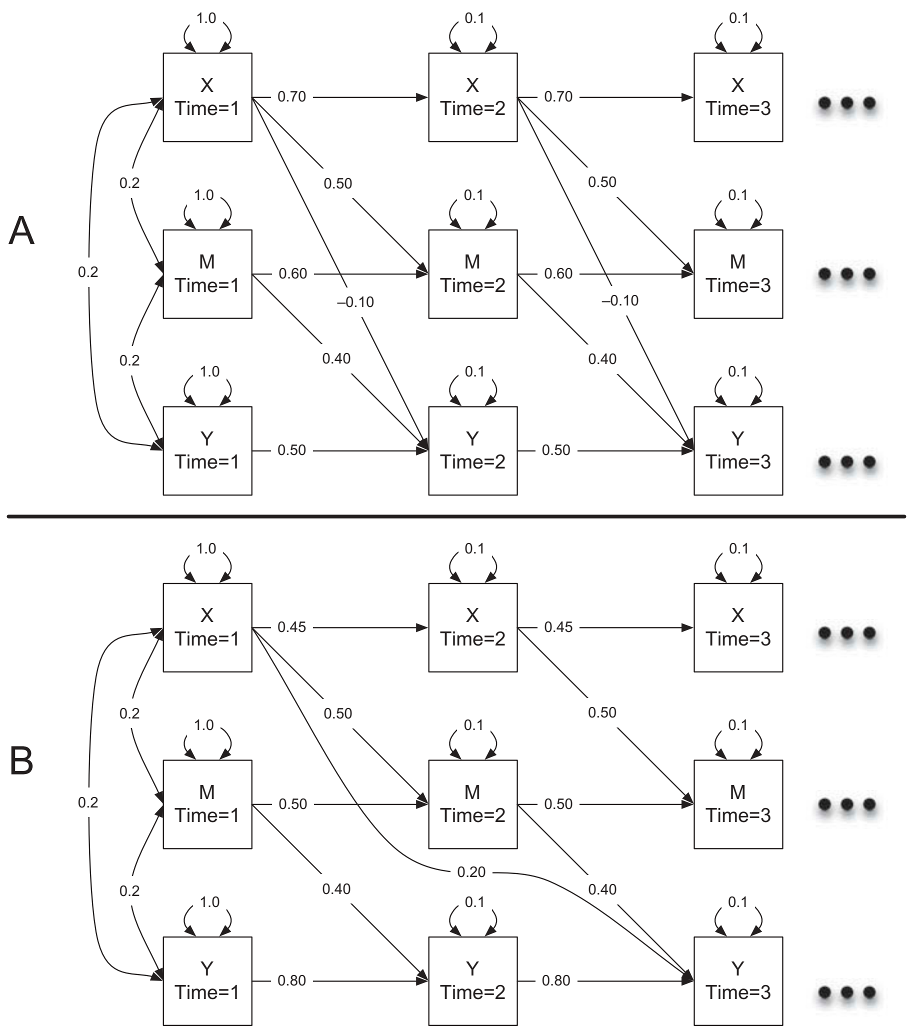

FIGURE 1 An example of the cross-lagged panel models with paths specified to test mediation; herein models A and B are referred to as $CLPM_A$ and $CLPM_B$, respectively. Full cross-lagged panel models would include relationships from M → X, Y → X, and Y → M. The path values listed for A and B correspond to the population parameters for the $CLPM_A$, and $CLPM_B$ in Simulation I.

effect (e.g., X to M) might differ from the lag(s) that should be used to estimate other effects (e.g., M to Y, or X to Y controlling for M).

Discrete and Continuous Time Models

The dependency of effects on lag is common to discrete time models. In discrete time models, the effect of time is considered only implicitly. In the CLPM, observations at one time (t_i) are regressed on observations at a previous time ($t_i - \Delta t_i$); the ordering of observations is retained but the model does not explicitly account for the time that elapses between observations (Voelkle, Oud, Davidov, & Schmidt, 2012). As the interpretation of an effect is specific to the lag(s) being examined, researchers collecting data at differing sampling rates might have difficulty comparing effects because differences could be due to differences in lag rather than to other possible attributions, such as differing populations. Differences in effects might include inconsistencies (a) in the relative contributions of two or more constructs to a construct of interest (Oud & Delsing, 2010); (b) in whether the relation between two variables is positive, negative, or zero (Oud, 2007); and (c) in selection of the best model (Bergstrom, 1988). The dependence on lag does not invalidate such results, but it limits conclusions to the specific lag(s) being examined.

In contrast, CTMs explicitly model time and consequently parameters of CTMs have the advantageous feature of being independent of lag. Hence, CTMs are promising in the context of mediation. Rather than placing focus on identifying the "correct" interval, or contending with the fact that the lags of most interest might differ for each effect, CTMs applied in a mediation context would allow researchers to gain insight into how key effects vary as a function of lag. To see how discrete and continuous time parameters differ, we present a first-order differential equation model with ongoing, stochastic inputs. This model could be a good first step for modeling many processes, because the changes postulated by this model bear resemblance to the description of the CLPM when used to test mediation hypotheses.

A First-Order Differential Equation Model

CTMs are often expressed using derivatives, which express the change in a variable with respect to change in another variable. For example, a linear slope, expressed as a derivative, can be written dx/dt (change in x with respect to time t). Differential equation models are models that include one or more derivatives. Differential equation models can be used to express the moment-to-moment changes that are occurring in a system of variables. Through integration, one can add up all of the moment-to-moment changes expressed in a differential equation model that occur from some time (t_i) to a later time ($t_i + \Delta t_i$).

Many functional forms could be specified to describe a mediation process. In examining the CLPM, one observes that variables are regressed both on themselves (*autoregressive effects*) and on other key variables (*cross-lag effects*). This is important, as the variance in M (t_i) explainable by X ($t_i - \Delta t_i$) logically is limited to the variability of M (t_i) that is not stable over time. By controlling for M ($t_i - \Delta t_i$) the stable variance is removed, leaving only the volatility in M (t_i) to be explained by predictors assessed at a previous occasion. The same logic follows for Y. Stated another way, the changes in M (with respect to time) $\frac{dM(t)}{dt}$ are related to the level of X (plus measurement error $\varepsilon(t)$). In matrix notation, the variables X, M, and Y can be combined into a matrix $\boldsymbol{x}$, with each column corresponding to one of the variables. Using matrix notation, one can express that the change in one or more variables with respect to time is related to one or more variables plus measurement error using the differential equation model:

$$\frac{dx(t)}{dt} = \boldsymbol{Ax}(t) + \boldsymbol{\varepsilon}(t). \tag{1}$$

In Equation 1, $\boldsymbol{A}$ represents the relationships of variables with themselves (*autoeffects*), and with other variables (*cross-effects*). Integration of this equation would allow one to sum all of the changes expected on each variable from some time to some later time.

To integrate Equation 1, a continuous-time stochastic process must be specified in place of $\boldsymbol{\varepsilon}(t)$. One commonly used process is the Wiener process (also called Brownian motion), $\frac{dW(t)}{dt}$. The Wiener process produces errors that are mutually independent, and the increments between two occasions are stationary[1] and normally distributed (Arnold, 1974; Gardiner, 2009). The variance of the Wiener process depends on the time over which it is integrated (i.e., the lag), such that the accumulation of errors over longer periods of time produces larger variances. Premultiplication by the constant $\boldsymbol{G}$, which is a matrix of estimated parameters, allows the variance to be scaled to the variable of interest. Incorporating the Wiener process into Equation 1:

$$\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{Ax}(t) + \boldsymbol{G}\frac{d\boldsymbol{W}(t)}{dt} \tag{2}$$

[1] In this context, *stationary* refers to the definition commonly used in the time series literature. A stationary process has a joint probability distribution that is constant. Consequently, the expected values of the mean and variance are constant over time and finite. Furthermore, the covariance between lagged observations is also constant and finite, which means the covariance between lagged observations can depend on the lag between observations but not on time. The first-order differential equation model presented inherits these assumptions, unless nonstationary components are explicitly modeled (e.g., continuous time autoregressive latent trajectory model; Delsing & Oud, 2008).

Through stochastic integration—integration of differential equations that include a stochastic process—one can solve for exact relations between Equation 2 and the equation

$$\boldsymbol{x}(t_i) = \boldsymbol{A}(\Delta t_i)\,\boldsymbol{x}(t_i - \Delta t_i) + \boldsymbol{w}(\Delta t_i)\,. \quad (3)$$

This is the matrix form of a CLPM, where $\boldsymbol{x}(t_i)$ and $\boldsymbol{x}(t_i - \Delta t_i)$ are the values on one or more variables at time (t_i) and some previous time $(t_i - \Delta t_i)$, $\boldsymbol{A}(\Delta t_i)$ is a matrix of autoregressive and cross-lag effects (the magnitudes of which depend on the lag Δt_i), and $\boldsymbol{w}(\Delta t_i)$, which consists of independent, normally distributed errors (which also depend on the lag).

The CTM autoeffects and cross-effects $\boldsymbol{A}$ are related to the autoregressive and cross-lag effects in $\boldsymbol{A}(\Delta t_i)$ through the equation:

$$\boldsymbol{A}(\Delta t_i) = \boldsymbol{e}^{\boldsymbol{A}\times\Delta t_i}. \quad (4)$$

This states that the matrix of autoregressive and cross-lag effects, for a particular lag Δt_i, is equal to the exponential of the product of the continuous time autoeffects and cross-effects matrix and the lag. Likewise, the continuous time parameters allow for calculation of the expected error covariance matrix for a particular lag. The covariance matrix can be written

$$\begin{gathered} irow\left\{\boldsymbol{A}_{\#}^{-1}\left[\boldsymbol{e}^{\boldsymbol{A}_{\#}\times(\Delta t_i)} - \boldsymbol{I}\right] row\left(\boldsymbol{GG}'\right)\right\},\ \text{and} \\ \boldsymbol{A}_{\#} = \boldsymbol{A}\otimes\boldsymbol{I} + \boldsymbol{I}\otimes\boldsymbol{A} \end{gathered} \quad (5)$$

where *row* and *irow* are operations to place a matrix row-wise into a column vector and the inverse of this operation, and ⊗ represents the Kronecker product. These equations have been presented in several domains (e.g., Bergstrom, 1988, 1990; Björk, 2009; Langevin, 1908; Phillies, 2000; van Kampen, 2007), but for readers in the social sciences interested in further explanation, particularly regarding the error covariance matrix, we refer them to Oud and Jansen (2000), Oud (2007), and Voelkle et al. (2012).

Taken together, Equations 2 through 5 constitute an exact discrete model (EDM)—a mathematically exact connection between a continuous first-order differential equation model and a discrete time autoregressive model (Bergstrom, 1988). Fitting the CTM in Equation 2 to data involves using Equation 4 to calculate the expected relations between subsequent observations in Equation 3 given the lag between observations. In SEM software these relations, combined with the error covariance matrix in Equation 5, can be used to calculate an expected covariance matrix to compare to an observed covariance matrix. Code implementing this model is available (e.g., Oud & Folmer, 2011; Voelkle et al., 2012).

Interpretation of Continuous Time Parameters

Because the interpretations of the continuous time parameters differ from those of discrete time parameters, we present several ways to conceptualize continuous time parameters, with particular attention to the context of mediation analyses. Rewriting Equation 4,

$$\boldsymbol{A} = \ln\left(\boldsymbol{A}(\Delta t_i)\right)/(\Delta t_i)\,. \quad (6)$$

That is, the continuous time parameters are equal to the matrix-natural-log of the discrete time parameters divided by lag.[2] Considering first a single variable (i.e., both $\boldsymbol{A}$ and $\boldsymbol{A}(\Delta t_i)$ are 1×1) with a discrete time autoregressive parameter of 0.9 and lag 1, the equivalent continuous time parameter would be $\ln(0.9)/1 = -0.105$. Calculating several values, one finds that as the discrete time autoregressive effect approaches 1, the continuous time autoeffect approaches zero; as the autoregressive effect approaches zero, the autoeffect approaches negative infinity. The interpretation of the cross-effects is more challenging than that of the autoeffects. If one calculates the exponential of a 2×2 (or larger) **A** matrix times lag, one observes that the discrete cross-lag effects will depend both on the relations between variables (cross-effects) and how highly variables relate to themselves (autoeffects).

There is one case when the off-diagonal element of $\boldsymbol{A}$ and $\boldsymbol{A}(\Delta t_i)$ will be equivalent. This occurs when the lag between measurements is 1, and (a) there is a one-directional relationship between two variables (e.g., X → Y), with no reverse relationship (e.g., Y → X) and no indirect effects (e.g., no X → M → Y); and (b) the diagonal elements of **A** equal 0; that is, the diagonal elements of $\boldsymbol{A}(\Delta t_i)$ equal 1.

As the diagonal elements of $\boldsymbol{A}(\Delta t_i)$ depart from a value of 1, the off-diagonal element of $\boldsymbol{A}(\Delta t_i)$ (cross-lag) at a lag of 1 will be closer to zero than the cross-effect in **A**. Consequently, when considering only the effect of one variable on another, off-diagonal elements of **A** follow the directions one expects: Positive effects suggest that moment-to-moment changes from one variable to another variable are positively related; the reverse is true for negative effects. Off-diagonal elements in **A** equal to zero suggest that the moment-to-moment changes in one variable are not related to the moment-to-moment changes in another variable—there is no effect that is accumulating over time. With more than one relationship (e.g., both direct and indirect effects), one must consider the contributions of all effects at every moment in time.

We now present several tools to aid in the interpretation of cross-effects: a graphical interpretation, an equation for the lag of maximal effect, and an equation for the maximal

[2]Equation 6 is provided as a method for understanding the values of **A**; in practice taking the logarithm of a matrix can be problematic as the solutions are not always unique (Higham, 2008).

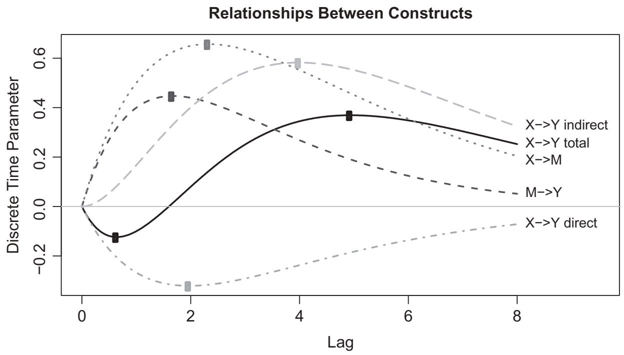

FIGURE 2 This figure plots the estimated discrete time parameters for the X → M, M → Y, total X → Y effect, indirect X → Y effect, and direct X → Y effect as a function of lag. All of these values are derived from a single continuous time matrix and Equation 3. Squares indicate the point at which the parameter is maximal or minimal for each relationship. This figure is based on the continuous time parameters shown in Equation 9.

discrete time parameter (at any lag). The graphical interpretation involves using Equation 4 to compute $\boldsymbol{A}\,(\Delta t_i)$ for a range of lags (i.e., using **A** and a range of values for Δt_i). An example is shown in Figure 2, which depicts the expected discrete time relationship, based on the continuous time matrix, as a function of lag; R code for producing such a plot for a mediation model is provided in online Appendix A.[3] Many features of this representation match theoretical expectations; for example, the effects of one variable on another take some time to accumulate (i.e., effects increase as lag increases from zero), but if one uses too large a lag the effects of one variable on another diminish (i.e., decreasing discrete time parameter for long lags). In addition, variables might differ in the lag at which an effect is maximal or minimal, as would be expected from the mediation literature (e.g., Cole & Maxwell, 2003). If we wish to identify the maximal relationship between a pair of variables, one can differentiate Equation 4 and solve for the point at which the first derivative equals zero. As shown in online Appendix B, the lag of maximal effect, Δt_{max}, is equal to[4]

$$\Delta t_{max} = -\ln\left(a_{ii}/a_{jj}\right) / \left(a_{ii} - a_{jj}\right). \tag{7}$$

In this equation a is used to represent elements from the continuous time matrix **A**, i and j correspond to the row and column of the effect of interest (predictor and outcome, respectively), and Δt_{max} is the lag at which the ij cross-lag effect is maximal. This lag is a function of the autoeffects of the two variables involved. Naturally, caution is warranted if the lag of maximal effect is much larger than the range of time over which data were collected or much shorter than the lag between the two most closely spaced observations. A related measure is the maximum discrete time parameter—the value of the discrete time relation $\boldsymbol{a_{ij}}\,(\Delta t_{max})$. For the cross-lag effect in row i and column j this parameter equals (see online Appendix B)

$$\boldsymbol{a_{ij}}\,(\Delta t_{max}) = \frac{a_{ij}\left(\frac{a_{jj}}{a_{ii}}\right)^{-\frac{a_{ii}}{a_{jj}-a_{ii}}}}{a_{jj}}. \tag{8}$$

This parameter is a function of the autoeffects as well as the continuous relation a_{ij}.

Calculating the Direct and Indirect Effects

We have not provided equations for calculating the maximal value of the indirect and direct effects, or the lag at which they occur, as the solutions for models with three variables are not as straightforward as with a pair of variables. Given values for the matrix **A**, however, numerical methods can be used to find such values.[5] Equation 4 can be used to compute the discrete time parameters for a specific lag given **A**. It is important to note that the elements of the resulting discrete time matrix correspond to the total effects. Consider, for example:

$$\boldsymbol{A} = \begin{bmatrix} -0.357 & 0 & 0 \\ 0.771 & -0.511 & 0 \\ -0.450 & 0.729 & -0.693 \end{bmatrix}. \tag{9}$$

In this matrix, –0.357, –0.511, and –0.693 are the continuous time autoeffects for X, M, and Y. The value –0.450 represents a negative direct effect of X → Y, whereas 0.771 and 0.729 represent positive effects of X → M and M → Y. Solving Equation 4 for the discrete time matrix at lag 1:

$$\boldsymbol{A}\,(\Delta t_i) = \begin{bmatrix} 0.700 & 0 & 0 \\ 0.500 & 0.600 & 0 \\ -0.100 & 0.400 & 0.500 \end{bmatrix}. \tag{10}$$

In this matrix, the –0.100 value is the total effect (i.e., the sum of the direct and indirect effects) of X → Y when $\Delta t_i = 1$. To obtain the indirect effect, we must set the direct effect in **A** to equal zero before solving for the discrete time parameters. That is,

[3]Online appendices are available online through both authors' Web sites.

[4]As the derivation of this equation depends on the calculation of the natural log of a matrix (see Footnote 2), we encourage users of this formula to compare results with the previous graphical interpretation to ensure a valid result.

[5]Online Appendix A provides estimates of the maximal direct and indirect effects and the lags at which they occur.

$$A = \begin{bmatrix} -0.357 & 0 & 0 \\ 0.771 & -0.511 & 0 \\ 0 & 0.729 & -0.693 \end{bmatrix}, \text{which produces}$$

$$A(\Delta t_i) = \begin{bmatrix} 0.700 & 0 & 0 \\ 0.500 & 0.600 & 0 \\ 0.167 & 0.400 & 0.500 \end{bmatrix}. \quad (11)$$

The 0.167 in the discrete time matrix, because the direct effect was set to zero, now represents the X → Y indirect effect, for a lag of 1 unit of time. If we wish to compute the direct effect, the effects of X on M and M on Y must be set equal to zero:

$$A = \begin{bmatrix} -0.357 & 0 & 0 \\ 0 & -0.511 & 0 \\ -0.450 & 0 & -0.693 \end{bmatrix}, \text{which produces}$$

$$A(\Delta t_i) = \begin{bmatrix} 0.700 & 0 & 0 \\ 0 & 0.600 & 0 \\ -0.267 & 0 & 0.500 \end{bmatrix}.$$

The direct effect –0.267 plus the indirect effect 0.167 is equal to the total X → Y effect –0.100. It should be noted that the discrete time value –0.100, which is often interpreted as the direct effect of X on Y controlling for the previous values of M and Y, in fact represents a combination of indirect and direct effects; this is a consequence of the CTM's assumption that X continues to affect M, which in turn affects Y, in the interim between observations. It should also be kept in mind that all of these values are specific to a lag of 1 and would differ for other lags. This is the process used to calculate the direct and indirect effects shown in Figure 2.

What Could Be Gained?

We have discussed the application of a CTM for assessing mediation and presented methods for interpreting continuous time parameters. The following sections present the results of a simulation comparing the EDM to two common specifications of the CLPM. Subsequently, a substantive example is presented in which we reanalyze a published covariance matrix. These presentations highlight that CTMs do not change the results observed from discrete time analyses for a particular lag. Rather, CTMs have the potential to provide additional information about how relations change over lags other than the lag used in the data collection.

SIMULATION: COMPARISON OF DISCRETE AND CONTINUOUS TIME MODELS

In this example, data with known characteristics are generated and used to produce two data sets with differing lags. The data are analyzed using two common discrete time analyses: (a) using a CLPM where the direct effect of X_t is on Y_{t+1} ($CLPM_A$; Figure 1a), and (b) using a CLPM where the direct effect of X_t is on Y_{t+2} ($CLPM_B$; Figure 1b). Both CLPMs are presented, as they appear to be similarly prevalent in applications of longitudinal mediation analysis. For both CLPMs, the discrete and CTMs are used to analyze the same data.

Method

Cross-lag panel data were simulated using M*plus* 6.1 (Muthén & Muthén, 1998–2010). The CLPM models corresponded to the models shown in Figure 1a ($CLPM_A$) and Figure 1b ($CLPM_B$). The parameter values in Figure 1 correspond to the population values used to generate the data; the specific values are inconsequential. For both models a single data set was generated with 10,000 subjects with seven measurements on each variable separated by a lag of 1; a large sample was used to approximate asymptotic results. To vary the lag, the data sets were downsampled such that every other observation was analyzed; that is, observations were separated by a lag of 2. This scenario parallels a substantive situation in which two researchers are assessing children from ages 4 to 10, but one researcher collects annual measurements and the other collects measurements in alternating years.

The two discrete time CLPMs and continuous time EDM were fit to both the lag 1 (seven observations) and lag 2 (four observations) data sets. For the CLPM, the model corresponding to the population model was fit. For the EDM the model described in the introduction, with effects from X → M, M → Y, and X → Y, was run for comparison to the $CLPM_A$ and $CLPM_B$ results. Error variances for X, M, and Y were constrained to be equal after the initial observation for both the CLPM and EDM, although this is not necessary for either model. The correlations between the errors, beyond the initial observation, were set to zero for both the CLPM and the EDM. Profile likelihood confidence intervals were generated.

Results: $CLPM_A$

The first section of Table 1 reports the population values, the parameters estimated by the lag 1 and lag 2 CLPMs, and confidence intervals for the parameters. As expected, the discrete time lag 1 model accurately recovers the population values. The lag 2 results differ from the lag 1 results in an expected manner; for example, for the X_{t-1} to X_t effect, the lag 2 estimate (0.490) equals the lag 1 estimate squared ($0.700^2 = 0.490$; Bollen, 1987, 1989). The final row of the discrete time analyses presents a paradox, as the effect of X → Y is negative in the lag 1 analyses and positive in the lag 2 analyses. This difference is a result that could be common with discrete time analyses (Oud, 2007); researchers using the same model, examining children over the same time span, could

TABLE 1
Simulation Results for the $CLPM_A$ and $CLPM_B$

Parameter	*Population Values*	*Lag 1*		*Lag 2*	
		Estimate	*95% CI*	*Estimate*	*95% CI*
Discrete time cross-lagged panel model results $CLPM_A$, data from Model A (Figure 1)					
X_{t-1} to X_t	0.700	0.700	[0.697, 0.704]	0.490	[0.484, 0.496]
M_{t-1} to M_t	0.600	0.600	[0.598, 0.602]	0.359	[0.354, 0.365]
Y_{t-1} to Y_t	0.500	0.499	[0.499, 0.501]	0.248	[0.242, 0.251]
X_{t-1} to M_t	0.500	0.500	[0.495, 0.504]	0.651	[0.647, 0.658]
M_{t-1} to Y_t	0.400	0.397	[0.397, 0.398]	0.439	[0.434, 0.444]
X_{t-1} to Y_t	−0.100	−0.098	[−0.100, −0.097]	0.077	[0.075, 0.083]
Continuous time cross-lagged panel model results $CLPM_A$, data from Model A (Figure 1)					
X → X	−0.357	−0.359	[−0.365, −0.353]	−0.357	[−0.362, −0.352]
M → M	−0.511	−0.511	[−0.518, −0.504]	−0.512	[−0.520, −0.506]
Y → Y	−0.693	−0.696	[−0.698, −0.687]	−0.698	[−0.707, −0.687]
X → M	0.771	0.770	[0.760, 0.779]	0.773	[0.760, 0.785]
M → Y	0.729	0.724	[0.714, 0.735]	0.732	[0.717, 0.748]
X → Y	−0.450	−0.446	[−0.457, −0.435]	−0.457	[−0.477, −0.438]
Discrete time cross-lagged panel model results $CLPM_B$, data from Model B (Figure 1)					
X_{t-1} to X_t	0.450	0.449	[0.445, 0.452]	0.199	[0.196, 0.204]
M_{t-1} to M_t	0.500	0.503	[0.501, 0.504]	0.257	[0.257, 0.259]
Y_{t-1} to Y_t	0.800	0.800	[0.798, 0.801]	0.654	[0.649, 0.656]
X_{t-1} to M_t	0.500	0.501	[0.500, 0.506]	0.479	[0.474, 0.486]
M_{t-1} to Y_t	0.400	0.401	[0.400, 0.404]	0.603	[0.596, 0.612]
X_{t-1} to Y_t	0.200	0.198	[0.193, 0.203]	0.094	[0.084, 0.105]
Continuous time cross-lagged panel model results $CLPM_B$, data from Model B (Figure 1)					
X → X	**	−0.794	[−0.806, −0.783]	−0.808	[−0.822, −0.793]
M → M	**	−0.684	[−0.692, −0.675]	−0.678	[−0.690, −0.667]
Y → Y	**	−0.222	[−0.227, −0.217]	−0.216	[−0.217, −0.213]
X → M	**	1.045	[1.031, 1.060]	1.057	[1.050, 1.084]
M → Y	**	0.731	[0.722, 0.740]	0.653	[0.641, 0.665]
X → Y	**	−0.283	[−0.295, −0.270]	−0.071	[−0.091, −0.059]

Note. Population values for discrete time models are based on a time lag of 1. Population continuous time values were calculated using Equation 3. **Population continuous time values are not reported for $CLPM_B$, and the exact discrete model are equivalent.

come to opposite conclusions about relationships between variables merely due to differing lags.

The second section of Table 1 reports the population values, the parameters estimated by the EDM analysis of the lag 1 and lag 2 data, and confidence intervals for the parameters. The EDM recovers the same continuous time population values, regardless of whether the lag 1 or lag 2 data are used. This is because the EDM estimates a continuous time parameter that is independent of lag.

The results for the EDMs and CLPMs closely correspond to each other. Note that in this model, the X → Y effect does not correspond to a direct effect, but rather to a combination of direct and indirect effects, as expected by the discrete time mediation literature (MacKinnon, Warsi, & Dwyer, 1995). To further understand this point, it might be helpful to note that the matrices in Equations 9 and 10 contain values corresponding to this simulation; the calculation of the total, indirect, and direct effects of X on Y are demonstrated in the introduction. Further evidence of the equivalence of these models is clear in the χ^2 values and degrees of freedom of the CLPM and EDM, which are equal within each of the lag conditions (–2LL = 180866.1, df = 209964 for lag 1; –2LL = 169276.9, df = 119973 for lag 2). Although it might be expected that the EDM would require more parameters than the CLPM, this is not the case.

Results: $CLPM_B$

The final two sections in Table 1 report the population values, the estimated values, and confidence intervals for the lag 1 and lag 2 data for the CLPM and the EDM, respectively. As with the previous model, the estimates for the CLPM match the population values (for lag 1). It is apparent that the continuous and discrete time models do not match

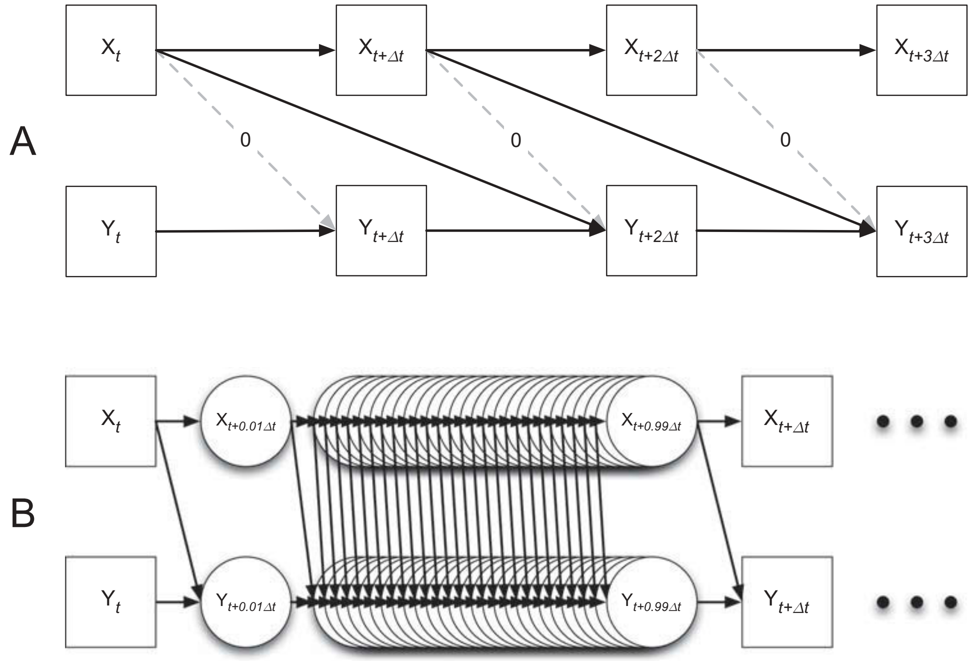

FIGURE 3 (a) The $CLPM_B$ posits a relationship between X and Y, such that there is no relationship over a lag of Δt_i (gray dashed lines), but that there is a relationship over a lag of $2\Delta t_i$ (black cross-lags). (b) The exact discrete model posits that variables affect each other at many unobserved times (circles) between measurement occasions (squares), so there is an effect of X on Y at every possible lag—even between measurement occasions.

in this case. Moreover, the EDM does not yield equivalent estimates for the lag 1 and lag 2 data; this is most apparent in the mismatch of the results for the parameters M $\rightarrow$ Y and X $\rightarrow$ Y. Further evidence of the nonequivalence of these models is clear in the χ^2 values, which differ despite equivalent degrees of freedom: $-2LL_{CLPM}(df = 209964) = 181842.0$, $-2LL_{EDM}(df = 209964) = 186033.6$ for lag 1; $-2LL_{CLPM}(df = 119973) = 177310.7$, $-2LL_{EDM}(df = 119973) = 168403.1$ for lag 2.

Figure 3 gives a visual interpretation of the mismatch between the $CLPM_B$ data that were generated and the EDM; to simplify the presentation Figure 3 focuses only on the direct effect. In the $CLPM_B$, there is an effect from X_t to $Y_{t+2\Delta t}$ (Figure 3a). One key aspect to note is that in this model the paths from X_t to $Y_{t+\Delta t}$ are constrained to equal zero; this means that in the $CLPM_B$ there is absolutely no direct relationship between X_t and Y_{t+1} expected, but that the effect of X_t skips over $Y_{t+\Delta t}$ to $Y_{t+2\Delta t}$. The EDM that was fit does not allow effects to skip over a period of time, and instead assumes that there is continuous interaction between the variables at many unobserved times (Figure 3b). The $CLPM_B$ posits a discontinuity in how the variables affect each other: There is no effect of X_t on Y between t and $t + \Delta t$, but there is an effect between $t + \Delta t$ and $t + 2\Delta t$. Although widely used, the CLPM in Figure 1b is not equivalent to the three-variable first-order differential equation model described in the introduction.

Discussion

In this simulation we examined two common specifications of the CLPM for longitudinal mediation analyses and the estimates produced by the EDM. The results for the $CLPM_A$ demonstrate equivalence between the discrete time CLPM and the continuous time EDM. In the $CLPM_A$ simulation, the discrete time results were valid for only a specific lag, whereas the EDM results could be used to compute the discrete time results for many possible lags. Although it provides related information, the discrete time model acts like a microscope focusing the researcher's attention on a particular lag, rather than giving a more macroscopic understanding of the underlying process. Although one could focus on locating the lag of maximal effect, even in this simple example there is no single lag at which all relationships are maximal simultaneously (Hoyle & Kenny, 1999; Shrout & Bolger, 2002). The issue is further complicated by relations such as the total effect of X on Y, which is negative at short lags, and positive at longer lags.

The results from the $CLPM_B$ do not suggest equivalence to the CTM presented. Although the $CLPM_B$ is commonly used, and the intent of the lag-2 X $\rightarrow$ Y relationship in the $CLPM_B$ is to clarify the direct and indirect effects, it posits a discontinuity between X and Y that is not easily reconciled with a model where variables continuously interact over time. The $CLPM_B$ might not be realistic, as it assumes that the effect of X on Y is exactly zero at a specific lag that exactly corresponds to the selected sampling rate.

The continuous time EDM can be used to provide the same information as a discrete time model. It can also provide a more macroscopic view of how the relationships between variables vary with lag. Whereas researchers are often subject to constraints that impose specific lags on data collection, the models fit to these data do not have to be similarly constrained.

COMPARISON OF APPROACHES: A SIMULATION BASED ON SUBSTANTIVE DATA

In this section we reanalyze data from a published longitudinal mediation study. We purposefully selected published mean, standard deviation, and correlation information to highlight that CTMs do not require special data above and beyond the kinds of data that are already being collected. We have generally refrained from commenting on the theoretical implications of the analyses performed; this exercise serves only to highlight differences in model interpretation.

Method

The substantive example is based on Grundy, Gondoli, and Blodgett-Salafia (2007). In the words of the authors, "The present study considered whether maternal knowledge mediated the relation between overt marital conflict and preadolescent behavioral competence" (p. 675). Mothers and preadolescents (133 pairs) completed measures annually for 4 years. Three measures are examined: mother-reported maternal conflict (conflict; O'Leary–Porter Scale; Porter & O'Leary, 1980), preadolescent-reported maternal knowledge (knowledge; based on scales used in Brown, Mounts, Lamborn, & Steinberg, 1993; Fauber, Forehand, Thomas, & Wierson, 1990), and preadolescent-reported behavioral competence (competence; Behavioral Conduct subscale of the Self-Perception Profile for Children; Harter, 1985). The sample of preadolescents was 55% female and primarily White (95.5%), with an average mother's age of 37.8 years (*SD* = 4.02), and typically from well-educated, middle-class participants (median annual household income = $69,000, mother's average years of postsecondary education = 3.4 years).

Grundy et al. (2007) presented a CLPM where conflict predicts knowledge, which in turn predicts competence. A reciprocal effect from competence to knowledge is also included, a departure from the traditional longitudinal mediation model. The autoregressive effects between variables are also included. The cross-lagged effects were allowed to vary between each pair of occasions. The direct effect of conflict on competence was removed from the final model, as it was not significant. Grundy et al. included only participants with complete data.

Two models were fit to the data: (a) the CLPM model fit by Grundy et al. (2007) including the direct effect of conflict → competence, the indirect effect of conflict → knowledge → competence, and the reciprocal effect of competence → knowledge; and (b) an EDM with equivalent cross-effects. Unlike Grundy et al. (2007), we constrained the relationships between observations to be constant over time, as well as the the error variances after the first observation; this is not a requirement of either model.[6] Covariances between the initial observations were estimated.

For illustrative purposes, we provide bootstrap confidence intervals. To obtain bootstrap confidence intervals the full data set is required, rather than only summary statistics. Making an assumption of multivariate normality, which is assumed when using SEM with maximum likelihood estimation, we generated a data set consisting of 133 observations with exactly the same mean and covariance structure as the data described by Grundy et al. (2007) using the mvrnorm() function in the statistical program R (R Development Core Team, 2013).

Results and Discussion

The model fit and degrees of freedom were the same for both the CLPM and EDM, as was the case in the first simulation (–2LL = 8209.742, *df* = 1,596). The results of the parameter estimates for the CLPM model are shown in Table 2. These results suggest that conflict affects knowledge (X → M) and that there is support for feedback from competence to knowledge (Y → M). Support was not found for knowledge affecting competence (M → Y) nor the direct effect of conflict on competence (X → Y). It should be highlighted that the CLPM results represent the total effects for a particular lag. For example, the effect of conflict on competence (X → Y) is a total effect; we cannot disentangle how much of this effect is due to the direct effect (X → Y) or the indirect effects (X → M → Y, X → M → Y → M → Y). The CLPM model has provided a snapshot of the total effect for a particular lag, but decomposing the total effects, and deriving how the effects differ for other lags, is not possible with these results.

Table 2 presents parameter estimates for the EDM **A** matrix. Unlike the CLPM estimates, these values do not represent total effects, but the direct effects of variables on each other, independent of lag. Based on bootstrap confidence intervals, there is support for a negative direct effect from conflict to knowledge (X → M), and a positive direct effect

[6]Often in implementation of the CLPM, parameters are allowed to change suddenly at exactly the times at which observations have been made; this creates a step-like function of the parameters between pairs of measurement occasions. This would be an unusual hypothesis for a continuous time model. The EDM can be extended, however, to allow the relationships between variables and the error variance to change as a function of time; for example, one could allow the parameters in **A** to change linearly with time (Oud & Jansen, 2000).

TABLE 2
Discrete and Continuous Time Analysis of Data Simulated from Grundy et al. (2007)

	Discrete Time Lag 1 Year		*Continuous Time*	
Parameter	*Estimate*	*95% CI*	*Estimate*	*95% CI*
Conflict to knowledge (X → M)	−0.084	[−0.145, −0.024]	−0.124	[−0.268, −0.029]
Knowledge to competence (M → Y)	0.054	[−0.020, 0.129]	0.115	[−0.042, 0.753]
Conflict to competence (X → Y)	−0.042	[−0.088, 0.004]	−0.057	[−0.133, 0.027]
Competence to knowledge (Y → M)	0.201	[0.110, 0.304]	0.434	[0.224, 1.607]
Autoregressive conflict (X)	0.871	[0.815, 0.916]	−0.138	[−0.200, −0.091]
Autoregressive knowledge (M)	0.432	[0.343, 0.513]	−0.865	[−2.143, −0.683]
Autoregressive competence (Y)	0.512	[0.432, 0.587]	−0.693	[−1.360, −0.553]

from competence to knowledge (Y → M). Determining whether X or Y has a larger effect on M is difficult because conflict, knowledge, and competence differ substantially in their autoregressive components.

To understand how the effects of variables change in a dynamic way over time, Equation 4 was solved for many possible lags to create Figure 4. This gives some insight into how the effects would have appeared had the data been collected with differing lags. As highlighted in the example in the introduction, using the same **A** matrix one can solve for the total, direct, or indirect effects of variables on each other; for Figure 4 each effect was solved for individually (other effects set to zero) so that we could solve for each of the direct effects rather than the total effects. As expected based on the continuous time parameters, there is a negative effect of conflict on knowledge (X → M) and a positive direct effect from competence to knowledge (Y → M) across many possible lags; the other effects have confidence intervals that contain zero.

The results suggest that this study might have been well designed to capture the maximal effect of some pairs of variables, but not others. The effects of knowledge on competence (M → Y) and competence on knowledge (Y → M) are expected to approach their maxima near lags of 1.1 and 1.2 years, respectively; nearly equivalent to the lag of 1 year used to collect the data. The effects of conflict on knowledge (X → M) and conflict on competence (X →Y) are not expected to reach their maximal effects until lags of 2.9 and 3.7 years; the results suggest it takes longer for the effects of marital conflict on adolescent competence and knowledge to accumulate than the other effects examined. The indirect effect of conflict to competence via knowledge (X → M → Y) does not reach its maximum within a lag of 4 years. As the study spanned 3 years, we should be cautious about drawing conclusions about lags that exceed the span of the study. The data, however, might suggest that researchers interested in the effect of conflict on knowledge (X → M), conflict on competence (X → Y), and conflict on competence via knowledge (X → M → Y) should consider studies longer than the 3-year span used in this study.

A few more general principles of implementing EDM can be drawn from this example. In considering the maximal effects, we have highlighted that whereas the EDM can be used to calculate discrete time parameters for any possible lag, lags that are much shorter than the smallest interval or much longer than the time span covered by one's data should be considered carefully; this recommendation is similar to the common admonition to be cautious when drawing conclusions outside of the range of observed values. It should also be noted that the EDM was fit to the same data as the CLPM, 133 mother–preadolescent dyads at four time points; whereas the EDM might appear more complex than the CLPM, fundamentally it is a different way of looking at the same data typically analyzed using a CLPM. We are unaware of any EDM papers that give specific guidance about power and sample size, but it is reasonable to expect that the data requirements do not appreciably differ from those for the CLPM; that is, with the exception that EDM can more easily handle situations such as unequally spaced observations.

One general point of interest to consider is the role of effect sizes when different variables might take less or more time to exert a cumulative effect. The idea of an effect size might be inappropriate, as the effect of one variable could change with time (Deboeck, Montpetit, Bergeman, & Boker, 2009; Oud, 2007). Depending on context, one might be interested in the maximum effect of one variable on another, or the effect at a particular time. A teacher preparing for national exams next year will want to know what methods will lead to the largest effect at a lag of 1 year. However, a teacher hoping to instill a love of learning might prefer methods that have the maximal effect in the long run, even if that effect might take a long time to accumulate.

GENERAL DISCUSSION

We have presented a first-order differential equation, a CTM, and compared it with commonly used, discrete time CLPMs, with specific consideration of mediation analyses. CTMs treat variables as if they continuously interact over time,

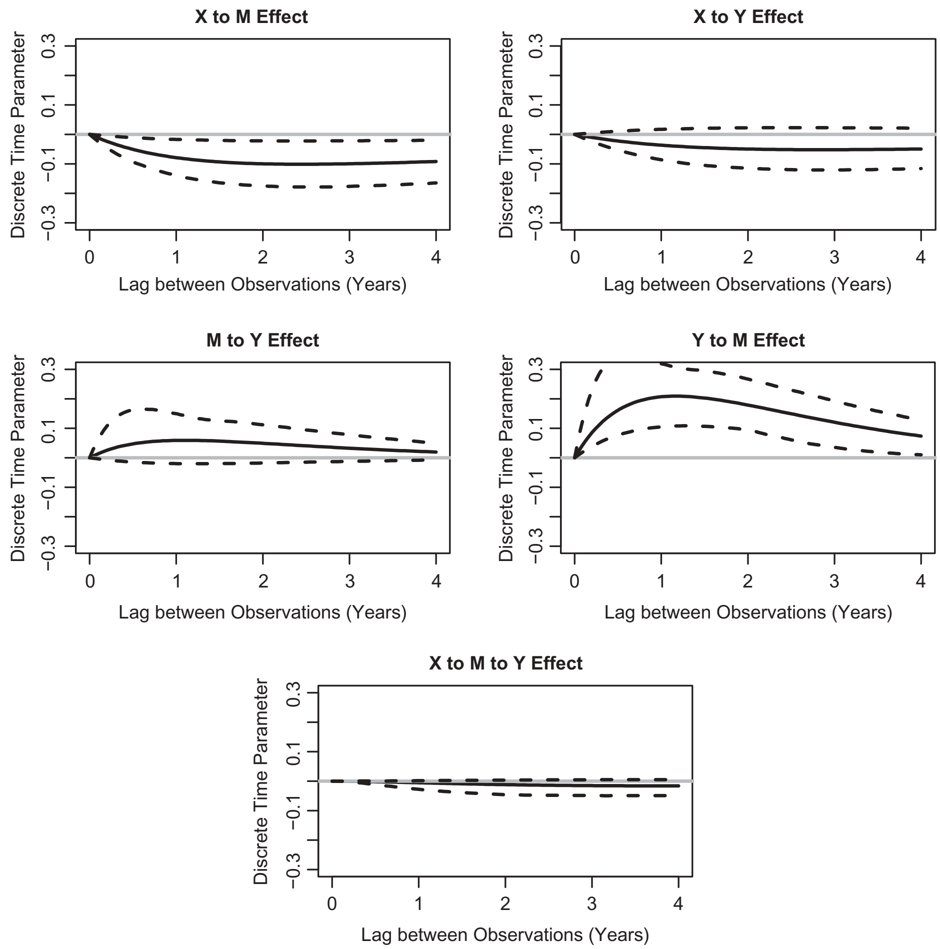

FIGURE 4 This plot depicts the estimated discrete time parameters as a function of lag, based on the data from the applied example. The estimated discrete time parameters (solid lines) and 95% bootstrap confidence intervals (dashed lines) are shown for each effect. The variables X, M, and Y represent conflict, knowledge, and competence. Note that these are plots only of the effects listed, not total effects; for example, the X → Y effect represents only the direct effect of X on Y, but not the indirect effects such as X → M → Y, X → Y → M → Y, or many other combinations of paths that could result in an X to Y relationship.

whereas discrete time models do not and consequently the results, including model selection and parameters, are limited to the specific lag between observations. CTMs have the potential to advance mediation analyses by providing a better match to theory, lag independent parameter estimates, and ways to handle data measured at irregular intervals.

A simulation and a substantive example were used to demonstrate the relations between the CTM and the CLPM. It was shown that there is an equivalence (i.e., same degrees of freedom, same model fit) between the EDM for a first-order stochastic differential equation model with errors described by the Weiner process and what was denoted $CLPM_A$, a cross-lagged panel model where all cross-effects occur between adjacent observations (i.e., Figure 1a). Thus, the EDM and $CLPM_A$ are examples of equivalent models in the sense described in the SEM literature (e.g., Lee & Hershberger, 1990; MacCallum, Wegener, Uchino, & Fabrigar, 1993; Stelzl, 1986). In this literature, equivalent models often represent incompatible or confounding causal explanations for the same observed data (e.g., X → M → Y and Y → M → X are equivalent yet causally incompatible mediation models).[7] The parameters estimated in the EDM and $CLPM_A$ were compared to highlight that these models are not proposing alternative explanations of the same data. Rather, these models can be thought of as expressing the same underlying model in two different manners, as is commonly done with reparameterizations of the same model (e.g., Cudeck & du Toit, 2002; Preacher & Hancock, 2012).

As with reparameterizations of other models, there is a difference in the interpretation of the parameters from the EDM and $CLPM_A$. Whereas the EDM parameters are potentially more challenging to interpret because familiarity with

[7]It is the authors' perspective that the EDM and $CLPM_A$ do not represent incompatible or confounding causal explanations for the same observed data, which is a common consideration in discussions of equivalent models in SEM. Other individuals familiar with equivalent models do not share the authors' viewpoint.

CTMs is not commonplace, the simulation and substantive example served to demonstrate the additional information CTMs can provide. In the examples, the CTM provided a macroscopic view of how the effect of one variable on another was expected to change as a function of lag. The CTM also showed how effects accumulate over time, that the rate at which effects accumulate might differ for different variables, and that effects might reach their maxima at different lags. Whereas the CLPMs provide a very static view of the values of parameters, the CTM models inherently consider the relationships between variables to be dynamic entities that will change depending on the lag over which they are observed. The substantive example showed how the same data that are typically required to fit a CLPM can be fit with a reparameterized model—the EDM—and how the EDM provides the potential to garner more information from the same data. The EDM and $CLPM_A$ therefore do not lie in opposition to each other as competing theoretical explanations; rather, they represent the same underlying theory. These models, however, might differ in the usefulness of their parameters.

CTMs for mediation analysis also could provide new insight into familiar concepts. One topic that warrants more discussion is the persistent distinction between partial mediation and complete (or full or perfect) mediation (Baron & Kenny, 1986; James & Brett, 1984). Mediation is said to be complete if the effect linking X and Y decreases to nonsignificance on the addition of M to the model, and partial if the effect is reduced, but not to nonsignificance. Thus, the distinction is based on p values of the X $\rightarrow$ Y effect before and after including M in the model. These qualifiers are used to convey the extent to which an effect has been explained by a mediator (and thus can be seen as crude measures of effect size), or as a stopping rule for deciding when an effect has been sufficiently explained by a set of mediators. This labeling of effects has been strongly criticized on logical and statistical grounds in recent literature (Preacher & Kelley, 2011; Rucker, Preacher, Tormala, & Petty, 2011). Viewing mediation through the lens of CTMs presents an additional reason to avoid using these terms to qualify mediation effects: The estimates and p values used to make the determination of whether a mediation effect is partial, complete, or absent are themselves "moving targets"; they change with lag when considered with discrete time models, as can be seen in the substantive example.

Limitations and Future Directions

The limitations to this presentation can be considered in three parts: the limitations to the current implementation of the first-order model discussed, the limitations imposed through selection of a first-order differential equation model, and limitation inherent to SEM.

Limitations of the current implementation. The model presented is relatively limited, in that it assumes no exogenous predictors (e.g., treatment implementation), no differences across subjects (i.e., intercept and slope random effects), use of observed variables only (i.e., no measurement model), and constant parameters (i.e., **A** does not vary as a function of time). A special case of EDM was selected to facilitate introduction of this model. More advanced models, including those with exogenous predictors, random effects, latent variables, and time-varying parameters have been presented in other papers (e.g., Oud, 2007, 2010; Oud & Jansen, 2000). Similarly, the simulation and substantive example held the time between observations constant and consistent for all individuals. The EDM is not limited to the analysis of such data and can be used in cases where the time between sequential observations differs and allows for data where there are individual differences in assessment timing (Voelkle & Oud, 2012).

Limitations of the first-order differential equation model. It is important to note that we began with the selection of a specific CTM—a first-order differential equation model with stochastic errors following a Wiener process. The relations between subsequent observations in the CLPM do not imply a specific functional form for changes over time—only that linear relations will be present between subsequent observations. In contrast, CTMs require the specification of a specific functional form that describes change over time, and many differing functional forms could be specified. Consider if one were examining the concentrations of chemicals in the bloodstream that vary with the day–night cycle, which in turn might affect behavior. In such a case, a model that allowed for the coupling of two oscillating functions, like two coupled pendula, might be more appropriate than the model presented here (e.g., Boker, Neale, & Rausch, 2004). Similarly, if one expects the functional form to describe a decay process, we might consider the mediation relations of an exponential decay model (Fritz, 2007), a first-order differential equation model where the stochastic errors are either absent or of relatively small magnitude. Both of these models also can be fit using the EDM (Oud & Folmer, 2011).

The CTM presented here might be a good first step for modeling many processes. As the changes in constructs are better understood, however, consideration of other models will be imperative. CTMs will require researchers to be increasingly specific about change processes. As we become more adept at modeling change, we will become better at selecting functional forms that will provide interesting, meaningful parameters. This might include specialized forms that are theoretically motivated, such as how the effect of a treatment on smoking cravings could be mediated by negative affect (Rivera et al., 2011; Trail, Timms, Piper, Rivera, & Collins, 2011) or the modeling of negative affect and stress (Deboeck & Bergeman, 2013). It is likely that to understand these functional forms, increased emphasis will need to be placed on the collection of intraindividual measurements, which could produce many benefits (Molenaar & Campbell, 2009).

Using the EDM

Admittedly, fitting the EDM is more complicated than fitting a CLPM, but resources are available to help with fitting the model presented in this article and other CTMs (e.g., Voelkle et al., 2012). Part of the challenge is that most SEM software is written such that it does not allow the user to perform matrix operations, which can make the application of the EDM challenging or impossible. The primary problem is in specifying Equations 4 and 5, which requires the ability to perform matrix operations while iteratively exploring parameters. Although there have been attempts to simplify the EDM through calculation of continuous time parameters from discrete time parameters (indirect method; Arminger, 1986) or linearizing Equations 4 and 5 (approximate discrete model; Oud, 2007, 2010), these approaches cannot be recommended over a wide range of conditions as can the EDM (Hamerle, Nagl, & Singer, 1991). As CTMs are embraced, however, further statistical innovations and software developments will aid in fitting these models (e.g., Singer, 2012). Newer methods will also allow for the fitting of stochastic differential equation models with nearly the exactness of analytic solutions, using common SEM conventions and without the need for stochastic calculus (Deboeck & Boulton, submitted).

One ongoing challenge with CTMs is the issue of parameter interpretation; few researchers are accustomed to interpreting continuous time parameters directly. We have introduced several ways that the continuous time parameters can be understood. To allow researchers to become more accustomed to interpretation of these parameters, several tools are provided to show the continuous time matrix **A** in terms of more familiar discrete time counterparts, including code to produce figures of discrete time parameters versus lag (online Appendix A) and equations for calculating the lag of maximal effect and the maximal discrete time parameter (regardless of lag). In providing these tools we want to discourage researchers from considering only the maximal and minimal effects, but instead encourage them to focus on understanding the relationships among variables as they evolve over time. There might be some benefits to considering the maximal and minimal effects when redesigning a study or when trying to express the maximal benefit of an intervention. But more generally, there is the need to acknowledge that relationships between variables are not static, but rather dynamic and that they require more nuanced description than simply "present or not" and "positive or negative." This is particularly true if one is considering the summation of two or more effects, as relationships might be present or not at differing time scales and could even change sign. When examining discrete time values calculated from a continuous time matrix, it is necessary to not reduce interpretation to a single value (e.g., the maximum).

CONCLUSION

This article addresses the elephant in the room when it comes to mediation analysis—the relationships among variables might change depending on lag. Much of the psychological literature is content to find relationships among variables, but less effort has been placed on defining the time over which such effects are likely to take place (Deboeck et al., 2009; Selig, 2009). We look for relationships among variables but little focus is placed on how long it takes for these effects to accumulate or dissipate. Statistical models often do little to help, often providing information about relationships for a very narrow time scale—a single lag. Continuous time methods have the potential to provide researchers a way to extract additional information, a more macroscopic view of the relationships between variables, using the same kind of data that are already being collected for longitudinal mediation research. Although data are constrained in terms of the lag(s) over which data are collected—lags that are as much a function of theory as of practical matters such as funding and participant burden—the EDM offers interpretations of results that are not similarly constrained. The key assumption of this article is that variables and participants could theoretically be measured at many times between observations—when modeling such data readers are encouraged to remember that there is no need to be discrete!

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