Results to accompany:

Cho, S.-J., Athay, M., & Preacher, K. J. (in press). Measuring change for a multidimensional test using a generalized explanatory longitudinal item response model. *British Journal of Mathematical and Statistical Psychology.*

This document reports the results of the following six models also shown in the paper using simulated data:

- Model 1: Embretson descriptive model for unidimensional tests,
- Model 2: Embretson explanatory model for unidimensional tests,
- Model 3: Descriptive longitudinal model for multidimensional tests (without fixed effects in Equation 2),
- Model 4: Explanatory longitudinal model for multidimensional tests (Equation 2),
- Model 5: Descriptive bi-factor longitudinal model for multidimensional tests (without fixed effects in Equation 3),
  and
- Model 6: Explanatory bi-factor longitudinal model for multidimensional tests (Equation 3).

The simulated data with the same indicators and the lmer code shown in the paper were also posted as separate files on the website. Item responses were generated based on Equation 2 and estimates reported in Table 2 of the paper were considered as true parameters in the model (i.e., explanatory longitudinal model for multidimensional tests).

Models 1-6 were fit to the same simulated data. Results of Models 1-6 are reported in Table 1-6 respectively below. Model fit results are shown in Table 7.

Table 1: Estimates and Standard Errors (SE) from Model 1

		Est.		SE
Fixed Effects				
β_1		1.285		0.164
β_2		1.614		0.169
β_3		-0.057		0.158
β_4		0.730		0.159
β_5		1.069		0.162
β_6		-0.355		0.159
β_7		1.012		0.161
β_8		0.917		0.160
β_9		1.880		0.174
β_{10}		0.418		0.158
β_{11}		-0.223		0.158
β_{12}		-0.112		0.158
β_{13}		-1.052		0.166
β_{14}		-0.488		0.160
Random Effects				
Persons				
		Var.	Corr.	
			Time 1	Time 2
Σ	Time 1	3.372		
	Time 2	1.690	-0.641	
	Time 3	5.116	-0.809	0.378

Table 2: Estimates and Standard Errors (SE) from Model 2

		Est.			SE
Fixed Effects					
$\zeta_{..}$[Intercept]		0.286			0.858
Persons					
ζ_{21}[Time 2 by Group]		-0.268			0.521
ζ_{31}[Time 3 by Group]		0.861			0.563
$\zeta_{2.}$[Time 2]		0.737			0.138
$\zeta_{3.}$[Time 3]		2.5187			0.158
$\zeta_{.1}$[Group]		-1.195			0.476
Items					
β_1[Operation]		-1.324			0.785
β_2[Measurement]		0.303			0.439
β_3[Representation]		-0.456			0.592
Random Effects					
Persons					
		Var.	Corr. Time 1	Time 2	
Σ	Time 1	1.219			
	Time 2	1.089	-0.410		
	Time 3	1.097	-0.330	-0.333	
Items					
σ_ϵ[Variance]		0.507			

Table 3: Estimates and Standard Errors (SE) from Model 3

		Est.		SE
Fixed Effects				
β_1		1.193		0.163
β_2		1.518		0.168
β_3		0.047		0.207
β_4		1.139		0.216
β_5		1.631		0.221
β_6		-0.348		0.206
β_7		1.547		0.220
β_8		1.409		0.219
β_9		2.169		0.202
β_{10}		0.632		0.245
β_{11}		-0.203		0.237
β_{12}		-0.182		0.157
β_{13}		-1.114		0.167
β_{14}		-0.554		0.159
Random Effects				
Persons				
		Var.	Corr.	
			Time 1	Time 2
Σ_1[Operation]	Time 1	3.527		
	Time 2	1.782	-0.773	
	Time 3	4.120	-0.862	0.720
Σ_2[Measurement]	Time 1	4.707		
	Time 2	3.639	-0.320	
	Time 3	4.314	-0.569	-0.514
Σ_3[Representation]	Time 1	4.569		
	Time 2	1.058	0.189	
	Time 3	2.972	-0.907	-0.585

Table 4: Estimates and Standard Errors (SE) from Model 4

		Est.			SE
Fixed Effects					
$\zeta_{..}$[Intercept]		0.616			0.930
Persons					
ζ_{21}[Time 2 by Group]		0.102			0.565
ζ_{31}[Time 3 by Group]		0.960			0.595
$\zeta_{2.}$[Time 2]		0.670			0.150
$\zeta_{3.}$[Time 3]		2.529			0.171
$\zeta_{.1}$[Group]		-1.418			0.496
Items					
β_1[Operation]		-1.619			0.861
β_2[Measurement]		0.384			0.493
β_3[Representation]		-0.620			0.660
Random Effects					
Persons					
		Var.	Corr.		
			Time 1	Time 2	
Σ_1[Operation]	Time 1	3.002			
	Time 2	0.448	0.909		
	Time 3	2.142	-0.992		-0.954
Σ_2[Measurement]	Time 1	2.620			
	Time 2	2.924	-0.165		
	Time 3	2.912	-0.405		-0.763
Σ_3[Representation]	Time 1	1.126			
	Time 2	0.838	-0.467		
	Time 3	0.507	-0.448		0.160
Items					
σ_ϵ[Variance]		0.602			

Table 5: Estimates and Standard Errors (SE) from Model 5

		Est.		SE
Fixed Effects				
β_1		0.177		0.169
β_2		0.494		0.173
β_3		-1.801		0.238
β_4		-0.744		0.235
β_5		-0.294		0.236
β_6		-2.202		0.240
β_7		-0.370		0.236
β_8		-0.496		0.235
β_9		1.041		0.235
β_{10}		-1.149		0.285
β_{11}		-1.984		0.289
β_{12}		-1.226		0.172
β_{13}		-2.260		0.192
β_{14}		-1.628		0.178
Random Effects				
Persons				
		Var.	Corr. Time 1	Time 2
Σ_0[Overall]	Time 1	0.613		
	Time 2	0.203	–	
	Time 3	6.278	–	–
Σ_1[Operation]	Time 1	0.302		
	Time 2	1.182	–	
	Time 3	2.022	–	–
Σ_2[Measurement]	Time 1	2.192		
	Time 2	2.012	–	
	Time 3	2.306	–	–
Σ_3[Representation]	Time 1	2.888		
	Time 2	0.366	–	
	Time 3	0.000	–	–

–: Not modelled

Table 6: Estimates and Standard Errors (SE) from Model 6

		Est.			SE
Fixed Effects					
$\zeta_{..}$[Intercept]		0.573			0.939
Persons					
ζ_{21}[Time 2 by Group]		-0.178			0.501
ζ_{31}[Time 3 by Group]		0.835			0.648
$\zeta_{2.}$[Time 2]		0.643			0.135
$\zeta_{3.}$[Time 3]		2.766			0.187
$\zeta_{.1}$[Group]		-1.273			0.461
Items					
β_1[Operation]		-1.611			0.866
β_2[Measurement]		0.184			0.504
β_3[Representation]		-0.566			0.669
Random Effects					
Persons					
		Var.	Corr. Time 1	Time 2	
Σ_0[Overall]	Time 1	0.474			
	Time 2	0.081	–		
	Time 3	1.037	–	–	
Σ_1[Operation]	Time 1	0.346			
	Time 2	0.604	–		
	Time 3	0.056	–	–	
Σ_2[Measurement]	Time 1	1.940			
	Time 2	1.240	–		
	Time 3	0.794	–	–	
Σ_3[Representation]	Time 1	2.649			
	Time 2	0.052	–		
	Time 3	0.000	–	–	
Items					
σ_ϵ[Variance]		0.614			

–: Not modelled

Table 7: Model Fit Comparisons using the Simulation Data

Model	Dimension	Explanatory	Num. of Parameters	Log-Likelihood	AIC	BIC
Model 1	Unidimensional	No	20	-2463.5	4967.0(6)	5095.6(6)
Model 2	Unidimensional	Yes	16	-2413.9	4859.9(5)	4962.8(5)
Model 3	Multidimensional	No	32	-2343.1	4750.1(3)	4955.8(3)
Model 4	Multidimensional	Yes	28	-2304.5	4664.9(1)	4844.9(2)
Model 5	Bi-Factor Multidimensional	No	23	-2373.7	4799.4(4)	4966.6(4)
Model 6	Bi-Factor Multidimensional	Yes	22	-2328.5	4701.0(2)	4842.4(1)

Table 8: Model Fit Comparisons using the Empirical Data (Table 5 of the paper)

Model	Dimension	Explanatory	Num. of Parameters	Log-Likelihood	AIC	BIC
Model 1	Unidimensional	No	20	-2462.9	4965.8(6)	5094.4(6)
Model 2	Unidimensional	Yes	16	-2413.7	4859.4(4)	4962.3(4)
Model 3	Multidimensional	No	32	-2338.8	4741.5(3)	4947.2(3)
Model 4	Multidimensional	Yes	28	-2302.7	4661.4(1)	4841.4(1)
Model 5	Bi-Factor Multidimensional	No	23	-2411.7	4869.4(5)	5017.3(5)
Model 6	Bi-Factor Multidimensional	Yes	22	-2328.0	4700.1(2)	4841.4(1)

Note. Values in parentheses in Tables 7 and 8 indicate the rank orders of AIC or BIC values (The smallest rank value means the smallest AIC or BIC). Rank orders of AIC and BIC values are similar between the empirical data reported in the paper and the simulated data.